Ch. 7 Exercise: Solow Model

Model:
Consider the Solow growth model without population growth or technological change. The parameters of the model are given by \( s = 0.2 \) (savings rate) and \( \delta = 0.05 \) (depreciation rate). Let \( k \) denote capital per worker; \( y \) output per worker; \( c \) consumption per worker; \( i \) investment per worker.

a) Rewrite production function \( Y = K^{\frac{1}{3}}L^{\frac{2}{3}} \) in per-worker terms.

Divide both sides by \( L \) to get output per worker on the left-hand side.

\[
\frac{Y}{L} = \frac{K^{\frac{1}{3}}L^{\frac{2}{3}}}{L} = \left( \frac{K}{L} \right)^{\frac{1}{3}} = k^{\frac{1}{3}}
\]

b) Find the steady-state level of the capital stock, \( k_{ss} \).

Write the steady-state condition for the Solow model and solve for the steady-state level of the capital stock, \( k_{ss} \).

\[
sf(k_{ss}) = \delta k_{ss}
\]

\[
sk_{ss}^{\frac{1}{3}} = \delta k_{ss}
\]

\[
k_{ss}^{\frac{2}{3}} = \frac{s}{\delta}
\]

\[
k_{ss} = \left( \frac{s}{\delta} \right)^{3} = \left( \frac{0.2}{0.05} \right)^{3} = 8
\]

c) What is the “golden rule” level of \( k \) for this economy? Recall that the golden rule level of the capital stock \( k_{gr} \) maximizes consumption per worker in steady-state. Report your answer to two decimal places.
Write consumption per worker as a function of the capital stock in steady-state.

\[ c(k) = f(k) - \delta k \]

We are maximizing steady-state consumption; take the first-order condition with respect to \( k \).

\[ c'(k) = f'(k) - \delta = 0 \Rightarrow f'(k_{gr}) = \delta \]

\[ \frac{1}{3} k_{gr}^{-\frac{2}{3}} = \delta \]

As per the question, report your numerical answer to two decimal places.

\[ k_{gr} = \left( \frac{1}{3\delta} \right)^{\frac{3}{2}} = \left( \frac{1}{3(0.05)} \right)^{\frac{3}{2}} = 17.21 \]

d) Let’s say that a benevolent social planner wishes to obtain \( k = k_{gr} \) in steady-state. What is the associated savings rate \( s_{gr} \) that must be imposed by the social planner to support \( k_{gr} \)?

To find the associated savings rate \( s_{gr} \), solve for \( s \) in the law of motion \( \Delta k = 0 \) (steady-state again) provided that \( k = k_{gr} \).

\[ \Delta k = s_{gr} f(k_{gr}) - \delta k_{gr} = 0 \]

Use the exact value for \( k_{gr} \) here; you will introduce error into your answer for \( s_{gr} \) if \( k_{gr} \) is rounded or truncated.

\[ s_{gr} = \frac{\delta k_{gr}}{f(k_{gr})} = \delta \left( \frac{1}{3\delta} \right)^{\frac{3}{2}} = \frac{1}{3} \]

Therefore, we conclude that the welfare-maximizing social planner sets \( s = \frac{1}{3} \).

e) Compare your result in the previous part with the assumed savings rate \( s \). To obtain \( k_{gr} \), do citizens need to save more or less?

\[ s = 0.2; \ s_{gr} = \frac{1}{3} \]
$s_{gr} > s$

Citizens need to save more under the “golden rule” policy regime, which implies that consumption per worker decreases in the short-run to obtain a long-run improvement in the standard of living (as measured by consumption per worker).

f) Plot the following on a single graph: $y = f(k)$, $\delta k$, $s f(k)$, and $s_{gr} f(k)$. Does the savings curve pivot up or down, relative to its initial position, when the planner’s $s_{gr}$ is implemented?

![Graph showing output, savings, depreciated capital per worker]

Relative to the $s = 0.2$ case, the savings curve pivots up as $s_{gr}$ is implemented by the social planner.

g) Discuss two to three economic policies that could help the social planner implement $s_{gr}$ in a real-world situation.

1. Savings tax credit for consumers.
2. Tax on consumer goods.
3. Social security or any type of mandatory savings program.
4. Education on the long-term benefits of saving for retirement.
5. Contractionary monetary policy $\Rightarrow$ real interest rate increases $\Rightarrow$ increased level of savings.