Notes on the Elasticities Approach

As noted on page 303 of Caves, et al. (2002), $TB^*$, in units of foreign currency, is given by:

$$TB^* = \left(\frac{P}{E}\right) \times X - P^* \times M$$  \hspace{1cm} (0)

Multiply both sides by $E$, and divide by $P$, to obtain:

$$\frac{E \times TB^*}{P} = \frac{TB}{P} = X - \left(\frac{EP^*}{P}\right) \times M$$  \hspace{1cm} (1)

$TB$ is the trade balance in nominal domestic currency terms. Define the trade balance ($\tilde{TB}$) in domestic widget terms as the difference between exports ($X$) and imports ($M$) in widgets:

$$\tilde{TB} = X - M$$  \hspace{1cm} (2)

However, imports are the foreign ($^*$) country's exports, so:

$$M = (EP^*/P) \times X^* \equiv qX^*$$  \hspace{1cm} (3)

where $q$ is a real exchange rate, measured in terms of number of home widgets needed to purchase one foreign widget. Then the trade balance is functionally defined as:

$$\tilde{TB} = X - qX^*$$  \hspace{1cm} (4)

(As in the textbook, we take the income levels as exogenously fixed.) To determine the response of the trade balance to a change in the real exchange rate, take the partials with respect to the real exchange rate:

$$\frac{\partial \tilde{TB}}{\partial q} = \frac{\partial X}{\partial q} - \left( q \frac{\partial X^*}{\partial q} + X^* \right)$$

$$\frac{\partial \tilde{TB}^*}{\partial q} = \left( \frac{\partial X}{\partial q} - q \frac{\partial X^*}{\partial q} \right) - X^*$$  \hspace{1cm} (5)

$$\frac{\partial \tilde{TB}}{\partial q} = \text{(volume effect)} + \text{(value effect)}$$
One wants to know if $\frac{\partial T \overline{B}}{\partial q}$ is greater than or less than zero. Solve for:

$$0 < \frac{\partial X}{\partial q} - q \frac{\partial X^*}{\partial q} - X^*$$  \hspace{1cm} (6)

Multiply both sides by the quantity $(q/X)$ to obtain:

$$0 < \frac{\partial X}{\partial q} \frac{q}{X} - q \frac{\partial X^*}{\partial q} \frac{q}{X} - X^* \frac{q}{X}$$  \hspace{1cm} (7)

Define the first term as $\varepsilon_X$, the export supply elasticity. Further note that if initial trade is balanced, then $X = qX^*$ such that $q/X = 1/X^*$.

$$0 < \varepsilon_X - \frac{\partial X^*}{\partial q} \frac{q}{X^*} - 1$$  \hspace{1cm} (8)

Finally, define the import elasticity:

$$-\frac{\partial X^*}{\partial q} \frac{q}{X^*} \equiv \varepsilon_M$$

then one obtains the Marshall-Lerner-Robinson condition:

$$1 < \varepsilon_M + \varepsilon_X$$  \hspace{1cm} (9)

which is appropriate for export demand and import supply elasticities that are infinite.