78. For a linear demand curve \( p = a - bq \), the surplus is 
\[
S(p) = \frac{\frac{3}{2}(a-p)^2}{b}.
\]
The fixed fee is the smaller surplus. Here \( a_1 = 3, b_1 = 1, a_2 = 4, b_2 = 2 \), so \( S_1(p) = \frac{1}{2}(3-p)^2 \) and \( S_2(p) = \frac{1}{4}(4-p)^2 \). When \( p \) is high, \( S_2 \) is larger, and \( S_1 = S_2 \) for \( (4-p) = \sqrt{2}(3-p) \), so that \( p = 2+\sqrt{2} \), with \( S_1 \) above \( S_2 \) for prices below this. It happens in this example that profit rises when \( p \) is reduced with \( A = S_2 \), and profit rises when \( p \) is increased with \( A = S_1 \), so the optimal choice is to reduce \( p \) until \( A = S_1 = S_2 \), so \( p^* = 2+\sqrt{2} = .586 \), which is below marginal cost. The fixed fee is \( A = 3/2+\sqrt{2} = 2.914 \), total quantity sold is \( 2+(3/2)\sqrt{2} \), and profit is \( 3+2\sqrt{2} - (\sqrt{2}-1)(2+(3/2)\sqrt{2}) = 2 + (3/2)\sqrt{2} = 4.121 \), per customer, so $412.1 altogether.

If \( p = mc \) then \( S_1 = 2 \) and \( S_2 = 9/4 \). So if \( A = 2 \) and \( p = 1 \) then \( q_1 = 2 \) and \( q_2 = 3/2 \). Profit is just 2 per customer and there are 200 customers. This is less than the optimal profit.

If the two markets could be separated, set \( p = 1 \) and \( A_1 = 2 \) and \( A_2 = 2+\sqrt{2} \), giving a total profit of $425. This gives an upper bound.

The reason why \( p^* \) is below \( mc \) here is that at the point where \( p = mc \), \( S_1 \) is the smaller surplus, and yet \( q_1 \) is bigger than \( q_2 \). So reducing price by \( \delta \) allows \( A \) to rise by \( 2\delta \), while the "operating" loss is \( (q_1+q_2)\delta \), which means profit rises on balance.

[Chris Adams] Offer two different packages, one intended for type 1, the other for type 2. Type one pays a high entry fee and a low price, and type two pays a low entry fee and a high price. Set \( A_1 = S_1 = 3/2+\sqrt{2} \) with \( p_1 = 2+\sqrt{2} \) and \( A_2 = S_2 = 2+\sqrt{2} \) with \( p_2 = 1 \). Total profit is then \( 2+\sqrt{2} - (\sqrt{2}-1)(2+(3/2)\sqrt{2}) = 2 + (3/2)\sqrt{2} = 4.164 \) per customer.

A two-part tariff is an affine pricing scheme: \( T = A + pq \). In general, the monopolist can do better with a nonlinear pricing scheme (see Tirole, page 148). In the nonlinear case with two types, the relevant constraints are the IR constraint for the low type, and the IC constraint for the high type.