Up or Over? The role of permanent associates in lawirms

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Abstract

Over the last two decades many lawirms switched from the traditional up-or-out system to retaining associates that don’t make partner. This paper addresses the reasons for this change by comparing using a permanent associates position to the up-or-out system in the context of a tournament. A model is described with heterogenous abilities among associates. Firms use a noisy signal of output to decide which associates are promoted. Promotions to the permanent associate position are interpreted as consolation prizes. Using the consolation prize as well as partnership, the firm is able to induce the optimal effort from all types; up-or-out rules alone can not accomplish this. Use of a consolation prize requires the firm to increase the partnership prize to prevent high ability workers from pooling with lower ability workers. Firms prefer to create a non-equity, permanent position, if the value of lawyers as a permanent associate is large relative their value in the secondary market. We claim that an increased emphasis on technical skills has increased the relative value of retaining associates, resulting in the abandonment of up-or-out rules.

1 Introduction

The Cravath system, named for the organizational philosophy of Paul Cravath, has long been the dominant paradigm of lawfirm organization within the United States. Its most salient features are profit sharing amongst partners and a rigid up-or-out promotion policy. Incoming associates are either invited to the partnership table or asked to leave the firm. Promotion prospects serve as the primary
mechanism of providing incentives to associates. Over the last 15 years, many law firms (most outside the top Wall Street firms) abandoned the traditional model in favor of a two-tiered partnership ("Getting on the Partnership Track"). In this organizational structure, not all associates who fail to make full partner are asked to leave the firm. The "off-track" partners (also referred to as non-equity partners and permanent associates) income will rise, but they will never become equity partners. While making it easier to stay with the firm, this has made partnership even more exclusive.

The motivation for this paper is to reconcile the previous dominance of the Cravath system with the dramatic changes of the past two decades. While the use of non-equity partners is becoming prevalent at many firms nationwide, top Wall Street firms continue to use the traditional system. Is there an economic reason why these firms may continue to use this system, but others do not?

This paper examines the profitability of the two promotion schemes. The main result shows that when the productivity of associates is large relative the secondary employment market, the two-tiered partnership is more profitable than the Cravath system. This is a consequence of an asymmetry of information. Firms hire a distribution of heterogeneous associates from the labor market, and observe an imperfect signal of their performance. Under the up-or-out rules, if the signal fails to surpass the partnership threshold the associate is fired and enters the secondary market. The secondary wage market is unable to observe any signal of the productivity of workers; therefore, workers who are not retained earn the same wage regardless of performance at the firm. As a result, there are no incentives for associates with little chance of making partner. The firm can create incentives by making it easier to become partner, but this just shifts the problem to more able workers who can now make partner with less effort. In contrast, the two-tiered system offers a consolation prize to create incentives for those unable to make partner.

The primary feature of both these approaches is the use of a tournament. This allows the firm to exploit the risk aversion amongst associates. Since workers are heterogeneous, the perceived likelihood of winning the tournament prize differs across abilities. When there is a single prize, as in the up-or-out framework, either the high-ability workers are able to win the prize without effort or the low-ability workers are unable to win the prize regardless of effort. No single prize is able to induce the efficient effort level from both types. The multi-tier partnership alleviates this incentive problem by providing a "carrot" for all types.
This comes at the price of higher compensation for those that make partner than the single standard system. When higher ability associates pool with lower ability workers they receive an informational rent— in the form of compensation for effort and risk that they don’t have to undertake. A higher partnership prize is needed to prevent high-ability workers from settling for the consolation prize. If the wage in the secondary market is high enough, the gains from increasing effort and lower separation rates are offset by the increase in wages for higher ability workers. Under these circumstances, the rm prefers to re workers even though their productivity might be above their wage. This differs from previous explanations of up-or-out rules in that they arise entirely from the moral hazard problem for workers.

We claim that an increased emphasis on technical skills in the legal profession is responsible for increasing the value of associate effort. This raised the relative value of keeping associates to their secondary market wage—making the multi-tiered partnership more profitable than the up-or-out system.

The structure of the paper is the following. The second section describes the related work on law rms and the basic predictions of the tournament literature. The two models for comparison are developed in section 3. We consider a simple tournament that is uses promotion prizes as a way of sorting the most able associates into the partnership, while providing incentives for associates to work hard at the associate position. Section 4 discusses the main results of the paper. Specifically, why we might or might not see each scheme in practice. The nal section compares the observable characteristics of the two approaches and how robust the model is to a continuum of types.

2 Related Literature

2.1 Why do rms use up-or-out rules?

Up-or-out rules have drawn the attention of economists primarily because of their apparent ine ciency. Presumably the rm would like to be able to retain workers who are not suited for promotion in their current position. Up-or-out rules force the rm to re otherwise productive workers.

One explanation, proposed by Kahn and Haberman (1988), is that up-or-out rules are used as a solution to two-sided moral hazard. The rm would like workers to make an unveri able investment in
...rm-specific human capital. If the ...rm paid for the investment up front, the worker has an incentive to not make the investment. By using promotion as the reward for investment, the ...rm has (implicitly) promised to promote qualified workers who make an investment. The ...rm’s incentive for cheating arises when, at the time of the promotion decision, the ...rm can renege and keep the worker in the current task. Since their human capital is not transferable, ...rms are able to offer workers a wage better than their outside option to stay on in the current task. They conclude that “bonding” the promotion decision with the retention of the worker forces ...rms to commit to promoting qualified workers. The ...rm may prefer not to promote qualified workers, but the ...rm prefers promoting qualified workers to renege them.

This explanation relies on the unverifiable nature of the investment and sufficiently low-reputational concerns for the ...rm. If future associates can observe the outcome of past associates, then the ...rm can be induced to honor its commitment to current associates. Partners nearing retirement may still wish to renege on promoting some associates (since they don’t care about incentives to future associates), but younger partners would prefer to live up to their promise. Law ...rms frequently use law school class graduation career benchmarking systems (“Partnership Track Survey”). By setting a fixed number of promotions in advance, future associates can confirm whether the ...rm is fulfilling its part of the contract by observing the number of previous promotions. If incoming associates can confirm the number of promotions to partner, why can’t incoming associates confirm the numbers of partners and non-equity partners? If the up-or-out system is not the only way to prevent cheating, why is it used instead of alternative mechanisms?

Screening explanations, such as O’Flaherty and Siow (1995), rely on a strict complementarity between associates and partners. They model the output technology as a sum of production units, where a unit is a pairing of a senior worker and a junior worker. Each period, as well as producing output, the senior worker learns information about the skill level of the junior worker. Even though an associate might be more productive than the market wage, the ...rm may prefer to hire them as a way of opening up the senior worker for work with future associates. In this setting, the change towards permanent associate positions reflects a change in the underlying production technology.

This explanation seems improbable for two reasons. First, one of the principal advantages of non-equity partners, noted by legal observers, is their ability to perform high quality work without much partner supervision. Since these workers are no longer being considered for partner, they don’t require
any further evaluation by the partners. They are also useful in training new associates, "Partners tend to manage senior associates, who in turn, train incoming associates much the same way professors manage graduate students who teach undergraduates (Solomon 2002)." This would imply, that senior associates instead of taking the place of younger associates, allow the firm to handle more young associates. Second, a cursory look at the associate to equity partner ratio, shows substantial variation across firms. Among the top 24 in average partner profits of New York firms, the ratio ranges from 1.23:1 to 4.34:1 (Solomon 2001). If the nature of the complementarity is quite strict, we would expect that similar law firms would have similar associate to partner ratios. If this complementarity isn't stringent, the firm will wish to retain some associates without promoting them to partner.

Gradual learning of workers about their abilities, as in Harris and Weiss (1984), show that workers will behave as-if there are up-or-out rules in place if they are slowly learning about the surplus in their current task. They show that less suited workers will slowly leave the firm for a secondary position until some date when no one leaves the firm.

The model presented here extends this literature by showing that up-or-out rules can arise entirely as a result of the moral hazard problem for workers of heterogeneous abilities.

2.2 Explanations of why up-or-out rules might disappear

Levin and Tadelis (2002) argue that up-or-out rules may disappear as firms switch from a partnership structure to that of a corporation. In their framework, partnerships have stronger incentives to guarantee a high quality of worker than corporations in the face of imperfect market monitoring. As firms switch to a corporate form, their commitment to quality is lessened. This allows the firm to retain lower quality workers. They interpret a disappearance of up-or-out rules as a lowering of the standard for employment. The key difference between their interpretation and this set-up is that they do not distinguish between associates and partners. This paper does not offer an explanation for why some workers may now be retained as associates.
2.3 Tournaments

Lazear and Rosen (1981) model promotions as a prize to the winner of a tournament within a firm. The winner of the tournament gets the promotion, raise, and prestige associated with it. They show that if agents are risk-averse and output is imperfectly observed, the tournament is able to induce the efficient effort level; while piece rate pay cannot. The principal designs this tournament by setting a winning prize, losing prize, and a decision rule for who is the winner. Effort and ability are not directly observable and some uncertainty to output exists. The risk-averse agent expends a high effort level to avoid the possibility of not getting the prize. This effort level is higher than the optimal choice the worker would make if effort was directly rewarded. Since a tournament only requires an ordinal measurement rather than a cardinal one, another advantage to tournaments over piece rate pay occurs when output or effort is more costly to observe than ranking. A final reason why tournaments might be used is as a commitment device. The performance of a particular worker might not be verifiable in court. The firm could then claim a worker's performance was poor even if this is not the case. However, even if performance is observable, but not verifiable to an outside party, it may be possible to verify the proportion of workers (on average) that a firm has awarded the prize. Under this constraint, the firm has no incentive not to reward the highest performing workers; therefore, the implicit contract between the workers and firms can be enforced.

Another prediction of the theory is that if workers are of unobserved heterogeneous ability, low quality workers will attempt to pool with higher quality workers, resulting in adverse selection. If standards are set high enough, no pooling will take place, but the disadvantaged workers will not actively participate in the tournament. Intuitively, if an agent views it as very unlikely that effort will result in winning the prize, he is less likely to exert effort.

Despite a rich treatment of tournaments in the literature, very little has been done about how firms and agents might behave in biased tournaments. This paper tries to do this by building a model that illustrates the above predictions and considers the aspects of consolation prizes.

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1. Associates in law firms, concerned that bonus plans may be manipulated by the firm, express reservations about performance based pay (Sheineman and Horvath, 2001).
2. For a complete review of the literature on tournaments see Prendergast (1999).
3 Model

Production within the law ...rm is modelled as a technology requiring two inputs—partners and associates. Typically partners spend more time on client contact and recruitment. Associates, depending on the level of experience, tend to spend more time on tasks like research and document drafts. Screening models use a strict complementarity between these two tasks to justify the up-or-out rules. Since the purpose here is to examine the incentive effects of the consolation prize on the tournament, we take the other extreme. This complementarity is ignored beyond the fact that these are two technologically different jobs. This allows us to measure the ability of workers in terms of the production they add to the ...rm—indepen dent of the associate to partner ratio. Each unit of output is sold at price $p$.

Skills come in the form of a vector with two components. The $rst$ is the productivity of the worker in the associate position with zero effort and no disturbance term. This is the inherent ability of the worker in the task of associate, and it takes on the values $\mu_h$ or $\mu_l$. The second is the value of the worker to the ...rm as partner, and it takes on the values either $v$ or $0$. The skills are distributed such that workers with productivity $\mu_h$ as associates have productivity $v$ as partners, while workers with productivity $\mu_l$ have productivity $0$ as partners. One way we might think of these values is that as an associate a worker is asked to do document discovery and other research that all associates are capable of doing. As a partner only the very capable bring value to the ...rm by attracting new business. This correlation between ability in the associate task with output as partner, allows the ...rm to use performance in the $rst$ task to sort more able workers into the partner position. While the ...rm is not able to perfectly observe the ability level of its workers, it knows the distribution of ability over its associates. The distribution of abilities is such that the proportion of workers with skills $(\mu_h; v)$ is equal to $\pi$.

When workers are $rst$ hired to the ...rm, they are assigned to the associate task: Certainly no partners are hired directly from law school without $rst$ spending time as an associate. Workers enter the ...rm aware of their own cost of effort, their outside utility option, the rules for promotion that the ...rm has established, and their own skills. Obviously associates select into the ...rm based on the lifetime expected utility of working for the ...rm. A worker may choose to accept a position at an alternative ...rm that has a lower threshold (and lower partnership prize) if the expected utility before accepting the position is higher. If low-ability workers are better suited to work in some ...rms
than others, we would expect this to have significant effects on the distribution of types across rms. In this model we do not allow for this heterogeneity across rms. We also assume the compensation of the associates in the secondary wage market will be higher than the opportunity cost of associates. For a model that focuses on these selection issues in an unbiased tournament, see Farrell (1996).

The worker chooses the optimal choice for effort $e_i \in [0, e]$ based on the structure of the tournament set out by rms and their own characteristics. The feasible effort range reflects the need for some effort for the worker to avoid being red and time constraints that limit effort choice. The implicit assumption is that if the associate puts in an effort below $e$ (for example, quits coming to work altogether) the rm will summarily dismiss the associate. For computational ease, a linear cost of effort and exponential utility function are assumed. Specifically, $U(w; e) = \int e w \exp -r(w - c(e)) g$ and $c(e) = ce$.

This particular representation has the property that the ratio between $U(w; e)$ and $U(w^0; e)$ does not depend on the effort level chosen. This follows from the constant trade-off between wealth and effort, regardless of wealth or effort levels. The key difference between this model and previous approaches is that each worker knows their own ability; therefore, workers of different abilities have different perceived likelihoods of promotion as a function of effort.

3.1 The Cravath Benchmark (up-or-out)

The essence of the system is to have a large number of incoming lawyers enter the rm as associates. Associates compete with one another to become full-partners in the rm. The system produces large partner salaries by successfully leveraging these associates. Associates are paid only a fraction of their billable hours, while the rest goes to the rm’s partners.\footnote{Using data from the Altman Weil Compensation Survey: Executive Summary, the mean associate salary is only 37% of the value of their billable hours (an imprecise estimate of their value to the rm). This is compared to 41% for non-equity partners and 61% for equity partners (See appendix, for full table).} A consequence of this hierarchical structure is that the rm’s profitability depends on the partner to associate ratio. Increasing the number of partners requires a commensurate increase in associates, and may potentially “water down” the profit pool if the
partner does not generate large amounts of revenue. This set-up provides strong incentives for ..rms to admit only a very small number of exemplary associates to partners.

Under the traditional set-up, the ..rm establishes a tournament by setting a task assignment rule for promotion to partnership and the wage partners receive \( (w_p) \): After a set number of years, usually between 7-10, the ..rm makes a decision based on a signal of output as to whether the associate should make partner. The associates observed productivity is denoted by output \( q_i \), where \( i \) is indexing the agent observed: Additionally,

\[
q = \mu + \varepsilon + \eta,
\]

where \( \varepsilon \) is the effort level chosen by individual \( i \) and \( \eta \) is an individual specific disturbance term drawn from the distribution \( F \) with density \( f \) (assumed to be uniform on \([0; 1]\)). In this setup there are no ..rm specific disturbance terms (or this is known by the ..rm). The ..rm specifies \( q \) as the task assignment rule (i.e. if \( q_i \geq q \), then the worker is promoted to partner).

This ..xed standard is slightly different than the typical tournament literature where this is the form of a number of promotions. However, if the distribution of incoming workers is stable, then a difference in incentive effects between tournaments and a ..xed threshold only exists in small ..rms (Malcomson 1984). Since the ..rm knows the distribution of workers and the optimal response of associates as a function of the threshold and wages, setting a ..xed cut-off is equivalent to setting an expected number of promotions. The difference in the incentive mechanisms is that the typical tournament exposes the agents to randomness in the signals of their competitors. Firms may use this randomness, in the same way as they use the disturbance term in output, to induce higher effort from workers. However, including this only serves to further complicate this framework without providing additional insight on the use of consolation prizes.

An alternative interpretation of this set-up is that output is perfectly observed by the ..rm, but the ..rm sets a random standard for each worker. Since the worker does not know exactly how much effort is needed to get the promotion, this can create the same incentive effects. This might be a more appealing interpretation in the context of law ..rms. Gilson and M nookin, as well as other legal observers, point out that the criteria for partnership is very subjective at most ..rms. Firms might create this uncertainty as a way of inducing greater effort from associates.
Associates that make partner are awarded \( w_p \); while associates that fail to make partner are asked to leave the firm and enter a secondary market with wage \( w_s \). We treat \( w_p \) as a choice variable. In a traditional partnership this is just a share of the total profits. In practice each firm has its own particular profit sharing system. Since these usually involve a chosen base partner compensation rate, it seems reasonable to view this as a choice variable. The value of \( w_s \) is not determined by the firm and does not depend on \( q \). Some firms attempt to influence \( w_s \) with an emphasis on outplacement, but for the most part this is not possible\(^4\).

It is worth noticing that the wage in the secondary market for associates tends to be dramatically lower than the partnership prize. Gilson and Mnookin (1989) report a surprising result that at “perhaps the foremost law firm in America, only 40% of workers who fail to make partner ever make partner at another large firm.”\(^5\) This is a reflection of law firms' reluctance to hire partners from outside the firm, even those with top pedigree. This result is less surprising when we consider that one of the primary functions of promotion to partnership is to create incentives amongst associates. Recall that at the time of the partnership decision the firm prefers to keep workers as associates, but the firm must promote some to partner to insure that future associates will provide effort. Hiring partners from outside the firm does nothing for internal incentives of future associates. If anything, lateral hires make the chances of partnership look bleaker and provides a disincentive to effort.

### 3.1.1 Optimal effort choice by associates

The incoming associate then faces the following expected utility maximization problem:

\[
\max_{\varepsilon \in \mathbb{R}} \left[ U(w_p; \varepsilon) - F(q - \mu_i - \varepsilon) U(w_s; \varepsilon) \right]
\]

The first term is the probability that a worker of type \( \mu_i \) who invests effort \( \varepsilon \) is above the partnership threshold, times the utility of the partnership. Similarly, the second term is the probability that the worker will not win the prize, times the value of entering the secondary market. For each possible

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\(^4\) The Cravath firm, in particular, prides itself on its relations with former workers. They use a referral process of clients to influence their outplacement (Gilson and Mnookin 1989).

\(^5\) There are obvious similarities between law firms' promotion policies and those of academic institutions, but this is a key difference. A professor that fails to make tenure at a top school can still go to another university and make tenure. This is not the case in the legal profession.
standard the optimal effort choice is not an interior solution (a complete analytical description of the workers choice is in the appendix).

To see why the solution is all-or-none, break down the marginal effect of effort on utility into two effects; one negative and one positive. First, increased effort lowers the utility of the prize that the associates actually receive. This effect is the weighted (by the probability of getting that wage) sum of the marginal disutility of effort at each wage. Because the cost of effort function is constant in terms of wealth, the marginal disutility of effort is the utility at that wage times $r c$. This simplifies the effect to a weighted sum of $U(w_p; e)$ and $U(w_s; e)$. Second, it improves the associates chance of winning the prize. Since the expected utility function is linear in the probabilities of winning the prize, and the probabilities are linear in effort (because of the uniform distribution), this effect is the difference between $U(w_p; e)$ and $U(w_s; e)$. The ratio of (absolute value of) the negative effect to the positive effect is an increasing linear function of the probability of not winning the prize—which is decreasing in effort. This implies that if these two terms are equal (i.e. the derivative of utility with respect to effort equals zero), the associate can increase utility by increasing or decreasing effort.

Graphically, the optimal choice of effort is depicted below.

\[ \text{Figure 1} \]

The lower upward sloping line represents the output that a worker is assured for a given effort choice. The upper line is the bound on observed ability—which depends on the error term from the distribution.
The promotion policy is a horizontal line set at $q$. When $\mu + \bar{e} < q < 1 + \mu + \bar{e}$, the probability of winning the prize is the distance between the cutoff and the upper line divided by the distance between the upper and lower lines (which is one). This corresponds to an effort level $e^f$ when the cutoff is $q_1$ in Figure 1. When the threshold is above both lines, for example if the worker chooses $e^g$ when the cutoff is $q_2$, the probability of winning the prize is zero. When it is below both ($e^f$ when the cutoff is $q_2$), the probability is one.

When the standard is set such that at any effort the worker will always win (or lose) the prize, the optimal choice is $e$. If you're going to get the same wage regardless of effort, why provide any? When this isn't the case, the optimal effort choice depends on whether $w_p$ is incentive compatible. If the wage is incentive compatible, associates will choose the effort level where the lower line is equal to $q$ (for example, $e^g$ when the standard is $q_1$). This corresponds to the choice $e^g = q - \mu$: At this standard, the associate has picked an effort just high enough to guarantee winning the prize. If the cutoff is high enough that this is impossible, the optimal choice is to pick $e^g = q^2$ when the cutoff is $q_2$: Because of the all or none nature of the effort choice problem, workers will never choose an effort level that leaves some chance of losing the prize unless its $e$ or $\bar{e}$: If $w_p$ is not incentive compatible the worker will pick $e$.

Characterizing the incentive compatible wage depends on where $q$ is set in relation to the ability of the associate. A wage that is incentive compatible to one type is not necessarily incentive compatible to the other. Restricting ourselves to things the firm might do in equilibrium, the incentive compatible wage simplifies to (see appendix):

$$w_p \colon \begin{cases} \frac{q}{\bar{e}} & \text{if } q \cdot \mu + \bar{e} \leq \frac{g}{\bar{e}} \\ \frac{\bar{w} + c(e^m) e}{\bar{e}} & \text{if } q \cdot \mu + \bar{e} > \frac{g}{\bar{e}} \end{cases}$$

In the first case, the worker faces no risk of missing the partnership threshold at their optimal effort choice. The incentive compatible wage only needs to compensate the associate for their effort level above their secondary option. When the standard is high enough that even at $e = \bar{e}$ the associate faces positive probability of losing the prize, the incentive compatible wage must compensate the worker for the higher effort choice and assuming the risk.

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6Since the effort choice is all or none by nature, an incentive compatible wage for effort $e^m$ is a $w_p$ that satisfies $E[U(w; e^m)] - E[U(w; \bar{e})]$. 

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3.1.2 The rm's optimal choice of $q$

The rm wishes to solve the following maximization problem:

$$\max_{\mathcal{A}, w_p} \mathcal{A}[p(\mu_h + e_h^i) + (1 - F(\mu_h; e_h^i; q))(v - w_p)] + (1 - \mathcal{A})[p(\mu_l + e_l^i) + (1 - F(\mu_l; e_l^i; q))(v - w_p)]$$

subject to, the incentive constraints implied by the worker choice described above. This is just the weighted productivity of each type as associate, plus the weighted probability of promotion times the value to the rm of the promotion. The difference in the value of the promotion between the two types is that promoted high types produce a value of output $v$.

Assuming $\mu_h - \mu_l$ is not too far apart, the rm is stuck with the choice predicted by tournament theory. Set a high standard where only high ability workers provide any effort or a lower standard where the high-type workers provide less than full effort and some low-type workers are promoted to partner.\(^7\)

When the rm chooses the former, it behaves exactly the same as if all workers are of the high-type. The rm will choose $q = \mu_h + e_i$. Low-type workers choose $e_i$ and are red. High-type workers will select $e_i$ just enough to guarantee winning the promotion. The lowest incentive compatible wage for this standard is $w_p = w_s + c(q - \mu_h - e_i)$:

At any standard below this and the rm can raise the standard and increase effort. The marginal value of an increase in effort to the rm is $p$. While, the marginal cost of this increase is the derivative of the incentive compatible wage with respect to the standard. When the standard is below $\mu_h + e_i$ this derivative is a constant $c$: Implying that if $p > c$; the rm can do better by raising its standard.\(^9\)

Raising the standard above $\mu_h + e_i$ does not increase the effort level of workers, because they are already choosing the maximum. Instead, increasing the standard changes the probability of winning the prize and the incentive compatible wage. If partners are being paid a wage greater than their value to the rm, the rm benefits by having fewer workers being promoted. This is reflecting the idea that at the time of the partnership decision rms prefer not to promote associates to partner. Since workers are risk averse the necessary increase in the partnership prize-to offset the risk faced by associates-is greater than the benefit (see appendix for details).

\(^7\)Where $e_i^q$ refers to the optimal effort choice of type $i$:

\(^8\)If the two types are far apart, the rm will select the same standard as when they don't induce effort from low-type associates.

\(^9\)If $c > p$ the rm never finds it profitable to induce effort from any employees.
If low-type workers are valuable enough (or there are enough of them), the firm does better by choosing the latter and inducing effort from both types. For the given choice of $q$, the firm is able to induce $e$ from the low-types and $e_h > e$ (illustrated below):

![Figure 2](image)

For the same reasons as before, the firm is never going to set a standard below $\mu + e$: Raising the standard increases effort from both types, while still only increasing the wage at rate $c$. Depending on the parameter values, the firm may choose a standard above this. Increasing the threshold above $\mu + e$ has three effects. First, the probability of low-types winning the prize decreases. Some low-types will fail to win the prize despite choosing the maximum effort level. Since low-types that are promoted to partner are certainly being paid more than their value to the firm, the firm benefits from fewer promotions. Just like before, the firm must increase the partnership prize to make this incentive compatible for the low-types. This made it unprofitable to make associates face risk when the firm only induced effort from high-types. However, since high type workers are choosing $e_h = q_1 - \mu_h$, increasing the standard also raises the chosen effort level for high-types. While the details are messy (see appendix), firms may set a standard $q > \mu + e$.

Notice that when both types are induced to provide effort, high-types are receiving an informational rent. They are paid a partnership wage that compensates low-types for risk and the low-types chosen effort level. The high-type workers are choosing a lower effort and not facing risk, but are rewarded anyway.
3.2 Consolation Prizes

The set-up for the ... is almost identical as the Cravath system, except now the ... is able to set two cut-offs, $q^0$ and $q^{00}$. If the output of an associate is below $q^0$ then the worker is ... red and gets $w_s$; the wage in the outside market. When the output observed is between $q^0$ and $q^{00}$, the worker is promoted to the job of permanent associate.

Notice the dynamics of this two-tiered approach. The very best associates are still promoted to partnership, but some that meet a lower standard are offered the position of permanent associate. Permanent associates are asked to do the same tasks as associates, but with a higher pay and no chance of becoming an equity partner. This position, as the name would suggest, is a terminal position in the ...’s hierarchy.

Although no future periods are modeled here, we assume that once in this position the worker will work just hard enough not to be ... red from the ... This serves to pin-down the benefit for the ... of retaining these workers. As a result, workers who are promoted to the job of permanent associate have productivity $\mu + e$. Alternative mechanisms to induce higher effort once in that position do not affect the outcome discussed here.

The ... also sets two wages, $w_a$ and $w_p$: $w_a$ is the wage paid to workers who are promoted to the permanent associate position, while $w_p$ is still the prize for partners.

3.2.1 Optimal response by workers

Regardless of the choice of $q^0$ and $q^{00}$, we only need to consider the workers problem as if there is a single standard for each type. To see why, suppose the ... asked each associate to sign-up for one of two tournaments in advance. In one tournament, you get the partnership prize if you meet the standard $q^{00}$, but are ... red otherwise. The other tournament offers a lower prize, but at a lower standard. If structured properly, the high-type workers will compete for the partnership prize, while the low-types attempt to win the secondary prize.\footnote{Some might be suspicious that law firms would adopt such a policy. Yet, two lawyers, Gilson and Mnookin, have suggested this approach as a way to deal with increased variance of associate abilities. This might be accomplished by dividing associates into two categories at the time they are hired. Those from the traditional pool of associates are treated as associates in an up-or-out system... (the associates) from the non-traditional pool are advised from the outset that they...} This could be implemented by assigning higher ability workers...
to duties that can demonstrate their partnership qualities, while assigning lower ability workers to more mundane tasks. This rules out the possibility that low-type workers trying to win the consolation prize might get lucky enough to win the partnership prize. It also rules out that unlucky high-types will still qualify for the consolation prize, but this is irrelevant when we restrict ourselves to standards the rm might actually set.

For high-types we have the same maximization problem as before, but now there is an additional incentive constraint. When high-types pool with the low-type workers they are rewarded for higher effort levels (and for any risk) than they must incur. For a wage to be incentive compatible it must now make the expected utility of the partnership tournament be higher than the best expected utility of the consolation tournament.\textsuperscript{11}

Simplifying this constraint for the relevant case implies that $w_p \cdot w_a + c(q^0_i - q^i)$.\textsuperscript{12} The difference between $q^0$ and $q^i$ is the additional effort in the partnership tournament beyond their optimal effort in the consolation tournament. This constraint seems in line with observations that "the partner entry compensation rate is largely affected by associate market salaries and may be only marginally better for new partners than top associate rates (Solomon 2001)."

The decision problem for the low-type workers is exactly the same as before. Entering into the partnership tournament is not appealing because $w_p$ is set just high enough to make high-type workers enter. Low-types will enter the consolation prize tournament and make the same decisions as if this were the only standard with prize $w_a$.

3.2.2 Optimal choice of $q^0$ and $q^i$

Using the mechanism where high-type workers sign-up for one tournament and low-type workers for the other, the rm's maximization problem can be written as:

\begin{equation}
\max_p E_p [U(w; e)] = \max_c E_c [U(w; e)],
\end{equation}

where the first expectation is over the partnership tournament and the second over the consolation tournament. This is the incentive compatible wage when $q^0 - \mu + c$. Like before, when the standard is higher than this the associate must be compensated. However, this will not be the case in equilibrium and is ignored here. Also, if the rm chooses two standards that are far apart (i.e., $q^0_i < q^0$, $\tilde{e} = \varnothing$) then the rm only needs to compensate the workers for the range of possible effort choices.
subject to the incentive compatibility constraints. Like the up-or-out system, high-type workers that fail to make partner are still red. Low-type workers that meet the standard \( q^0 \) are kept on as permanent associates; they have a productivity of \( \mu_l + e_l \) and are paid \( w_a \). Low-types are not considered for the partnership position.

Since the choice of \( q^0 \) will not have any incentive effects on the low-type workers, the optimal standard is the same as when the rm only tries to induce effort from the high-types in the up-or-out system. Any standard below \( \mu_h + e \) and the rm can increase effort at a marginal cost of \( c \). Any standard above and the cost of compensating associates for this risk outweighs the benefits of fewer associates making partner. All high-types provide effort \( e \) and win the partnership prize. Substituting this standard into the incentive compatible wage, partners are paid \( w_p = w_a + c(\mu_h + e - q^0) \):

The choice of \( q^0 \) is a question of whether all low-types should make permanent associate. Raising a standard above \( \mu_l + e \) lowers the probability of workers being promoted to permanent associate. Depending on the value of workers in this position and the secondary wage, this may or may not benefit the rm. As a result of this increased risk, \( w_a \) must increase to compensate the associates. Changes in \( w_a \) affect \( w_p \) by changing the value for high-types of entering the consolation tournament. The higher standard in the consolation tournament increases the prize in the consolation tournament, but high-types must make a higher effort choice to guarantee winning this prize. This is illustrated on the figure below.
The shaded line segment represents the additional effort that high-types provide in the partnership tournament compared to the effort they would choose if they were in the consolation tournament. The difference in $w_p$ and $w_a$ is the length of this segment times $c$: Raising $q_0$ raises $w_a$ but shortens this distance. For choices of $q_0$ that involve small chances of low-types not winning the prize, this effect is negative. When we combine these three effects, the firm's optimal choice of $q_0$ might be larger than $\mu + \varepsilon$ (as depicted). In the equilibrium, both types are providing the maximum effort choice. This was something the single standard was unable to accomplish. Some low-types may still be let-go by the firm as a way of keeping down the wages needed to induce effort from high-types.

4 Main Results

Why the change?

Perhaps the most perplexing question has been why such a dominant system seems to have changed so quickly. Gilson and Mnookin suggest that in the mid-80's law firms began to place increased importance on "specialized technical skills." They describe these skills as those necessary to support a practice, instead of developing relationships with clients. One interpretation is that this shift is relecting a change in the way law is practiced. Imagine that the legal production process requires a certain proportion of
critical thinking and leg work. The leg work is certainly skilled work, but isn’t as crucial as the critical
thinking. Implied that a competent associate or partner is capable of performing this task. Over the
last 20 years, the typical client requires an increased proportion of leg work. This increased the value
of associates to the rm.

This would likely raise both the value of keeping associates in the rm and their secondary wage. If
the secondary market employs them to use similar skills, its wage should also rise when these become
more valuable. However, it is not going to increase their values equally. As we demonstrated in this
paper, retaining associates as permanent associates can increase the effort of associates. The value
of retaining associates in the rm is their future productivity and the extra effort. The secondary
wage market is unable (and unwilling) to reward effort undertaken in the rm. The shift described
above increases the value of this effort. This implies the relative value of retaining an associate to the
secondary wage is increasing.

How does this change affect the profitability of the two-tier system? Comparing the profits of the
two systems, a sufficient condition for the rm to prefer the multi-tiered partnership to inducing effort
from high types is

\[ \frac{p(e_1 \mu + e) \cdot w_s}{1 - \Phi} \cdot 0. \\
\]

The first term in this expression is the value of retaining an associate. It is the sum of the future
productivity of the associate and the additional effort that the associate is providing. Clearly, a change
that increases this value relative to \( w_s \) could cause this expression to change signs.

Why are many Wall Street rms reluctant to change?

One of the other interesting features of this transition has been the reluctance of large Wall Street
rms to adopt this system (Solomon 2002). A natural explanation might be that these rms are hesitant
to change from a system that has procured such large profits. The idea is that if changing systems
involves some uncertainty about future profits, rms with higher current profits are less likely to take
the leap. If this argument holds, then as time passes Wall Street rms will become more sure of the

\[ \text{This condition is only for when the rm prefers to induce effort from the high-types in the Cravath set-up. However,}
\]

\[ \text{the qualitative reasons for using the two-tiered system to the up-or-out system are the same for either case except for the}
\]

\[ \text{reversals of } \Phi \text{ and } 1. \\
\]

19
pro...tability of the new system. Eventually the uncertainty about pro...ts will be small enough that these ...rms will make the change as well. This is certainly plausible, but another reason may exist.

Wall Street ...rms have been uniquely successful at using outplacement as a form of a consolation prize. By using business referrals to place workers in other ...rms, they are able to reward their associates who don't make partner, but did provide e...rt. The ...rms use of outplacement ...lls a similar role as the permanent associate position. It requires an increase in the incentive compatible wage for high-types and is able to provide incentives to both types. Firms are still ...ring otherwise productive workers, but someone else is now rewarding low-type employees for their e...rt.

Why have these ...rms been able to successfully use outplacement while other ...rms can not? This seems to be a result of the high concentration of corporate activity in New York. We might suspect that since ...rms have become increasingly national and international, location does not matter. However, to the extent that referrals are an important method of facilitating outplacement, it is di¢cult to imagine a Houston ...rm placing workers in Denver. What level of concentration is needed to make outplacement viable? We have no way of answering this question, but it could be that only the nations largest market surpasses this threshold.

5 Discussion

Comparison of partnership thresholds

Recall the two possible initial situations. When the ...rm originally chose to just provide incentives to the high-type workers, adding a secondary prize did not change the threshold for partnerships. Before the secondary prize was added all low-type associates were ...red in equilibrium, now some will remain on as permanent associates. When compared to the situation where the ...rm was inducing e...rt from both types, the addition of the secondary prize raised the threshold for partnership as well as the partnership prize. In the two-tiered scheme, only high-types made partner. While some low-types are still ...red with the addition of the secondary prize, the proportion ...red will never be larger than under the one promotion system. Thus, the additional prize made it more di¢cult to make partner, but easier to stay

14Outplacement is also stressed in non-legal professions (for example, McKinsey management and (formerly) Arthur Anderson consulting) supporting an up-or-out system.
This is consistent with the observation made by some legal observers, that the use of permanent associates has had two consequences. "First, is a lessening of the non-equity partnership entry threshold, providing legitimate partner prospects for many associates. Accompanying this has been an increase in the equity partnership threshold (Solomon 1996)."

Besides these predictions about the probability of promotion, we also can say how this standard affects the compensation levels. Adding a prize increased the compensation for partner regardless of the initial situation. Compared to the initial case where both types provided effort, the consolation prize $w_s$ is smaller than the initial partnership prize. However, the permanent associate position pays better than the case where only high-types were induced to provide effort.

How robust is this to continuous types?

The two-types framework was useful in showing how a single consolation prize was able to provide incentives for heterogeneous workers. When we extend this set-up to continuous types we will see a result that looks similar. To modify the model above to accommodate a continuum of types, the productivity of type $\mu$ as a partner is now $\mu$:

The firm's optimal strategy is to set-up a continuum of tournaments (similar to what it did in the two-tier system). Each tournament has a different wage. A worker signs up in advance for one tournament and gets the wage if successful, otherwise they enter the secondary market. The task assignment rule assigns workers to the more productive task (i.e. partner if $\mu > \mu$). Like the two-tiered system, only one standard is relevant for each worker. Providing incentives to a lower ability worker, requires increasing the wage for higher ability workers to keep them from settling for a lower prize. The resulting incentive compatible wage\(^{15}\) (for a worker claiming to be type $\mu$) is described by $w(\mu) = w_s + cE[\mu - \mu]$ where $\mu$ is the lowest type that is actively participating in the tournament. Again, this is just compensating workers for effort above their opportunity cost.

The firm selects the lowest type that it will provide incentives for in equilibrium (see appendix for maximization problem). The lower the type, the greater the cost in increased wages. As a result, any

\(^{15}\text{For a wage to be incentive compatible for tournament } \mu \text{ it must be the case that } E_\mu[w; \mu] \geq \max_{\mu} E_\mu[w; \mu]$. Basically, this says that for a worker of type $\mu$; they will enter into tournament $\mu$; if the expected utility is higher than the expected utility at the optimal effort choice in the best alternative tournament.
type below $\mu$ will choose the lower bound on effort and be red. Types in the range of $[\mu, \frac{e}{1-p}]$ will choose maximum effort and be assigned to the task of permanent associate. Any type higher chooses maximum effort and is assigned to the partner task.

In this set-up, when the rm’s optimal $\mu > \frac{e}{1-p}$; the rm appears to be using an up-or-out system. The only types that are induced to provide effort are those that are promoted to the partnership task. The use of permanent associates occurs when this lower bound is below the task assignment rule. Just like before, when the value of the associate at the permanent task position is high relative the secondary option, $\mu$ falls.

We should be a little careful here about why types that will have no chance of winning the prize will ever enter the rm. Certainly, we would think that some of these would be better off going to a different rm with a lower partnership threshold. There are two possible explanations. First, it could be that the utility of entering this prestigious rm, providing the minimum effort, and entering the secondary market is higher than the outside utility option. This is what we assumed in the two-type set-up. Second, it might be the case that when selecting a rm, each worker only has a signal of their true type. As a result, some choose to go to a rm, only to find out later that they were better off elsewhere.

6 Conclusion

In this paper, we consider law-rms facing a group of associates of heterogeneous abilities. The rm is unaware of the exact abilities of individual workers, but observes a signal of their output. When the rm uses one promotion standard to partnership, some types will not have effective incentive mechanisms. Using a secondary prize can solve this incentive problem, but increases the cost of providing incentives. If the value of the keeping associates on in a permanent capacity is high relative their secondary market wage, the rm prefers the multi-tiered promotion system.

We contend that changes in the way the law is practiced has made the value of retaining associates greater relative the secondary wage market. This has led to rms abandoning the traditional model for the multi-tiered partnership. This has not resulted in changes at the nation’s elite Wall Street rms. This might be the result of the unique ability of these rms to use outplacement.
Table 1

<table>
<thead>
<tr>
<th>Classification</th>
<th>Billable hours</th>
<th>Rate</th>
<th>Gross receipts</th>
<th>Compensation</th>
<th>Comp/receipts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity partners</td>
<td>1759</td>
<td>$265</td>
<td>$466,135</td>
<td>$285,227</td>
<td>61%</td>
</tr>
<tr>
<td>Associates</td>
<td>1827</td>
<td>$179</td>
<td>$327,033</td>
<td>$119,973</td>
<td>37%</td>
</tr>
<tr>
<td>Non-equity partners</td>
<td>1744</td>
<td>$247</td>
<td>$430,768</td>
<td>$177,851</td>
<td>41%</td>
</tr>
</tbody>
</table>

Data comes from the Altman Weil Survey of Law Firm Economics. It is based on information from 20,437 lawyers in 401 US law firms. All numbers are the mean of the survey. Rate is the mean billing rate of each classification of lawyer. Gross receipts is the billing rate times the mean billable hours. Compensation is the mean total compensation of each classification of lawyers.

Worker’s Choice of e

A: $q_1 = \mu + e$. The worker will win the prize regardless of the effort level chosen. Increasing effort does not change the probability of winning the prize. Clearly, the optimal response is for the worker to choose $e^* = e$.

B: $\mu + e < q_1$; $1 + \mu + e$ then the agent has a positive probability of winning the prize regardless of the effort level chosen, but could guarantee getting the prize by choosing $e = q_1 - \mu$: Solving, analytically by differentiating with respect to effort we find that there is no interior maximum.

\[
FOC : (1, \mu + e - U(w_p; \mu) + \mu + e)U(w_p; e) + F(q_1 - \mu - e)U(w_s; e) > 0
\]

then increase $e$.

Note: Since utility is of the exponential form,
\[
\frac{\partial U(w_p; e)}{\partial e} = U(w_p; e)rc, \quad \frac{\partial U(w_s; e)}{\partial e} = U(w_s; e)rc
\]

\[(1, \mu + e)U(w_p; e)rc + U(w_p; e) + (q_1 - \mu + e)U(w_s; e)rc > 0
\]

divide by $U(w_s; e) < 0$:

Note: Since utility is of the exponential form,
\[
\frac{U(w_p; \mu) - U(w_s; e)}{U(w_p; e)} = \exp f_i r(w_p; i) w_s g
\]

\[(1, q_1 - \mu + e)\exp f_i r(w_p; i) w_s g + \exp f_i r(w_p; i) w_s g + (q_1 - \mu + e)rc > 0, \quad 1 < 0
\]

\[(1, \mu + e)(rc + \exp f_i r(w_p; i) w_s g) + (q_1 - \mu + e)\exp f_i r(w_p; i) w_s grc
\]

\[(1, \exp f_i r(w_p; i) w_s g + \exp f_i r(w_p; i) w_s g] + \exp f_i r(w_p; i) w_s g
\]
guarantee winning the prize by choosing

each of these two choices, we get the following condition:

\[ 1 > (\overline{q}_i \cdot \mu + \varepsilon) rc + r c \frac{\exp(1 \cdot r(w_p, w_s))}{\overline{q}_i \cdot \exp(1 \cdot r(w_p, w_s))} \]

\[ (\overline{q}_i \cdot \mu + \varepsilon) < \frac{1}{rc} i \frac{\exp(1 \cdot r(w_p, w_s))}{\overline{q}_i \cdot \exp(1 \cdot r(w_p, w_s))}, \]

then increase \( \varepsilon \). So, the optimal choice is to pick a corner solution for \( \varepsilon \).

So, we know the optimal choice of \( \varepsilon \) will be either \( \varepsilon \) or \( \overline{q}_i \cdot \mu \): Comparing the expected utilities of each of these two choices, we get the following condition:

\[ e^2 = \overline{q}_i \cdot \mu \text{ if } w_p > w_s, \quad 1 + \frac{1}{r} \ln \frac{1 + \mu + \varepsilon}{\varepsilon} \] \[ g = \text{ otherwise} \]

\[ \text{where } z = \overline{q}_i \cdot \mu + \varepsilon \]

The optimal choice is to choose \( e^2 = \overline{q}_i \cdot \mu \) if

\[ E[U(w; e) | e = \overline{q}_i \cdot \mu] > E[U(w; e) | e = \varepsilon] \]

\[ E[U(w; e) | e = \overline{q}_i \cdot \mu] = i \cdot \exp f_i r(w_p, c(\overline{q}_i \cdot \mu))g \]

\[ E[U(w; e) | e = \varepsilon] = (1 + z)(i \cdot \exp f_i r(w_p, c(\varepsilon))g + z(i \cdot \exp f_i r(w_s, c(\varepsilon))g) \]

divide by \( i \cdot \exp f_i r(w_p, c(\varepsilon))g \)

\[ \varepsilon^2 = (1 + z) + z(\exp f r(w_p, w_s))g \]

\[ \varepsilon^2 = 1 + z < z \exp f r(w_p, w_s) \]

\[ w_p > \frac{1}{r} \ln \frac{1 + \mu + \varepsilon}{\varepsilon} g; \]

\[ C: 1 + \mu + \varepsilon < \overline{q}_i \cdot \mu + \varepsilon \] The agent has no chance of winning the prize if \( \varepsilon \) is chosen, but can guarantee winning the prize by choosing \( e = \overline{q}_i \cdot \mu \): The maximization problem is the same as in the previous case, again we have a corner solution. The optimal choice of \( e \) will be either \( \varepsilon \) or \( \overline{q}_i \cdot \mu \):

Comparing the expected utilities of these possibilities we get:

\[ 8 \quad e^2 = \overline{q}_i \cdot \mu \text{ if } w_p > w_s, \quad \frac{g}{\varepsilon} \text{ otherwise} \]

The optimal choice is to choose \( e^2 = \overline{q}_i \cdot \mu \) if

\[ i \cdot \exp f_i r(w_p, c(\overline{q}_i \cdot \mu))g > i \cdot \exp f_i r(w_s, c(\varepsilon))g \]

\[ w_p > c(\overline{q}_i \cdot \mu) > w_s > c(\varepsilon) \]

\[ D: \mu + \varepsilon < \overline{q}_i \cdot 1 + \mu + \varepsilon \] The agent will have a positive probability of winning the prize if a high enough effort is chosen, but will not be able to avoid facing some chance of losing the prize. Again, the
maximization problem is the same, giving us a corner solution. The optimal choice of $e$ will be either $e$ or $\bar{e}$. Comparing expected utilities we get:

$$
e^* = \begin{cases} 
8 & \text{if } w_p > \frac{1}{\gamma} \ln \frac{1}{\gamma^{\gamma + 1}} y \text{, where } y = z_i (\bar{e} | \bar{e}) \\
\bar{e} & \text{otherwise}
\end{cases}
$$

The optimal choice is to choose $e^* = \bar{e}$ if

$$E[U(w; e) | e = \bar{e}] > E[U(w; e) | e = \bar{e}]$$

$$E[U(w; e) | e = \bar{e}] = (1 - y) \exp f_i r(w_p | \bar{e} | \bar{e}) + y \exp f_i r(w_s | \bar{e} | \bar{e})$$

Evaluating the FOC:

$$E: \bar{q} > 1 + \mu + \bar{e} \text{ The agent will never win the prize regardless of exert choice, clearly the optimal }$$

$$e^* = \bar{e}$$

FOC for $\bar{q} > \mu + \bar{e}$

This is the maximization problem when the .rm is considering setting a standard where some high-types lose the promotion even at the maximum exert.

The FOC: $\frac{\partial F}{\partial y} = \pi(v_i, w_p | \bar{q}) + (1 - y) F(\bar{q} | \mu_i | \bar{e})$ $\pi(v_i | \bar{q}) \frac{\partial \pi}{\partial y} = 0$

Substituting the appropriate $w_p$: Recall, $y = F(\bar{q} | \mu_i | \bar{e})$

$$\frac{\partial w_p}{\partial y} = \frac{1}{r} \left( \frac{e^i r c(\bar{e} | \bar{e})}{1 + y} \right) \left( \frac{e^i r c(\bar{e} | \bar{e}) + y + 1}{(e^i r c(\bar{e} | \bar{e}) + y)^2} \right)$$

i (v_i, w_s) + \frac{1}{r} \ln \frac{1}{\gamma^{\gamma + 1}} y g + (1 - y) \pi(v_i | \bar{q}) \pi(w_p | \bar{q}) \pi(w_s | \bar{q}) = 0$

i (v_i, w_s) + \frac{1}{r} \ln \frac{1}{\gamma^{\gamma + 1}} y g + (1 - y) \pi(w_p | \bar{q}) \pi(w_s | \bar{q}) = 0$

Notice:
\[
\inf_{y} \left( \frac{1}{y} \right) \left( \frac{\exp \left( \frac{yc}{r} \right)}{\exp \left( \frac{yc}{r} \right) + 1} \right) < \left( \frac{1}{y} \right) \left( \frac{\exp \left( \frac{yc}{r} \right)}{\exp \left( \frac{yc}{r} \right) + 1} \right) \text{ for all } y < e^{r\mu} \cdot \frac{1}{e^{r\mu} + 1}; \text{ implying the sum of the last two terms is negative.}
\]

If the value of a qualified partner is greater than the outside option this expression is always negative. When this is negative, the rm prefers to lower the standard and set \( q = \mu + \varepsilon \).

Maximization problem for choosing \( q \) in the pooling equilibrium:

\[
\begin{align*}
\max_{q} & \quad \mathbb{E}[q + y_i w_p] + (1 - \mathbb{E}[q | i=0]) \mathbb{E}[y_i | w_p] \\
\text{FOC:} & \quad \frac{\partial}{\partial q} \mathbb{E}[q + y_i w_p] + (1 - \mathbb{E}[q | i=0]) \mathbb{E}[y_i | w_p] = 0
\end{align*}
\]

Substituting the appropriate \( w_p; y = F(q | i=0, \varepsilon) \):

\[
\frac{\partial}{\partial q} \mathbb{E}[q + y_i w_p] + (1 - \mathbb{E}[q | i=0]) \mathbb{E}[y_i | w_p] = 0
\]

Unfortunately, this does not have an analytic solution, but we do know when \( y = 0 \), i.e. \( q = \mu + \varepsilon \); the FOC simplifies to:

\[
\frac{1}{r} \left( e^{r\mu} \frac{\varepsilon}{r} + 1 \right) + \mathbb{E}(\varepsilon | i) + (1 - \mathbb{E}[q | i=0, \varepsilon]) w_p + \mathbb{E}[y_i | w_p] = 0
\]

This expression is positive or negative depending on the parameter values. If this is negative, that implies the optimal choice of \( q = \mu + \varepsilon \); otherwise it will be greater than \( \mu + \varepsilon \).

Incentive Compatibility of high-types in two-tiered system

The only region that needs to be considered is that of region 1. Since, the rm will never select a standard in any of the other regions.

\( C : 1 + \mu + \varepsilon < q^0 < q^0 \cdot \mu + \varepsilon \). The agent has no chance of winning the prize if \( \varepsilon \) is chosen, but can guarantee winning the prize by choosing \( \varepsilon = q^0 \cdot \mu \). A choice of \( \varepsilon = q^0 \cdot \mu \) will guarantee the secondary prize. The optimal choice of \( \varepsilon \) will be either \( q^0 \cdot \mu \) or \( q^0 \cdot \mu \). The optimal choice is to choose \( e^* = q^0 \cdot \mu \) if

\[
\begin{align*}
\exp( w_p | c(q^0 \cdot \mu)) | g > & \exp( w_s | c\varepsilon) | g \\
& \quad w_p | c(q^0 \cdot \mu) > w_s | c\varepsilon \\
& \quad w_p | w_s > c\varepsilon
\end{align*}
\]

and if \( \exp( w_p | c(q^0 \cdot \mu(g)) | g > \exp( w_s | c(q^0 \cdot \mu)) | g \\
& \quad w_p | w_s > c(q^0 \cdot \mu | q^0 \cdot \mu)
\]

Comparing the expected utilities of these possibilities we get:
Maximization for $q^0$ in the multi-tiered partnership

$$\max_{q^0} w_0 + c(q^0_i q^0) \text{ if } q^0 > \mu_h + \varepsilon$$

$$= \begin{cases} w_p + \frac{1}{r} \ln \frac{1}{e^{r c(w, \mu_h, \varepsilon)}} & \text{if } q^0 > \mu_h + \varepsilon \end{cases}$$

Unfortunately, this does not have an analytic solution, but we do know when $y = 0$, i.e. $q^0 = \mu + \varepsilon$; the FOC simplifies to:

$$\mu \left( \frac{d}{d \mu} \left( \frac{1}{r} \ln \frac{1}{e^{r c(w, \mu_h, \varepsilon)}} \right) \right) + \varepsilon + \left( 1 \mu \right) \left( p(\mu + \varepsilon) + w_s + \frac{1}{r} \ln \frac{1}{e^{r c(w, \mu_h, \varepsilon)}} \right)$$

This expression is positive or negative depending on the parameter values. If this is negative, that implies the optimal choice of $q^0 = \mu + \varepsilon$; otherwise it will be greater than $\mu + \varepsilon$.

Maximization problem for continuous types

$$\max_{\mu} \int_{-\infty}^{\infty} p(t)dt + \int_{-\infty}^{\infty} c(\varepsilon)dt + \int_{-\infty}^{\infty} \frac{c(t + \mu)}{1 - r \varepsilon} dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(t + \mu) \varepsilon c(\varepsilon) c(t + \mu) dt$$

8 Bibliography

References


