Chapter 11: Applying the IS / LM Model

Consider the IS / LM model.
Consumption function:
\[ C = a + b(Y - T) \] (1)
Investment function:
\[ I = c - dr \] (2)
Real money demand:
\[ L(r, Y) = l_1 Y - l_2 r \] (3)
Parameters:
\( a > 0, \ 0 < b < 1, \ c > 0, \ d > 0, \ l_1 > 0, \ l_2 > 0 \)

Given the information above, please answer the following questions:

a) Given equations (1) and (2), solve for \( Y \) as a function of \( r, \ G, \ T \), and parameters (IS curve).
\[
Y = C + I + G \\
Y = (a + b(Y - T)) + (c - dr) + G \\
Y(1 - b) = a - bT + c - dr + G \\
Y = \frac{1}{1 - b}[a - bT + c - dr + G]
\]

b) How does the slope of the IS curve depend on \( d \), the interest rate sensitivity of investment?
\[
m_{IS} \equiv \frac{\partial Y}{\partial r} = \frac{-d}{1 - b} \\
\frac{\partial}{\partial d} m_{IS} = -\frac{1}{1 - b} < 0
\]

c) Which will cause a larger horizontal shift in the IS curve, a $100 tax cut or a $100 increase in government spending?
\[
|\frac{\partial Y}{\partial T}| = |\frac{-b}{1 - b}| = \frac{b}{1 - b} \\
|\frac{\partial Y}{\partial G}| = |\frac{1}{1 - b}| = \frac{1}{1 - b} \\
0 < b < 1 \Rightarrow |\frac{\partial Y}{\partial G}| > |\frac{\partial Y}{\partial T}|
\]

Provided that \( \Delta G = -\Delta T \), \( \Delta G \) shifts the IS curve more than \( \Delta T \).
d) Given equation (3), solve for $r$ as a function of $Y$, $M$, $P$, and parameters (LM curve).

\[
\left( \frac{M}{P} \right)^d = L(r, Y) = l_1 Y - l_2 r
\]
\[
\left( \frac{M}{P} \right) = \frac{M}{P}
\]
\[
\left( \frac{M}{P} \right)^d = \left( \frac{M}{P} \right)^s \Rightarrow l_1 Y - l_2 r = \frac{M}{P}
\]
\[
r = \frac{1}{l_2} [l_1 Y - \frac{M}{P}]
\]

e) Using your answer from the previous part, how does the slope of the LM curve depend on $l_2$, the interest rate sensitivity of real money demand?

\[
m_{LM} = \frac{\partial r}{\partial Y} = \frac{l_1}{l_2}
\]
\[
\frac{\partial}{\partial l_2} m_{LM} = -\frac{l_1}{(l_2)^2} < 0
\]

$l_2 \uparrow \Rightarrow m_{LM} \downarrow$ (flatter)

$l_2 \uparrow \Rightarrow$ money demand more sensitive to real interest rates $\Rightarrow r$ responds less strongly to changes in income to achieve money market equilibrium; with some $\Delta Y$, smaller $\Delta r$ needed to return to equilibrium

e) How does the size of the shift in the LM curve resulting from a $100$ increase in $M$ depend on $l_1$? What about $l_2$?

\[
\mu = \frac{\partial r}{\partial M} = -\frac{1}{P l_2}
\]
\[
\frac{\partial}{\partial l_1} \mu = 0
\]
\[
\frac{\partial}{\partial l_2} \mu = \frac{1}{P (l_2)^2} > 0
\]
g) Use your answers from parts (a) and (d) to derive an expression for the aggregate demand curve. You should solve for \( Y \) as a function of \( P \), \( M \), \( G \), \( T \), and parameters; the resulting expression should not depend on \( r \).

From part (a):

\[
Y = \frac{1}{1-b} [a - bT + c - dr + G]
\]

From part (d):

\[
r = \frac{1}{l_2} [l_1 Y - \frac{M}{P}]
\]

AD curve (substitute LM curve into IS curve):

\[
Y = \frac{1}{1-b} [a - bT + c - \frac{d}{l_2} (l_1 Y - \frac{M}{P}) + G]
\]

\[
Y(1 + \frac{dl_1}{l_2} - b) = a - bT + c + \frac{dM}{Pl_2} + G
\]

\[
Y = \frac{1}{1 + \frac{dl_1}{l_2} - b} [a - bT + c + \frac{dM}{Pl_2} + G]
\]

h) Using your answer from the previous part, show that the aggregate demand curve is downward-sloping (negative slope).

\[
\frac{\partial Y}{\partial P} = -\frac{dM}{(1 + \frac{dl_1}{l_2} - b)Pl_2} < 0
\]

\( \frac{\partial Y}{\partial P} < 0 \Rightarrow \text{aggregate demand curve is downward-sloping} \)

i) Use your answer from part (g) to show that increases in \( G \) and \( M \), and decreases in \( T \), shift the aggregate demand curve to the right. How does this result change if parameter \( l_2 = 0 \) (real money demand does not depend on the real interest rate)?

Case \( l_2 \neq 0 \):

\[
\frac{\partial Y}{\partial M} = \frac{d}{(1 + \frac{dl_1}{l_2} - b)Pl_2} = \frac{d}{P((1-b)l_2 + dl_1)} = \frac{1}{P(l_1 + \frac{1-b}{d}l_2)} > 0
\]

\[
\frac{\partial Y}{\partial G} = \frac{1}{1 + \frac{dl_1}{l_2} - b} = \frac{l_2}{dl_1 + (1-b)l_2} > 0
\]

\[
\frac{\partial Y}{\partial T} = \frac{-b}{1 + \frac{dl_1}{l_2} - b} = \frac{-bl_2}{dl_1 + (1-b)l_2} < 0
\]

Case \( l_2 \to 0 \):

\[
\frac{\partial Y}{\partial M} \to \frac{1}{Pl_1} > 0
\]

\[
\frac{\partial Y}{\partial G} \to 0
\]

\[
\frac{\partial Y}{\partial T} \to 0
\]