Problem Set 2 Answers

Due in lecture on Monday, February 20th. Be sure to put your name on your problem set. Put “boxes” around your answers to the algebraic questions.

1. Suppose the economy is described by the following equations (so we are looking at a closed economy):

   **Real Sector**
   
   (1) \( Y = Z \)  
   Output equals aggregate demand, an equilibrium condition
   
   (2) \( Z = C + I + G \)  
   Definition of aggregate demand
   
   (3) \( C = c_0 + c_1 Y_D \)  
   Consumption fn, \( c_1 \) is the marginal propensity to consume
   
   (4) \( Y_D = Y - T + Tr \)  
   Definition of disposable income
   
   (5) \( T = t_i Y \)  
   Tax function; \( t_i \) is marginal tax rate.
   
   (6) \( Tr = TR_0 - \lambda Y \)  
   Transfer payments; \( TR_0 \) is lump sum transfers.
   
   (7) \( I = b_0 + b_1 Y - b_2 i \)  
   Investment function
   
   (8) \( G = GO_0 \)  
   Government spending on goods and services, exogenous

   **Asset Sector**
   
   (9) \( \frac{M^d}{P} = \frac{M^s}{P} \)  
   Equilibrium condition
   
   (10) \( \frac{M^s}{P} = M_0 \)  
   Real money supply
   
   (11) \( \frac{M^d}{P} = \mu_0 + Y - hi \)  
   Real money demand

1.1 Solve for the IS curve (\( Y \) as a function of \( i \)).

\[
Y = Z = C + I + G
\]

 substitute in for \( C, I, G \)

\[
Y = c_0 + c_1 Y_D + b_0 + b_1 Y - b_2 i + GO_0
\]

 substitute in for \( Y_D \)

\[
Y = c_0 + c_1 Y - (T + Tr) + b_0 + b_1 Y - b_2 i + GO_0
\]

 substitute in for tax, transfers functions

\[
Y - c_1 (Y - t_i Y - \lambda Y) = Y(1 - b(1 - t - \lambda)) = c_0 + c_1 TR_0 + b_0 + GO_0 - b_2 i
\]

 bring the "Y" terms to left hand side.

\[
Y - c_1 (Y - t_i Y - \lambda Y) = Y(1 - b(1 - t - \lambda)) = c_0 + c_1 TR_0 + b_0 + GO_0 - b_2 i
\]

 divide both sides by \( (1 - c_1 (1 - t_i - \lambda)) \) and let \( A_0 \equiv c_0 + c_1 TR_0 + b_0 + GO_0 \)

\[
\bar{Y}_0 = \frac{fA_0}{1 - c_1 (1 - t_i - \lambda)}
\]

1.2 Solve for the LM curve (\( i \) as a function of \( Y \)). What is the channel by which monetary influences affect the real goods sector in this model?

Solve by substituting into the equilibrium condition:
Solving for the interest rate, \( i \), yields the LM curve:

\[
R = \frac{\mu_0}{h} - \left( \frac{1}{h} \right) \left( \frac{M_0}{P_0} \right) + \left( \frac{1}{h} \right) Y
\]

Monetary policy influences (in part) interest rates. Interest rates in turn affect investment, and via the simple Keynesian multiplier (\( \gamma \)) affects the entire real sector.

1.3 Solve for the equilibrium values of \( Y \).

To solve for the equilibrium value of income, substitute the LM into the IS equation from 1.1:

\[
Y = \left( \frac{1}{1-c_1(1-t_1-\lambda)} \right) \times \left[ A_0 - b_2 \left( \frac{\mu_0}{h} - \frac{1}{h} \frac{M_0}{P_0} + \frac{1}{h} Y \right) \right]
\]

Move the term in parentheses (\( ) \) and the \( (dk/h)Y \) term to the LHS; factoring out the \( Y \)'s on the LHS yields:

\[
Y \left( 1-c_1(1-t_1-\lambda) + \frac{b_2}{h} \right) = \left[ A_0 - b_2 \left( \frac{\mu_0}{h} - \frac{1}{h} \frac{M_0}{P_0} \right) \right]
\]

Dividing both sides by the term in the parentheses yields:

\[
Y_0 = \gamma \left[ A_0 - \frac{b_2 \mu_0}{h} + \left( \frac{b_2}{h} \right) \left( \frac{M_0}{P_0} \right) \right]
\]

where \( \gamma = \frac{1}{1-c_1(1-t_1-\lambda) + \frac{b_2}{h}} \)

1.4 Graph the IS and LM curves on one diagram. Clearly indicate the intercepts and the slopes.
1.5 What are the exogenous and endogenous variables?

**Exogenous:** $G, M$ ($P$ might be thought of as exogenous, but technically it is "pre-determined" as we will find out in lecture)

**Endogenous:** $Y, i, Z, Y_{D}, C, I, T$.

1.6 What is the government spending multiplier? What is the monetary policy multiplier?

Take the total differential of the answer to 1.3:

$$\Delta Y = \gamma \left[ \Delta A - \frac{b_2 \mu_0}{h} + \left( \frac{b_2}{h} \right) \Delta \left( \frac{M}{P} \right) \right]$$

To find the government spending multiplier, set the changes in real money to zero and the money constant, and divide both sides by $\Delta GO$; To find the monetary policy multiplier, set the change in government spending equal to zero, and divide both sides by $\Delta (M/P)$. This will yield, respectively:

$$\Delta Y = \gamma \Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \gamma \equiv \frac{1}{1-c_1(1-t_1-\lambda)+b_2/h}$$

$$\Delta Y = \gamma \left( \frac{b_2}{h} \right) \Delta \left( \frac{M}{P} \right) \Rightarrow \frac{\Delta Y}{\Delta (M/P)} = \gamma \left( \frac{b_2}{h} \right)$$

2. Suppose the equations in the model above assume the following values:

$c_0 = 1000; \; c_1 = 0.8 \quad t_1 = 0.20 \quad TR_0 = 800; \; \lambda = 0.05; \; GO_0 = 600$

$b_0 = 2000; \; b_1 = 0; \; b_2 = 10 \quad h = 100; \; \mu_0 = 200 \quad M_0 = 10000; \; P_0 = 1$

2.1 Calculate the equilibrium values of $Y, i, and I$ (call them $Y_0, i_0, and I_0$, respectively).

Substituting the given numerical values into the expression in part 1.3., one obtains:

$$Y_0 = 1/(1-0.8(1-0.2-0.05)+ 10/100) \left[ 1000+0.8(800)+2000+600+( 10/100)(10000) - 10/100(200)\right] = 2\times5220$$

$Y_0 = 10440$

One can either substitute in the relevant numbers into the expression for $i_0$, or just substitute the numerical value for $Y_0$ into the LM curve. This latter approach yields:

$$i_0 = (1/100)(10440) + (1/100)(200) - (1/100)(10000) = 104.4 + 2 - 100$$

$i_0 = 6.4\%$

To find the level of investment, take the investment function and substitute in the numerical values for the parameters and the equilibrium interest rate $i_0$.

$$I_0 = 2000 - 10(6.4)$$

$I_0 = 1936$

2.2 Assume $G$ increases to 1100, and is completely bond financed (no portfolio effects here). Calculate the new level of income, $Y_1$, and hence calculate the numerical value of the government spending multiplier, $\Delta Y/\Delta GO$ (OR calculate $\Delta Y/\Delta GO$ and then find $Y_1$).

Assuming $G$ increases to 1100 (i.e., $\Delta GO = 500$), the most obvious way to figure out the new level of income, $Y_1$, is to substitute this new value in for $G$. This yields:
\[ Y_1 = \frac{1}{1-0.8(1-0.2-0.05)+ \frac{10}{100}} \times 1000 + 0.8(800) + 2000 + 1100 = 2 \times 5720 \]

\[ Y_1 = 11440 \]

From 1.6, we know that the government spending multiplier is merely \( \hat{\gamma} \); so substituting in the numerical values of the parameters yields:

\[
\frac{\Delta Y}{\Delta GO} = \hat{\gamma} + \frac{1}{1 - c_1(1 - t_1 - \lambda) + b_2 / h} = \frac{1}{1 - 0.8(0.75) + (10/100)} = 2
\]

This leads to an alternative means of calculating the new level of income:

\[ Y_1 = Y_0 + \Delta Y = Y_0 + \hat{\gamma} \Delta GO \]

\[ Y_1 = 10400 + (2 \times 500) = 11400 \]

2.3 Calculate how much investment has changed by the increase in \( G \). What is the amount of investment “crowded out” by higher interest rates? Explain the crowding out briefly using words and a graph.

To calculate the amount of investment crowded out, note that the total differential of the investment function is:

\[ \Delta I = \Delta b_0 - b_2 \Delta i \quad \text{(recall } b_1 = 0) \]

The new interest rate can be obtained by solving out for \( R_1 \), or one can use the LM curve:

\[ i = \frac{\mu_0}{h} - \left( \frac{1}{h} \right) \left( \frac{M_0}{P_0} \right) + \left( \frac{1}{h} \right) Y \]

\[ \Delta i = \frac{\mu_0}{h} - \left( \frac{1}{h} \right) \Delta \left( \frac{M}{P} \right) + \left( \frac{1}{h} \right) \Delta Y \]

and recall \( \Delta \mu = 0 \), so by substituting in numbers obtain:

\[ \Delta i = (1/100)1000 = 10\% \]

Then the change in investment is:

\[ \Delta I = (-10)10 = -100 \]

\[ I_1 = I_0 + \Delta I = 1936 + (-100) \]

\[ I_1 = 1836 \]

**Crowding out**: This form of crowding out will be referred to as "Transaction Crowding Out" because it works through the transaction demand for money. An increase in government spending will, if interest rates were to remain constant, increase income (through the usual multiplier) which in turn increases the desired demand for money beyond the given money supply (how far depends on the size of \( k \), the income sensitivity of money demand). This puts the money market in excess demand. Therefore the bond market must be in excess supply (by Walras' Law). If the bond market is in excess supply, the price of bonds will fall. Therefore the rate of return on bonds - the interest rate - will rise due to the inverse relationship of the price of bonds and the interest rate. This increase in the interest rate will choke off the excess money demand until the money market is in equilibrium but will also decrease investment spending (how much depends on the size of \( d \), the interest sensitivity of investment). This decrease in investment will, through the usual spending multiplier, decrease output. This last effect on income works against the stimulus of the initial increase in government spending. The net result is that government spending is not as expansionary as it was in the old Keynesian cross model (except for extreme cases - you should think about these cases) since the initial stimulus provided by increased
government spending will be partially offset by how money demand and interest rates work to reduce investment and hence output. This latter effect -- the fall in investment -- is called the transaction crowding out of investment; the resulting fall in income is called transactions crowding out of income.

2.4 Suppose the $G$ remains at 600, but $M/P_0$ increases to 10500. Calculate the new equilibrium $Y$ and $i$ (call them $Y_2$ and $i_2$).

2.5 Calculate the monetary policy multiplier, $\Delta Y/\Delta (M/P)$.

To calculate the new equilibrium income $Y_2$, there are once again two ways; (i) substitute into the expression for equilibrium income the new real money stock, or, (ii) using the monetary policy multiplier. From the answer to 1.6, and using route (ii):

$$\frac{\Delta Y}{\Delta (M/P)} = \partial \frac{b_2}{h} \times (10/100) = 0.2$$

Note that $Y_2 = Y_0 + \Delta Y$:

$$Y_2 = 10440 + 0.2 \times 500$$

$$Y_2 = 10540$$

3. Suppose that $G$ is increased to 1100, and $M/P_0$ is also increased to 10500 (so that the fiscal policy is money-financed).

3.1 What is the new equilibrium $Y$ and $i$ (call them $Y_3$ and $i_3$)?

One can solve this easiest using the multipliers. First generate the change in income:

$$\Delta Y = \partial \left[ \Delta A + \frac{b_2}{h} \Delta \left( \frac{M}{P} \right) \right]$$

$$\Delta Y = 2(500) + 2(0.1)(500) = 1100$$

Then add the change to the original level of income:

$$Y_3 = 11540$$

To obtain the new interest rate, one can then substitute this new income level into the LM curve.

3.2 What is the new level of investment (call it $I_3$)? Relative to what happens in question 2.3, why is the change in investment different in this case?

Substituting this interest rate into the investment function yields:

$$I_3 = 2000 - 10(12.4\%)$$

$$I_3 = 1876$$

Since monetary policy is expansionary, the increase in money supply mitigates the equilibrium increase in interest rates. Hence, the extent of crowding out is reduced.
4. Using the algebraic model provided in question 1, draw the IS-LM diagrams for the following situations:

4.1 Money demand is very insensitive to the interest rate.

4.2 Investment is very sensitive to the interest rate.

4.3 The marginal tax rate is very high.