Nominal vs. Real

In the past, we have focused on nominal interest rates, ignoring the effects of price level changes, i.e., inflation. However, when inflation is significant, ignoring the distinction is important.

The real (expected, or “ex ante”) interest rate is given by:

\[(14.1) \quad 1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}}\]

Note that:

\[(14.2) \quad \pi^e_{t+1} = \frac{(P_{t+1}^e - P_t)}{P_t}\]

And that:

\[
(14.3) \quad 1 + r_t = (1 + i_t) \left(1 + \frac{1}{1 + \pi^e_{t+1}}\right) = \left(\frac{1 + i_t}{1 + \pi^e_{t+1}}\right)
\]

or from hereon \(r_t \approx i_t - \pi^e_{t+1}\)  or from hereon \(r_t = i_t - \pi^e_{t+1}\)  and \(i_t = r_t + \pi^e_{t+1}\)

The real side of the economy should depend on the real interest rate. This means the IS curve is re-written, and so the IS-LM system is rewritten:

\[(5.2') \quad Y = C(Y - T) + I(Y, r) + G \quad \text{<IS schedule>}\]

\[(5.3) \quad \frac{M}{P} = YL(i) \quad \text{<LM schedule>}\]

\[Y_n = C(Y_n - T) + I(Y_n, r_n) + G\]

\[i_t = r_t + \pi^e_{t+1}, \quad i = r_n + \pi, \quad \pi = r_n + g_m \quad \text{if} \quad g_y = 0 \quad \text{since} \quad \pi = \bar{g}_m - \bar{g}_y\]
### Present value formulae

<table>
<thead>
<tr>
<th>This year</th>
<th>Next year</th>
<th>2 years from now</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$1(1+i)</td>
<td>$1</td>
</tr>
<tr>
<td>$ \frac{1}{1+i}</td>
<td>$1</td>
<td>$1(1+i)(1+i_{+1})</td>
</tr>
<tr>
<td>$ \frac{1}{(1+i)(1+i_{+1})}</td>
<td>$1$</td>
<td></td>
</tr>
</tbody>
</table>

\[
V_t = z_t + \frac{1}{(1+i)} z_{t+1} + \frac{1}{(1+i)(1+i_{t+1})} z_{t+2} + \ldots
\]

(14.5)

If interest rates are constant:

\[
V_t = z_t + \frac{1}{(1+i)} z^{e}_{t+1} + \frac{1}{(1+i)^2} z^{e}_{t+2} + \ldots
\]

If interest rates and payments are constant:

(i)

\[
V_t = z + \frac{1}{(1+i)} z + \frac{1}{(1+i)^2} z + \ldots = z \left(1 + \frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \ldots + \frac{1}{(1+i)^{n-1}}\right)
\]

(ii)

\[
V_t = z \left(1 - \frac{1}{(1+i) + [1/(1+i)]}\right) \text{ for } n \text{ equal to infinity, (i) becomes: } V = z \left(\frac{1+i}{i}\right)
\]

(iii)

\[
V_t = z \left(1 + \frac{1}{(1+i)} \right)^t + \frac{1}{(1+i)^2} + \ldots
\]

(iv)

\[
V_t = z \left(1 \right)^{t} \left(\frac{1}{1+i} \right) + \frac{1}{(1+i)} + \ldots
\]

To obtain real present values, divide the nominal expression in 14.5 by the price level to obtain (14.A1)

(14.A1)

\[
\frac{SV_t}{P_t} = \frac{z_t}{P_t} + \frac{1}{(1+i)} \frac{z^{e}_{t+1}}{P_{t+1}} + \frac{1}{(1+i)(1+i_{t+1})} \frac{z^{e}_{t+2}}{P_{t+2}} + \ldots
\]

Rewrite:

\[
\frac{SV_t}{P_t} = \frac{z_t}{P_t} + \frac{1}{(1+i)} \frac{P^{e}_{t+1}}{P_{t+1}} \frac{z^{e}_{t+1}}{P_{t+1}} + \frac{1}{(1+i)(1+i_{t+1})} \frac{P^{e}_{t+2}}{P_{t+2}} \frac{z^{e}_{t+2}}{P_{t+2}} + \ldots
\]

Using equation (14.3)

\[
1 + r_t = \left(1 + \frac{i_t}{1 + r^{e}_{t+1}}\right) \frac{SV_t}{P_t} = V_t \cdot \frac{z_t}{P_t} = z^{e}_{t} \text{ and one obtains (14.7)}
\]

\[
V_t = z_t + \frac{1}{1 + r_t} z^{e}_{t+1} + \frac{1}{(1+r_t)(1+r_{t+1})} z^{e}_{t+2} + \ldots
\]

If real interest rates and real payments were constant on into infinity (and the payments mass next period), then:

\[
V_t = \frac{z}{r}
\]