Problem Set 3 Answers

Due in lecture on Monday, March 26th. Be sure to put your name on your problem set. Put “boxes” around your answers to the algebraic questions.

1. Given the following closed economy:

   (1) \( Y = Z \)  
   Output equals aggregate demand, an equilibrium condition

   (2) \( Z = C + I + G \)  
   Definition of aggregate demand

   (3) \( C = c_o + c_1 Y_D \)  
   Consumption function, \( c_1 \) is the mpc

   (4) \( Y_D \equiv Y - T \)  
   Definition of disposable income

   (5) \( T = t_0 \)  
   Tax function; \( t_0 \) is lump sum taxes, exogenous

   (6) \( I = b_0 - b_2 i \)  
   Investment function

   (7) \( G = GO_0 \)  
   Government spending on goods and services, exogenous

   (8) \( \frac{M^d}{P} = \frac{M^*}{P} \)  
   Equilibrium condition

   (9) \( \frac{M^*}{P} = \frac{M_0}{P} \)  
   Money supply

   (10) \( \frac{M^d}{P} = \mu_0 + Y - hi + j \left( \frac{\text{wealth}}{P} \right) \)  
   Money demand

   Note further that the budget deficit is linked to bond supply:

   (11) \( -BuS \equiv BuD \equiv G - T = \Delta(B/P) \)

1.1 Derive equilibrium income (for period 0).

Solve the IS as usual: (note; the marginal tax rate is zero, ie. \( t_1 = 0 \))

\[
Y = \left( \frac{1}{1 - c_1} \right) [\Lambda_0 - b_2 i]
\]

Then solve for the LM:

\[
\frac{M^d_0}{P_0} = \frac{M^*}{P} = \frac{M^d}{P} = \mu_0 + Y - hi + j \left( \frac{\text{wealth}}{P} \right)
\]

Solving for the interest rate, \( i \), yields the LM curve:

\[
i = \frac{\mu_0}{h} - \left( \frac{1}{h} \right) \left( \frac{M_0}{P_0} \right) + j \left( \frac{M_0}{P_0} + \frac{B_0}{P_0} \right) + \left( \frac{1}{h} \right) Y
\]

<LM' curve>
Substitute the LM’ equation into the IS, and solving yields:

\[ Y_0 = \hat{\gamma} \left[ \Lambda_0 + \frac{b_2}{h} \left( \frac{M_0}{P_0} \right) - \frac{b_2}{h} \left( \frac{M_0}{P_0} + B_0 \right) \right] \]

where \[ \hat{\gamma} \equiv \frac{1}{1 - c_1 + (b_2 / h)} \]

1.2 Analyze the implications of running a budget surplus for one period (in period 1) induced by decreasing government spending by \( \Delta GO \), starting from budget balance. Draw an IS-LM diagram clearly indicating what happens.

The IS curve shifts out by \( \frac{1}{1 - c_1 - b_1} \Delta GO \), the LM curve shifts up down by \( \left( \frac{j}{h} \right) \Delta \left( \frac{B}{P} \right) \).

The change in income is given by:

\[ \Delta Y = \hat{\gamma} \left[ \Delta \Lambda_0 + \frac{b_2}{h} \Delta \left( \frac{M}{P} \right) - \frac{b_2}{h} \Delta \left( \frac{M_0}{P_0} + B_0 \right) \right] \]

Where the change in money is zero, and the change in real wealth is equal to the change in real bonds which is equal to the change in government spending:

\[ \Delta Y = \hat{\gamma} \left[ \Delta GO - \frac{b_2}{h} \Delta GO \right] \]

The multiplier is:

\[ \frac{\Delta Y}{\Delta GO} = \hat{\gamma} \left[ 1 - \frac{b_2}{h} \right] \]

1.3 Suppose after that period of running the deficit surplus, government spending is increased (in period 2) so as to balance the budget. Show what happens, in an IS-LM diagram. What is true about output relative to what it was before the government spending increase?
The IS curve shifts back to where it started from. However, since the stock of bonds is permanently lower, and is unchanging in the next period, the LM curve remains where it was in period 1. In this example, given the original shift in the LM, output is above where it started from. \( Y_2 \) is above \( Y_1 \) and above \( Y_0 \).

2. Suppose that we are operating under the old system (pre-October 2008), where the Fed does not pay interest on reserves, and the required reserve ratio is 0.12 for deposits and there are no excess reserves. Suppose also that the total demand for currency is equal to 0.2 times deposits money demand.

2.1 If central bank money is $40 billion, what is the level of the money supply?

\[
M = \left[ \frac{1}{c + \theta (1-c)} \right] \times H
\]

\[
M = \left[ \frac{1}{0.2 + 0.12(0.8)} \right] \times H = $135.1
\]

2.2 By how much does the money supply change if the Fed increases the required reserve ratio to 0.20? Assume that total central bank money is unchanged at $40 billion.

Now \( \theta = 0.20 \), \( M = \left[ \frac{1}{(0.2 + 0.12(0.8))} \right] \times H = $111.1 \). Hence, the money supply drops from 135.1 to 111.1, i.e., 24 billion.

2.3 By how much does the money supply change if the Fed buys $1 billion of government bonds in the open market? (Keep the required reserve ratio at 0.12).

The money supply increases by \( \left[ \frac{1}{(c + \theta (1-c))} \right] \times \Delta H = (1/0.36) \times 1 = 2.8 \)

3. Consider an economy with a “credit channel”, i.e., one with a banking system. This economy is described in the handout “The Financial and Economic Crisis…”. The CC curve is given by:

\[
Y = \bar{\alpha} (A_0 - dR - \gamma p)
\]
\[ \rho = \varphi_0 + \varphi_1 i + \varphi_2 Y - \varphi_3 [m(Res)(1 - r)] + \varphi_4 Z \]  

(7)

Notice that an increase in \([m(Res)]\) will decrease \(\rho\) and hence shift out the CC curve; and an increase in \(Z\) will increase \(\rho\) and hence shift in the CC curve.

The LM curve is given by:
\[ R = \frac{\mu_0}{h} - \frac{1}{h} \left( \frac{m(Res)}{P} \right) + \frac{k}{h} Y \]  

(4)

Notice that like the standard IS-LM model, the LM curve is shifted out by anything that increases \([m(Res)]\).

3.1 Show graphically what happens when government spending is increased.

The CC curve shifts out, as in a standard IS curve.

3.2 Show graphically what happens if all physical investment projects are suddenly perceived to be more risky than they previously were.

When \(Z\) rises, \(\rho\) rises, (according to equation 7); this means that loan supply decreases, which in turn means that the CC curve shifts in.
3.3 Show graphically what happens if the Fed increases the amount of reserves in the economy by undertaking open market operations. You can assume the interest rate paid on reserves is zero, and the money multiplier is constant.
The increase in reserves increases the money stock, in the usual fashion, thus shifting out the LM curve. However, increased reserves also results in increased credit supply (decreased lending rate). This means the CC curve shifts out as well. Economic activity rises to $Y_3$, the interest rate falls to $R_3$.

4. Suppose the markup of goods over wages is 10% (0.1) and the wage setting equation is $W = P(1-2u + z)$, where $u$ is the unemployment rate, and $z$ is also 10% (0.1).

4.1 What is the real wage, as determined by the price setting equation?
\[
P/W = 1+\mu \\
W/P = 1/(1+\mu)
\]

4.2 Solve for the natural rate of unemployment.
\[
W/P = (1-2u+z), \quad (1-2u+z)=1/(1+\mu) \\
1-2u+0.10 = 1/(1.1) \\
u = 0.095
\]

4.3 Solve for the natural rate of unemployment if $z$ falls to 5% (0.05).
\[
W/P = (1-2u+z), \quad (1-2u+z)=1/(1+\mu) \\
1-2u+0.05 = 1/(1.1) \\
u = 0.070
\]

The decrease in the unemployment benefits/minimum wage causes a fall in the natural rate of unemployment. Intuitively, an decrease in the $z$ reduces wages for any given level of unemployment, so the equilibrium level of unemployment falls.

5. Textbook Chapter 7, Problem 5.

a. SR: short run 
   MR: medium run

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b-c.

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<td>MR</td>
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In the medium run, consumption must lower than its original level because disposable income is unchanged, but consumer confidence is lower.

The short-run change in investment is ambiguous, because the interest rate falls, which tends to increase investment, but output also falls, which tends to reduce investment. In the medium run, investment must rise (as compared to its short-run and original levels), because the interest rate falls but output returns to its original level.

Since the budget deficit does not change in this problem, the change in private saving equals the change in investment. It is possible that private saving will fall in the short run, but private saving must rise (above its short-run and original levels) in the medium run.

6. Textbook Chapter 7, Problem 11.
   a. The unemployment rate rises in the short run and rises further in the medium run. The real wage falls immediately to its new medium-run level.
   
   b. The unemployment rate falls in the short run but returns to the original natural rate in the medium run. The real wage is unaffected, but after-tax income rises.
   
   c. In our model, the real wage depends only on the markup. A fall in the markup increases the real wage. Policy measures that improve product market competition—for example, more vigorous anti-trust enforcement—could increase the real wage.
   
   d. The fall in income taxes tended to increase the after-tax real wage. The increase in oil prices tended to reduce the after-tax real wage. Intuitively, the immediate effect of an oil price increase is to reduce the real wage by increasing gas prices. Thus, the increase in gas prices tends to absorb the extra after-tax income provided by the tax cut.

7. Textbook Chapter 8, Problem 3.
   a. \( u_n = 0.1/2 = 5\% \)
   
   b. \( \pi_t = 0.1 - 2(0.03) = 4\% \) every year beginning with year t.
   
   c. \( \pi^* = 0 \) and \( \pi = 4\% \) forever. Inflation expectations will be forever wrong. This is unlikely.
   
   d. \( \theta \) might increase because inflation expectations adapt to persistently positive inflation. The increase in \( \theta \) has no effect on \( u_n \).
   
   e. \( \pi_5 = \pi_4 + 0.1 - 2(0.03) = 4\% + 4\% = 8\% \)
   
   For \( t > 5 \), \( \pi_t = 8\% + (t - 5)(4\%) \). So, \( \pi_{10} = 28\% \); \( \pi_{15} = 48\% \). Inflation increases by four percentage points every year.
   
   f. Inflation expectations will again be forever wrong. This is unlikely.