104. The prices of consumer goods in Los Angeles and Miami are different: some things are cheaper in Miami, while others are cheaper in Los Angeles. Suppose the price of every consumer good in Houston is exactly halfway between the Miami price of that good and the Los Angeles price. A market research firm surveys 1000 consumers, who have different preferences over consumption bundles, and different incomes. The consumers are asked to rank these three cities in terms of the consumption bundles that they could afford in each place. The result of the survey is that 450 consumers rank Los Angeles first, 350 rank Houston first and 200 rank Miami first. Is this consistent with the Weak Axiom of Revealed Preference?

105. Consider the utility function
\[ u = 2x_1^2 + 4x_2^{1/2} \]
(a) Find the demand functions for goods 1 and 2 as they depend on prices and wealth.
(b) Find the compensated demand function \( h(\cdot) \).
(c) Find the expenditure function, and verify that \( h(p, u) = \nabla_p e(p, u) \).
(d) Find the indirect utility function, and verify Roy’s identity.

106. Suppose that a preference ordering on \( \mathbb{R}_+^L \) can be represented by the utility function
\[ u(x) = \theta_L x_L x_1 + \sum_{i=1}^{L-1} \theta_i x_i x_{i+1} \]
where \( \theta \in \mathbb{R}_+^L 
(a) Is this preference ordering homothetic?
(b) Is this preference ordering separable? For example, if \( x_L \) is fixed at some level \( a \), the utility function defines a preference ordering on \( \mathbb{R}_+^{L-1} \), and if \( x_L \) is fixed at \( b \), the utility function defines another preference ordering on \( \mathbb{R}_+^{L-1} \). Are these two orderings actually the same?
(c) Find the Walrasian demand function.

107. Suppose that a preference ordering on \( \mathbb{R}_+^2 \) can be represented by the utility function
\[ u(x) = \frac{1}{2} x_1^2 + \ln(x_2) \]
(a) Suppose the prices are \( p_1 = 2, p_2 = 1 \). If wealth is \( w = 4 \), what is the optimal consumption plan?
(b) Suppose that wealth increases to 5. Now what is the optimal consumption plan?
(c) Can you find a price vector and a wealth level such that \( x_1(p, w) = 1 \)?

108. Suppose that a consumer’s preference ordering on \( \mathbb{R}_+^L \) can be represented by the utility function

\[
u(x) = \sum_{\ell=1}^L \frac{\alpha_\ell (x_\ell - \delta_\ell)^{\rho_\ell} - 1}{\rho_\ell}
\]

where, for all \( \ell \), \( \rho_\ell < 1 \), \( \alpha_\ell > 0 \) and \( \delta_\ell \geq 0 \).

(a) Find the Frisch demand function.

(b) Show how the Walrasian demand function is obtained from the Frisch demand function.

(c) Suppose the parameters \( \rho_\ell, \alpha_\ell, \delta_\ell \) are given. Is it possible that this consumer chooses the same consumption plan from two different budget sets?

(d) Now suppose the assumption \( \rho_\ell < 1 \) is replaced by \( \rho_\ell \leq 1 \). How does this small modification affect the Frisch and Walrasian demand functions?