\[
\text{Merged firm: } \max_{w_e, w_c} p_e f_e(w_e) + p_c f_c(w_c) - (w_c + w_e)p_w \\
\text{F.O.C.'s } \Rightarrow p_e f'_e(w_e) = p_c f'_c(w_c) = p_w
\]

\[W_c = 400 - L_c \]
\[W_a = 400 - (420 - L_c) \]
\[W_c - W_a = 420 - 2L_c \quad (1)\]

\[L_c = 420 \int_0^{W_c-W_a} \frac{1}{42} \, dx = 10(W_c-W_a) \Rightarrow W_c - W_a = \frac{L_c}{10} \quad (2)\]

a) \((1), (2) \Rightarrow L_c = 200 \quad W_c = 200 \quad W_a = 180 \quad \Delta W = 20\)

b) Fred gains from diversity. In diverse economy \(W_c - W_a = 20\) ⇒ Fred works in auto industry and earns \(W_a = 180\). In homogenous economy, \(W_c - W_a = 21\) ⇒ Fred is indifferent between earning 200.5 in coal and earning 179.5 in auto. But since 180 > 179.5 Fred does better in diverse economy.
c) If $q$ excluded from coal industry we know all 126 $q$ will work in the auto industry ($120 \times , 294 \sigma$).

Thus we must determine the equilibrium wage differential and occupational distribution such that both the firms and the $\sigma$ are profit (utility) maximizing.

**Demand Side**

$$W_c = 400 - L_c$$

$$W_a = 400 - L_a = 400 - (126 + (294 - L_c))$$

$$\{ W_c - W_a = 420 - 2L_c \} \quad (1)'$$

**Supply Side**

$$L_c = 294 \int_0^{\frac{W_c - W_a}{42}} \frac{1}{42} \, dx = 7(W_c - W_a) \Rightarrow W_c - W_a = \frac{L_c}{42} \quad (2)'$$

\[
\begin{align*}
(1)', (2)' & \\
L_c &= 196 \quad W_c = 204 \\
L_a &= 126 + 98 = 224 \quad W_a = 176 \\
\Delta W &= 28
\end{align*}
\]

Average wages before:

$$\sigma: \left( \frac{294}{420} \cdot 200 + \frac{294}{420} \cdot 220 \right) \left( \frac{1}{294} \right) = 189.5$$

$$q: \left( \frac{126}{420} \cdot 200 + \frac{126}{420} \cdot 220 \right) \left( \frac{1}{126} \right) = 189.5$$

Average wages after:

$$\sigma: (196, 204 + 98, 176) \left( \frac{1}{294} \right) = 194.7$$

$$q: (126, 176) \left( \frac{1}{126} \right) = 176$$

\[\therefore \text{on average, men's wages rise and women's wages fall.}\]
**winners and losers**

\[ \sigma^* : \]
1. Were in coal before and still in coal after if \( x < 20 \)  
   \[ \Rightarrow \text{wage increases by 4} \Rightarrow \text{better off} \]

2. Were in auto before and still in auto after if \( x \geq 28 \)  
   \[ \Rightarrow \text{wage decreases by } 4 \Rightarrow \text{worse off} \]

3. In auto before and in coal after \( 20 \leq x \leq 28 \)  
   \[ \Rightarrow \text{wage decreases by } 24 \]
   
   but "earn" their differential \( x \)  

   \[ \Rightarrow \text{worse off if } x > 24 \]
   
   \[ \Rightarrow \text{better off if } x < 24 \]

\[ \text{for } \sigma^* \]

if \( x \geq 24 \) the same or worse off

if \( x \leq 24 \) the same or better off

\[ \tau : \]
1. Were in coal before but in auto now \( x \leq 20 \)  
   \[ \Rightarrow \text{wage decreases by } 24 \]
   
   but "earn" their differential \( x \)  

   \[ \Rightarrow \text{worse off if } x < 24 \Rightarrow \text{all } \tau \text{ with } x \leq 20 \]

2. Were in auto before and still in auto after  
   \[ \Rightarrow \text{wage decreases by } 4 \Rightarrow \text{worse off} \]

\[ \text{for } \tau \]

all worse off
d) damages will consist of lost wages and lost surplus differential

\[ \text{no restriction}\{\text{wages paid to auto and coal workers}} \]
\[ \text{wages paid to coal workers in excess of equalizing differential}} \]
\[ \text{restriction}\{\text{wages paid to auto workers under labor market restriction}} \]

\[ \text{surplus with no restriction} = 126 \cdot 180 + \left[ \frac{126 \cdot (200)(20)}{420} \right] = 23,280 \]
\[ \text{surplus with restriction} = 126 \cdot 176 = 22,176 \]
\[ \Delta \text{surplus} = \text{damages} = 1104 \]

\[ d) \sigma^{2} \text{ are in the majority. We have already found that if } x > 24 \text{ men are worse off. Since less than half the } \sigma^{2} \text{ have } x > 24, \text{ the majority of } \sigma^{2} \text{ would vote in favor of the restriction.} \]