Having pointed out yesterday that the strategies posited in Milgrom and Weber (1999) for bidding in sequential English (ascending) auctions are not an equilibrium, I thought it would be worth showing what is an equilibrium in that setting.

First, recall the symmetric equilibrium in the “one-shot” $k$-unit English auction. At the stage where there are $m+1$ bidders still active, and where the other $n-m-1$ bidders have revealed their signals to be $y_{m+1}, y_{m+2}, \ldots, y_{n-1}$ by when they dropped out, a bidder with signal $x$ drops out at the price

$$d_m(x; y_{m+1}, \ldots, y_{n-1}) = E(V_1|X_1 = x, Y_k = x, Y_{k+1} = x, \ldots, Y_m = x, (Y_{m+1}, \ldots, Y_{n-1}) = (y_{m+1}, \ldots, y_{n-1}))$$

That is, your dropout point is your expected valuation, conditional on

- $k-1$ of your opponents having signals higher than yours
- the other $m-k+1$ of your opponents have signals equal to yours
- the true types revealed by the dropouts

Note that $d_m$ need not be defined for $m < k$, since at this point, the auction would be over and bidding strategies would be irrelevant; I think this is where the confusion lies.

Now consider a sequence of $k$ single-unit English auctions. The following, I believe, is an equilibrium:

In the $l^{th}$ auction ($l = 1, 2, \ldots, k$), while $m+1$ bidders are still active in that auction and $n-m-l$ bidders have dropped out at prices revealing types $y_{m+l}, \ldots, y_{n-1}$,

- If $m+l > k$, stay in until the price reaches $d_{m+l-1}(x; y_{m+l}, \ldots, y_{n-1})$
- If $m+l \leq k$, drop out immediately

That is, as long as the number of active bidders, $m+1$, is greater than the number of unclaimed prizes, $k-(l-1)$, bid like you would in the ascending auction. Once the number of active bidders falls to the number of unclaimed prizes, everyone remaining drops out immediately, leaving the prize to one of them (presumably chosen at random); then rinse and repeat.

Under these strategies, the “winners” learn they will be winners when the $k+1^{st}$ bidder drops out in the first auction; since they all drop out immediately, their types are not revealed. In each successive auction, the “losers” drop out at the same point as they did in the first one, and so the price paid is the same in each auction.

You can verify that these are indeed equilibrium strategies, and that they seem “sensible” when applied to the special case of IPV.