The 1st conditions for a social welfare max are just \( p_1 = c_1 \) and \( p_2 = c_2 \). Clearly, revenue does not cover fixed cost.

For revenue maximization, \( p \) is replace by \( m \), where \( m \) is marginal revenue.

Second-best is a mixture of these: \( p_1 - c_1 + \lambda(m_1 - c_1) = 0 \), \( p_1 - c_1 + \lambda(m_1 - c_1) = 0 \)

The multiplier for this problem is \( \lambda = \frac{F}{(c_1 - m_1)q_1 + (c_2 - m_2)q_2} \)

The Ramsey number is \( \rho = (p-c)/(p-m) \). So the first-order condition is \( p-c+\lambda(m-p)+\lambda(p-c) = 0 \), and this can be written as \( \rho = \lambda/(1+\lambda) \). For the social welfare problem, \( \lambda = \rho = 0 \); for the monopoly problem \( \lambda = \infty \), and \( \rho = 1 \)

In the second-best problem, \( \lambda = \frac{\rho_1}{1-p_1} = \frac{\rho_2}{1-p_2} \)

so \( \rho_1 = \rho_2 \). In the linear case, \( A_1 = 2p_1 - m_1 \), so \( \frac{A_1 - p_1}{A_1 - c_1} = \frac{p_1 - m_1}{p_1 - m_1 + p_1 - c_1} \), and then equality of the Ramsey numbers implies equality of the ratios stated in part (d):

\[
\frac{A_1 - p_1}{A_1 - c_1} = \frac{A_2 - p_2}{A_2 - c_2}
\]

(c) Shutdown is socially optimal if the consumer surplus when \( p = mc \) is insufficient to meet the fixed cost. With linear demand this means

\[
\frac{(A_1 - c_1)^2}{2M_1} + \frac{(A_2 - c_2)^2}{2M_2} < F
\]

Shutdown is second-best optimal if a monopolist can't break even (because the monopolist must charge the same price for every unit sold). This happens if

\[
\frac{A_1^2 - c_1^2}{4M_1} + \frac{A_2^2 - c_2^2}{4M_2} < F
\]

There is no fix in this situation unless nonlinear pricing is feasible.