Notes on the Phillips Curve

Take the combination of the price and wage setting equations:
\[ P = P^e (1 + \mu) F(u, z) \]

Linearize the \( F(.) \) function:
\[ F(u, z) = 1 - au + z \]

Substitute the second equation into the first (which yields equation 8.1), and put in time \( t \) subscripts:
\[ P_t = P_t^e (1 + \mu)(1 - au_t + z_t) \]

Divide both sides by \( P_{t-1} \):
\[ \frac{P_t}{P_{t-1}} = \frac{P_t^e}{P_{t-1}} (1 + \mu)(1 - au_t + z_t) \]

(8A.1)

Then:
\[ (1 + \pi_t) = (1 + \pi_t^e)(1 + \mu)(1 - au_t + z_t) \]

Divide both sides by \( (1 + \pi_t^e)(1 + \mu) \) to obtain:
\[ \frac{1 + \pi_t}{(1 + \pi_t^e)(1 + \mu)} = (1 - au_t + z_t) \]

Approximate \( \frac{1 + \pi_t}{(1 + \pi_t^e)(1 + \mu)} \approx 1 + \pi_t - \pi_t^e - \mu \). Then:
\[ 1 + \pi_t - \pi_t^e - \mu = 1 - au_t + z_t \]

Rearranging:
\[ \pi_t = \pi_t^e + (\mu + z_t) - au_t \]  (8.3)

If expected inflation is always zero, one obtains the original Phillips curve:
\[ \pi_t = (\mu + z_t) - au_t \]  (8.4)

This seems to apply to the 1948-69 period (shown below). However, it breaks down post-1969.
How can one explain the post 1969 data? If instead of zero expected inflation, expectations are formed adaptively (backward looking), one could write expected inflation as:
\[
\pi_t^e = \theta \pi_{t-1}
\]
Which yields a Phillips curve of the form:
\[
\pi_t = \theta \pi_{t-1} + (\mu + z_t) - \alpha u_t
\]
When \( \theta = 1 \), one obtains the accelerationist hypothesis:
\[
\pi_t = \pi_{t-1} + (\mu + z_t) - \alpha u_t \quad \text{implies} \quad \pi_t - \pi_{t-1} = (\mu + z_t) - \alpha u_t
\]
It would be useful to express the Phillips curve as a function of the “gap” between unemployment and the natural rate of unemployment. One can solve for the natural rate of unemployment by setting the change in inflation in equation (8.6) equal to zero.
\[
0 = (\mu + z_t) - \alpha u_t
\]
Then
\[
\frac{u_n}{\alpha} = \frac{(\mu + z_t)}{\alpha} \quad \text{also} \quad \alpha u_n = (\mu + z_t)
\]
Substitute (8.8) into (8.3):
\[
\pi_t = \pi_t^e + (\mu + z_t) - \alpha u_t \quad \text{Bring expected inflation to the left hand side.}
\]
\[
\pi_t - \pi_t^e = \alpha u_n - \alpha u_t
\]
\[
\pi_t - \pi_t^e = -\alpha (u_t - u_n)
\]
Using the accelerationist hypothesis:
\[
\pi_t - \pi_{t-1} = -\alpha (u_t - u_n)
\]