Notes on the Elasticities Approach
(corrected 3/27/05)

As noted on page 303 of Caves, et al. (2002), $TB^*$, in units of foreign currency, e.g., yen (¥), is given by:

$$TB^* = (P/E) \times EX - P' \times IM$$  \hspace{1cm} (0)

(where $EX$ is the same as the $X$ in the textbook, and $IM$, in Japanese widgets) is the same as the $M$ in the textbook). Multiply both sides by $E$, and divide by $P$, to obtain:

$$\frac{E \times TB^*}{P} = \frac{TB}{P} = EX - \left( \frac{EP'}{P} \right) \times IM$$  \hspace{1cm} (1)

$TB$ is the trade balance in nominal domestic currency terms; $TB/P$ is the trade balance denominated in US widgets. Define the trade balance ($\tilde{TB}$) in domestic widget terms as the difference between exports ($EX$) denominated in US widgets and imports ($IM$) denominated in US widgets:

$$\frac{TB}{P} \equiv \tilde{TB} = EX - IM$$  \hspace{1cm} (2)

However, imports are the foreign (*) country's exports, so:

$$IM = (EP^* / P) \times EX^* \equiv qEX^*$$  \hspace{1cm} (3)

where $q$ is a real exchange rate, measured in terms of number of home widgets needed to purchase one foreign widget. Then the trade balance is functionally defined as:

$$\tilde{TB} = EX - qEX^*$$  \hspace{1cm} (4)

(As in the textbook, we take the income levels as exogenously fixed.) To determine the response of the trade balance to a change in the real exchange rate, take the partials with respect to the real exchange rate:
\[
\begin{align*}
\frac{\partial \overline{T\overline{B}}}{\partial q} &= \frac{\partial EX}{\partial q} - \left( q \frac{\partial EX^*}{\partial q} + EX^* \right) \\
\frac{\partial \overline{T\overline{B}}}{\partial q} &= \left( \frac{\partial EX}{\partial q} - q \frac{\partial EX^*}{\partial q} \right) - EX^* \\
\frac{\partial \overline{T\overline{B}}}{\partial q} &= \text{(volume effect)} + \text{(value effect)}
\end{align*}
\]

One wants to know if \( \frac{\partial \overline{T\overline{B}}}{\partial q} \) is greater than or less than zero. Solve for:

\[
0 < \frac{\partial EX}{\partial q} - q \frac{\partial EX^*}{\partial q} - EX^*
\]

(6)

Multiply both sides by the quantity \((q/EX)\) to obtain:

\[
0 < \frac{\partial EX}{\partial q} \frac{q}{EX} - q \frac{\partial EX^*}{\partial q} \frac{q}{EX} - EX^* \frac{q}{EX}
\]

(7)

Define the first term as \( \varepsilon_{\text{EX}} \), the export supply elasticity. Further note that if initial trade is balanced, then \( EX = qEX^* \) such that \( q/EX = 1/EX^* \).

\[
0 < \varepsilon_{\text{EX}} - \frac{\partial EX^*}{\partial q} \frac{q}{EX^*} - 1
\]

(8)

Finally, define the import elasticity:

\[
- \frac{\partial EX^*}{\partial q} \frac{q}{EX^*} \equiv \varepsilon_{\text{IM}}
\]

then one obtains the Marshall-Lerner-Robinson condition:

\[
1 < \varepsilon_{\text{IM}} + \varepsilon_{\text{EX}}
\]

(9)

which is appropriate for export demand and import supply elasticities that are infinite.

Caveats:

1. What if export supply and import supply elasticities are not infinite.
2. What if initial trade is not balanced.