Midterm Exam

You have 65 minutes to complete this 60 minute exam. Be sure to “box in” your answers. Show your work (so that partial credit can be granted if the final answer is incorrect).

1. [20 minutes] Suppose one is examining the term structure of a 3 year discount bond, and the expectations hypothesis of the term structure holds.

   \[ i_{3t} = \frac{(i_t + i_{t+1} + i_{t+2})}{3} \]  

   (1)

   Suppose yesterday,

   \[ i_{3t} = 0.15 \]

   \[ i_t = 0.05 \]

   1.1 (5 minutes) Calculate the average value of \( i_{t+1} \) and \( i_{t+2} \).

   1.2 (5 minutes) Returning to equation (1), suppose today the yield to maturity on the 3 year discount bond has increased by \( \Delta i_{3t} \), while the yield to maturity on a one year bond is unchanged from yesterday. Derive the algebraic expression for the implied change in the expected one year yield, assuming that the short term expected yield on the one year bond in period \( t+2 \) is unchanged.

   1.3. (5 minutes) Assume the 3 year bond yield is given by:

   \[ i_{3t} = \frac{(i_t + i_{t+1} + i_{t+2})}{3} + rp_{3t} \]  

   (2)

   And the yield on the 3 year discount bond has increased by \( \Delta i_{3t} \). Calculate the implied increase in the liquidity premium for the three year bond, if it is assumed that all the expectations of future one year interest rates are unchanged. Can you draw a picture of how the yield curve has changed (you can assume the liquidity premium on one year interest rates are always zero)?

   1.4 (5 minutes) Assume \( rp_{nt} \) is always zero, so the pure expectations hypothesis of the term structure holds. Can the central bank affect the long term interest rate by purchasing additional amounts of long term bonds? Explain your answer in words and/or equations.

2. [20 minutes] Suppose the stock price is given by:

   \[ P_t = \frac{D_{t+1}}{1 + rp + rf} + \frac{E_t P_{t+1}}{1 + rp + rf} \]  

   (3)

   2.1 (5 minutes) Show how to derive the current stock price as a function of stock prices at time \( t+3 \).

   2.2 (5 minutes) Derive the expression for the stock price as a function of only the present discounted value of expected dividends. Explain what assumption(s) you need to make in order to obtain this answer.
2.3 (5 minutes) Assume the log (E(X)) = E(log(X)), and assume no dividends are paid out. Derive a mathematical expression that describes the evolution of (log) stock prices over time. Describe in words the behavior of log stock prices.

2.4 (5 minutes) Returning to your answer to equation (3), can you tell whether stock prices at time t will go up or down if the economy booms in period t+3? Show why or why not.

3. [20 minutes] Suppose the economy is described by the following equations:

Real Sector

1. \( Y = Z \) Output equals aggregate demand, an equilibrium condition
2. \( Z = C + I + G \) Definition of aggregate demand
3. \( C = c_o + c_1Y_d \) Consumption fn, \( c_1 \) is the marginal propensity to consume
4. \( Y_d = Y - T \) Definition of disposable income
5. \( T = t_1Y \) Tax function; \( t_1 \) is marginal tax rate.
6. \( I = b_0 - b_2i \) Investment function
7. \( G = GO_0 \) Government spending on goods and services, exogenous

And the LM curve is given by:

\[
 i = \frac{\mu_0}{h} - \left( \frac{1}{h} \right) \left( \frac{M_0}{P_0} \right) + \left( \frac{1}{h} \right) Y
\]

3.1 (5 minutes) Solve for the IS curve (\( Y \) on the left hand side)
3.2 (5 minutes) Solve for equilibrium income.
3.3 (5 minutes) Show what the multiplier is for a change in government spending on goods and services.
3.4 (5 minutes) Show, either mathematically or graphically, what the multiplier is if the Federal Reserve Board targets the interest rate.