The Optimal Capital Stock

\[ K_t = (R_t + d_t)P_t^K \]

\[ R_t^K = (R_t + d_t)P_t^K - \Delta P_t^K \]

\[ R_t^K = (R_t - \pi_t^K + d_t)P_t^K \]

Where

\[ \pi_t^K = \frac{P_t^K - P_{t-1}^K}{P_t^K} \]

Relating the Optimal Capital Stock to Investment

\[ K_t^* = 0.5 \left( \frac{W_t}{R_t^K} \right) Y_t \]
The definition of investment is:
(11.4) \( I_t \equiv K_t - K_{t-1} \)
Assume actual investment undertaken is a function of desired capital stock relative to the previous capital stock:
(11.5) \( I_t = K^*_t - K_{t-1} \)
Substituting in the optimal capital stock yields:
(11.6) \( I_t = 0.5 \left( \frac{W_t}{R^K_t} \right) Y_t - K_{t-1} \)
If \( v = 0.5 \left( W / R^K \right) \), and the capital stock always equals the optimal capital stock, then:
(11.8) \( I_t = vY_t - vY_{t-1} = v\Delta Y_t \)
If there are lags in putting into place the capital stock, then one obtains a partial adjustment model:
(11.9) \( I_t = s(K^*_t - K_{t-1}) \)
Where \( s \) is the fraction of the gap between desired and lagged capital stock put in place each period.

\textbf{Tax Policy and the Capital Stock}

Let \( u \) be the tax rate on rental income (related to the corporate tax rate), and \( z \) is the investment incentive for each dollar of capital purchased (investment tax credit, accelerated write-offs or “depreciation for tax purposes”). Then firms equate:
(11.10) \( (1-u)R_t^K = (R_t + d)(1-z_t)P^K_t \), (11.11) \( R^K_t = \frac{(R_t + d)(1-z_t)P^K_t}{(1-u_t)} \)

\textbf{Housing}