Private Information, Wage Bargaining and Employment Fluctuations

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1. Introduction

Shimer (2003) pointed out that the basic Mortensen-Pissarides (1994) model does not generate nearly enough volatility in unemployment and vacancies, for plausible parameter values. Hall (2003) argued that this problem can be fixed if the Nash bargaining component of the model is dropped: Hall assumed that wages are sticky in the sense that the wage level in a previous contract establishes a “social norm” that largely determines the wage in the next contract. In the absence of a theory of social norms, this solution effectively requires the introduction of a free parameter. The question in this paper is whether an extension of the Mortensen-Pissarides model to allow for informational rents can explain the volatility of unemployment in a more parsimonious way.

2. A Model of Sticky Wages with Private Information and Aggregate Shocks

The model is a simplified version of the model analyzed in Kennan (2003). A successful job match generates a surplus to be divided between the worker and the employer. The value of the worker’s output is modeled as a binary random variable whose realization (“L” for low or “H” for high) is observed privately by the employer when the match is made. The probability of drawing a high surplus is a publicly observed Markov pure jump process with two states (“b” for bad and “g” for good), and exit hazards $\lambda_b$ and $\lambda_g$. For simplicity, it is assumed that the flow surplus is always low in the bad state, while there is a positive probability $p$ of a high surplus in the good state. Thus the expected value of a match is higher when the aggregate state is good.

Job and worker flows are modeled in the standard way, following Mortensen and Pissarides (1994). When the joint continuation value from a match falls below the joint opportunity cost, the match is destroyed. The job destruction hazard rate is a constant, $\delta$, and there is a constant returns matching function that generates a flow of new matches $M(N_U,N_V)$ from unemployment and vacancy stocks $N_U$ and $N_V$. There is an infinitely elastic supply of potential vacancies, and the actual number of vacancies posted is such that the expected profit from a vacancy is zero.

The match surplus is divided in the following way. Either the employer or the worker is randomly selected to make an offer, and if this offer is rejected the match dissolves. Clearly, the employer’s offer will just match the worker’s reservation level, which is the value of searching for another match. The worker has two choices: an offer that exhausts the low surplus, with a sure acceptance, or an offer that exhausts the high surplus, with acceptance only if the high surplus has actually been realized. It is assumed that the parameters are such that the worker always finds it optimal to demand the low surplus.
The match surplus depends on whether the employer draws a high or low value from the output distribution, and it also depends on the aggregate state, because this affects the worker’s value of continued search. Let $S^b_L$ be the surplus when the output value is low, and the aggregate state is bad, and similarly for the other configurations. In the low-output state, the surplus values are determined by the following asset pricing equations

$$
\begin{align*}
    rS^b_L &= y_L - rU^b - \delta S^b_L + \lambda_b \left( S^g_L - S^b_L \right) \\
    rS^g_L &= y_L - rU^g - \delta S^g_L - \lambda_g \left( S^g_L - S^b_L \right)
\end{align*}
$$

where $U$ denotes an unmatched worker’s (state-contingent) continuation value. This implies

$$
\begin{align*}
    (r + \delta) S^b_L &= y_L - rU^b - \frac{r \lambda_b \Delta U}{r + \delta + \lambda_b + \lambda_g} \\
    (r + \delta) S^g_L &= y_L - rU^g + \frac{r \lambda_g \Delta U}{r + \delta + \lambda_b + \lambda_g}
\end{align*}
$$

where $\Delta U = U^g - U^b$. Similarly, for a high-output match, the surplus values are given by

$$
\begin{align*}
    (r + \delta) S^b_H &= y_H - rU^b - \frac{r \lambda_b \Delta U}{r + \delta + \lambda_b + \lambda_g} \\
    (r + \delta) S^g_H &= y_H - rU^g + \frac{r \lambda_g \Delta U}{r + \delta + \lambda_b + \lambda_g}
\end{align*}
$$

The effect of the aggregate state on the match surplus is given by

$$
S^g_L - S^b_L = S^g_H - S^b_H = -\frac{r \Delta U}{r + \delta + \lambda_b + \lambda_g}
$$
Thus if an unmatched worker has better prospects when the aggregate state is good, the match surplus for each output value is lower when the aggregate state is good. On the other hand the probability of drawing a high output value is better in the good aggregate state.

The effect of the output draw on the match surplus is given by

\[ S^b_H - S^b_L = S^g_H - S^g_L = \frac{\Delta y}{r + \delta} \]

where \( \Delta y = y_H - y_L \).

The rate at which unemployed workers find new matches is \( M(N_U, N_V)/N_U = m(\theta) \), where \( \theta = N_V/N_U \) represents market tightness, and \( m(\theta) = M(1, \theta) \). If the employer makes the offer when a match is made, the worker gets the reservation level \( U \) and the employer gets the whole surplus. If the worker makes the offer, the worker gets half of the low-output surplus, and the employer gets an informational rent if the match value is high. Thus an unmatched worker’s continuation values are determined by the asset pricing equations

\[
\begin{align*}
  rU^b &= w_0 + m(\theta^b) \frac{1}{2} S^b_L + \lambda_b (U^g - U^b) \\
  rU^g &= w_0 + m(\theta^g) \frac{1}{2} S^g_L - \lambda_g (U^g - U^b)
\end{align*}
\]

where \( w_0 \) is the flow value of unemployment (including unemployment benefits and the value of leisure). Thus

\[
\begin{align*}
  rU^b &= w_0 + \frac{\lambda_b}{r + \lambda_b + \lambda_g} m(\theta^g) \frac{1}{2} S^b_L + \frac{r + \lambda_g}{r + \lambda_b + \lambda_g} m(\theta^b) \frac{1}{2} S^b_L \\
  rU^g &= w_0 + \frac{\lambda_g}{r + \lambda_b + \lambda_g} m(\theta^g) \frac{1}{2} S^b_L + \frac{r + \lambda_b}{r + \lambda_b + \lambda_g} m(\theta^b) \frac{1}{2} S^g_L
\end{align*}
\]
Employers post new vacancies to the point where the net profit from doing so is zero. When a match is made, the employer gets an informational rent if the match value is high, and also gets half of the low-output surplus (in expectation). Thus the zero-profit conditions implied by free entry are

\[
0 = r\nu^b = -c + \frac{m(\theta^b)}{\theta^b} \sqrt{2} S_L^b
\]

\[
0 = r\nu^g = -c + \frac{m(\theta^g)}{\theta^g} \left( \frac{1}{2} S_L^g + p \frac{\Delta y}{r + \delta} \right)
\]

where \(c\) is the flow cost of maintaining a vacancy.

The model can be solved as follows. For given values of \(\theta^b\) and \(\theta^g\), the free entry conditions determine \(S_L^b\) and \(S_L^g\):

\[
S_L^b = 2c \frac{\theta^b}{m(\theta^b)}
\]

\[
S_L^g = 2c \frac{\theta^g}{m(\theta^g)} - \frac{2p \Delta y}{r + \delta}
\]

The asset pricing equations for the low-output surplus values can be rearranged to give \(U^b\) and \(U^g\) as linear functions of \(S_L^b\) and \(S_L^g\), so \(U^b\) and \(U^g\) can be expressed in terms of \(\theta^b\) and \(\theta^g\) as

\[
rU^b = \gamma_L - (r + \delta + \lambda^b) \left( \frac{2c \theta^b}{m(\theta^b)} \right) + \lambda^b \left( \frac{2c \theta^g}{m(\theta^g)} - \frac{2p \Delta y}{r + \delta} \right)
\]

\[
rU^g = \gamma_L - (r + \delta + \lambda^g) \left( \frac{2c \theta^g}{m(\theta^g)} - \frac{2p \Delta y}{r + \delta} \right) + \lambda^g \frac{2c \theta^b}{m(\theta^b)}
\]

Next the above equations for \(S_L^b\) and \(S_L^g\) can be substituted in the asset pricing equation for the unemployment continuation values, giving
After eliminating $U^b$ and $U^g$ and rearranging terms, this gives the following two equations determining $\theta^b$ and $\theta^g$

$$
\begin{align*}
 r U^b &= w_0 + c \theta^b + \frac{\lambda_b}{r + \lambda_b + \lambda_g} \left( c \theta^g - \frac{p \Delta y m(\theta^g)}{r + \delta} - c \theta^b \right) \\
 r U^g &= w_0 + c \theta^g - \frac{p \Delta y m(\theta^g)}{r + \delta} - \frac{\lambda_g}{r + \lambda_b + \lambda_g} \left( c \theta^g - \frac{p \Delta y m(\theta^g)}{r + \delta} - c \theta^b \right)
\end{align*}
$$

These equations can be rewritten as

$$
\begin{align*}
 \theta^b &= -\frac{\Omega}{m(\theta^b)} + \frac{2 \lambda_b \Omega \theta^g}{m(\theta^b)} + \left( \frac{y_L - w_0}{c} - \frac{2 \lambda_b \Omega p \Delta y}{c(r+\delta)} \right) \\
 \theta^g &= \frac{2 \lambda_g \Omega \theta^b}{m(\theta^b)} - \frac{(r+\delta + \lambda_b \Omega)2 \theta^b}{m(\theta^g)} + \left( \frac{y_L - w_0 + 2p \Delta y}{c} + \frac{2 \lambda_g \Omega + m(\theta^g)}{c(r+\delta)} \right) p \Delta y
\end{align*}
$$

where

$$
\Omega = \frac{2r+\delta + \lambda_b + \lambda_g}{r}
$$

These equations can be rewritten as

$$
\begin{align*}
 \frac{2 \lambda_b \Omega \theta^g}{m(\theta^b)} &= \theta^b + \frac{(r+\delta + \lambda_b \Omega)2 \theta^b}{m(\theta^b)} - \frac{y_L - w_0}{c} + \frac{2 \lambda_b \Omega p \Delta y}{c(r+\delta)} \\
 \frac{2 \lambda_g \Omega \theta^b}{m(\theta^b)} &= \theta^g + \frac{(r+\delta + \lambda_g \Omega)2 \theta^g}{m(\theta^b)} - \frac{y_L - w_0 + 2p \Delta y}{c} - \frac{2 \lambda_g \Omega + m(\theta^g)}{c(r+\delta)} p \Delta y
\end{align*}
$$

(14)
In the numerical examples considered below with \( m(\Theta) = a\sqrt{\Theta} \), these equations have a unique solution.
The first equation gives \( \sqrt{\Theta} \) as a quadratic function of \( \sqrt{\Theta} \), while the second gives \( \sqrt{\Theta} \) as a quadratic function of \( \sqrt{\Theta} \). Substituting the second equation into the first then gives an equation of the form \( \sqrt{\Theta} = h(\sqrt{\Theta}) \), where \( h \) is a quartic function, and this equation has only one real positive root in the numerical examples.

**Unemployment Volatility**

Standard parameter values are used as far as possible, following Shimer (2003a,b) and Hall (2003). The simplest choice for the matching function is a constant-returns Cobb-Douglas function that is symmetric in unemployment and vacancies. This implies \( m(\Theta) = a\sqrt{\Theta} \), and \( a \) is set at 6.8, per annum (Shimer uses \( a = 1.7 \) for quarterly data). The job destruction rate \( \delta \) is set at .42 per annum, so that the quarterly rate is 10\% (i.e. \( \exp(-.25\delta) = .1 \)). In the NBER postwar data, the average duration of a recession is 10 months, and the average duration of an expansion is 57 months. This implies that the exit hazards are \( \lambda_g = 12/57 \) and \( \lambda_b = 12/10 \).

The low output value is normalized to 1. Since all matches produce low output in the bad aggregate state, the aggregate output level in the bad state is also 1. The difference between high and low output then determines the variability of output. Let \( Y_b \) and \( Y_g \) denote aggregate state-contingent productivity levels. The invariant distribution has mass \( \lambda_b/\Lambda \) on the bad state, and \( \lambda_b/\Lambda \) on the good state, where \( \Lambda = \lambda^b + \lambda^g \). Expected productivity is

\[
\mu_Y = Y_b + \frac{\lambda_b}{\Lambda} \Delta Y
\]

where \( \Delta Y = Y_g - Y_g = p(y_H - y_L) \). The variance is given by

\[
\sigma_Y^2 = \frac{\lambda_b}{\Lambda} (Y_g - \mu_Y)^2 + \frac{\lambda_g}{\Lambda} (Y_b - \mu_Y)^2
\]

\[
= \frac{\lambda_b \lambda_g}{\Lambda^2} \Delta Y^2
\]
This implies

\[
\frac{\Delta Y}{\sigma_y} = \sqrt{\frac{\lambda_b}{\lambda_g}} + \frac{\lambda_g}{\lambda_b}
\]

If the process is symmetric, the standard deviation is half of the difference between \(Y_b\) and \(Y_g\). Otherwise, the standard deviation is less than half of the difference. If the ratio of the transition rates is far from 1, the standard deviation can be made arbitrarily small, for any fixed difference (because the process spends virtually all of its time in one state). Setting \(\Delta Y = .053\), with \(\lambda_g = 12/57\) and \(\lambda_b = 12/10\) and \(Y_b = 1\) gives \(\sigma_y/\mu_y = .018\), which matches the coefficient of variation of U.S. aggregate labor productivity, as reported by Shimer (2003).

The parameter values are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>matching function</td>
<td>(m(\theta))</td>
<td>6.8(\sqrt{\theta})</td>
<td>Shimer</td>
</tr>
<tr>
<td>recession exit hazard</td>
<td>(\lambda_b)</td>
<td>12/10</td>
<td>recession duration (10 months)</td>
</tr>
<tr>
<td>expansion exit hazard</td>
<td>(\lambda_g)</td>
<td>12/57</td>
<td>expansion duration (10 months)</td>
</tr>
<tr>
<td>unmatched flow payoff</td>
<td>(w_0)</td>
<td>0.4</td>
<td>Shimer</td>
</tr>
<tr>
<td>low output</td>
<td>(y_L)</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>informational rent</td>
<td>(p\Delta y)</td>
<td>.05</td>
<td>volatility of labor productivity</td>
</tr>
<tr>
<td>vacancy flow cost</td>
<td>(c)</td>
<td>.54</td>
<td>Shimer</td>
</tr>
<tr>
<td>separation rate</td>
<td>(\delta)</td>
<td>.42</td>
<td>Shimer (see text)</td>
</tr>
<tr>
<td>interest rate</td>
<td>(r)</td>
<td>.05</td>
<td></td>
</tr>
</tbody>
</table>

The steady-state unemployment levels are determined in the usual way as
The equilibrium values of $\theta_b$ and $\theta_g$ for the parameters in Table 1 are obtained from equation (14), which is evaluated as

\[
\begin{align*}
\frac{22008}{1615} \sqrt{\theta_b} &= \frac{17785}{1292} \sqrt{\theta_b} + \frac{244322}{13395} \\
\frac{14672}{6137} \sqrt{\theta_g} &= \frac{172729003}{155757060} \sqrt{\theta_g} - \frac{2393519}{509010}
\end{align*}
\]

Rewrite these equations as

\[
\begin{align*}
\frac{17785}{1292} \sqrt{\theta_b} &= \frac{22008}{1615} \sqrt{\theta_b} - \frac{244322}{13395} \\
\frac{172729003}{155757060} \sqrt{\theta_g} &= \frac{14672}{6137} \sqrt{\theta_g} + \frac{2393519}{509010}
\end{align*}
\]

This is an equation of the form $x = f(x)$, where $x = (\sqrt{\theta_b}, \sqrt{\theta_g})$, and $f$ is a function which is concave, and quasi-increasing, (meaning that $f_1$ and $f_2$ are both concave functions, with $f_1$ increasing in $x_2$, and $f_2$ increasing in $x_1$). Thus, by the uniqueness theorem in Kennan (2001), there is at most one positive solution. And there is a positive solution, at $(\sqrt{\theta_b} = .6207655033, \sqrt{\theta_g} = 1.993820223)$. The steady state unemployment levels are

\[
\begin{align*}
\frac{1 - \frac{1}{1 + \frac{m(\theta_b)}{\delta}}}{1 - \frac{1}{1 + \frac{m(\theta_g)}{\delta}}} &= 9.0\%, \quad \frac{1 - \frac{1}{1 + \frac{m(\theta_g)}{\delta}}}{1 - \frac{1}{1 + \frac{m(\theta_b)}{\delta}}} = 3.0\%
\end{align*}
\]

Thus the variability of the unemployment rate for these parameter values is clearly large enough to match the data.

To illustrate the importance of informational rents in generating this result, consider an alternative parameter set that matches the variance of aggregate productivity by letting the match surplus depend on the aggregate state, with no idiosyncratic variation. The parameter values are as in Table 1, but with $y_b = 1$, $y_g = 1.053$, and $\Delta y = 0$. The steady state unemployment rates in this case are $u_b^* = 6.5\%$, $u_g^* = 5.6\%$. 

8
References


