So. The first half of this course was basically using game theory to model what happens once an auction begins – that is, taking the environment as given, and assuming everyone plays by the rules, what do the rules of the auction suggest about bidder strategies and outcomes.

Paul Milgrom said that one of the most important lessons he took away from all his time studying auctions was that the game is always bigger than you think. The last couple of lectures, we’ve begun enlarging the game to include not only the auction itself, but what happens before the auction. Last Thursday, we looked at information acquisition; this Tuesday, we looked at endogenous entry.

Today, we go back inside the auction itself, and consider what happens when players don’t follow the rules. That is, we relax the assumption that bidders each independently play according to their own self-interest, and allow for the possibility that they collude.

Obviously, collusion is more important in some settings than in others. We don’t worry about two bidders colluding in an eBay auction – if two friends decide not to bid against each other in an auction for a digital camera they both want, it doesn’t really matter, because there are lots of other bidders. Anonymity will make it almost impossible to collude with all the bidders interested in a particular auction, so the problem pretty much goes away.

On the other hand, there are some settings where the universe of potential bidders is small, they all know each other, and they expect to face each other over and over in a large number of auctions. This is the case with timber auctions – there are a large number of auctions for logging rights held every year, with the same competitors facing each other over and over. Similarly with government procurement auctions – reverse-auctions to provide a service, with the lowest price winning.

In these settings, we should take seriously the possibility that bidders collude, either explicitly or tacitly.

Hendricks and Porter (1989), “Collusion in Auctions,” point out that from 1979 to 1988, over four-fifths of the cases filed by the U.S. DOJ’s under the section 1 of the Sherman Antitrust Act were in auction markets.
Collusion in auctions could occur in one of several ways. For example:

- The colluding bidders could agree to all bid the exact same amount, say, the reserve price, and rely on the seller to randomly give the object to one of them

- (McAfee and McMillan (1992) show this is the optimal collusion mechanism for “weak cartels,” cartels which are unable to make cash transfers among themselves)

- Or the colluding bidders could agree to have one of them submit a low winning bid and the others bid still lower

- With common values, any rule for picking the designated winner will work; with private values, the bidders must decide among themselves who values the object most, which can be done by holding a second, private auction either before or after the “real” auction. (Such an auction is referred to as a “knockout” auction.)

Of course, since colluding bidders submit bids that are lower than their static best-responses to each others’ bids, they also need some means of punishing each other for deviating from the collusive arrangement. This could be outside of the auction framework (the organized crime approach), or it could be via a “grim trigger”-type punishment in a repeated-games framework.

A simple example will help...

**Example**

I’m guessing you all remember the idea of tacit collusion in a repeated game, but let’s do a simple example to see how it would work in an auction. Consider a simple common-value auction with $n$ bidders: $t_i$ are independently $U[0, 1]$, and $V = \sum_i t_i$. Suppose the seller is holding a first-price auction with no reserve price, and that the same auction is to be repeated infinitely many times, with independent draws of $t_i$ each time, and a discount factor of $\delta$.

The equilibrium in the first price auction is symmetric, so a bidder with type $t_i$ wins with probability $t_i^{n-1}$, so by the envelope theorem,

$$V_i(t_i) = \int_0^{t_i} s^{n-1} ds = \frac{1}{n} t_i^n$$

Taking the expectation over $t_i$,

$$\pi_i = \int_0^1 \frac{1}{n} t_i^n dt_i = \frac{1}{n(n+1)}$$

Suppose that both bidders’ bids were to be announced publicly after the auction, and that in the event of a tie, the winner would be chosen at random. Suppose the two bidders decided to collude by all bidding 0, letting the seller pick the winner randomly. If anyone ever deviated from this, they would revert to playing the Nash equilibrium thereafter.
Since the object is worth an average of $\frac{n}{2}$, colluding like this leads to an expected payoff of $\frac{1}{2}$ in every period, since each bidder wins with probability $\frac{1}{n}$ and gets an object for free which is on average worth $\frac{n}{2}$.

The most either bidder would ever be tempted to deviate was when he got the highest possible signal, 1, and considered outbidding his opponent by a minimal amount, $\epsilon$, to win with probability 1 instead of probability $\frac{1}{n}$. Given $t_i = 1$, the object is worth, in expectation, $1 + \frac{n-1}{2} = \frac{n+1}{2}$. So colluding is an equilibrium as long as

$$\frac{n+1}{2} + \frac{1}{2} (\delta + \delta^2 + \ldots) \geq \frac{n+1}{2} + \frac{1}{n(n+1)} (\delta + \delta^2 + \ldots)$$

Multiplying through by $2n$ gives

$$n + 1 + n (\delta + \delta^2 + \ldots) \geq n(n+1) + \frac{2}{n+1} (\delta + \delta^2 + \ldots)$$

or

$$\left(\frac{n - \frac{2}{n+1}}{n+1}\right) \frac{\delta}{1 - \delta} \geq (n+1)(n-1)$$

$$\left(\frac{n^2 + n - 2}{n+1}\right) \frac{\delta}{1 - \delta} \geq (n+1)(n-1)$$

$$\left(\frac{(n-1)(n+2)}{n+1}\right) \frac{\delta}{1 - \delta} \geq (n+1)(n-1)$$

$$\left(\frac{n+2}{n+1}\right) \frac{\delta}{1 - \delta} \geq (n+1)$$

$$\delta(n+2) \geq (1 - \delta)(n+1)^2$$

$$\delta \left((n+1)^2 + (n+2)\right) \geq (n+1)^2$$

$$\delta \geq \frac{(n+1)^2}{(n+1)^2 + (n+2)}$$

So for $\delta$ sufficiently big, tacit collusion in the repeated auction is sustainable as an equilibrium; but not surprisingly, collusion is harder to sustain for higher $n$, since the short-term gains from deviating are so much larger.

**Some Easy Conclusions**

A number of simple conclusions fall out of this example will a bit of thought.

- Sealed-bid auctions are more robust to collusion than ascending auctions. This is because in ascending auctions, a deviating bidder is noticed immediately, and can be punished immediately, so his gains from deviating can be made very small. In a sealed-bid auction, a deviating bidder could at least win that one auction at a low price, so the temptation to deviate is much greater; or, to put it another way, punishment is delayed, and therefore less effective.
• If the seller releases less information after the auction, it makes it harder for the bidders to sustain collusion. Announcing the winner and the winning bid allows the colluding bidders to verify that the winner did what he was supposed to do. Announcing only the identity of the winner (keeping the winning bid secret) is disruptive when bidders were colluding via the first strategy, since it is unclear whether the winner was chosen at random or deviated to a higher bid. (Of course, in many government auctions, the seller is required by law to disclose the winning bid, to reduce the possibility of collusion between the seller and one of the bidders.)

• Similarly, choosing a tiebreaking rule other than randomization makes an auction more robust to that type of collusion. (Not quite on topic, but I believe this is how Rockefeller broke the railroad cartel. He was shipping a lot of steel, and the various railroads agreed among themselves to all charge him the same high rates. Rockefeller picked one of them at random and used them exclusively; the other railroads assumed that one had cut its rates, and the cartel crumbled.)

• Similarly, if the bidders are colluding by pre-selecting the winner and allowing the others to knowingly submit losing bids, it may be optimal for the seller to sometimes award the object to one of the losing bidders.

• (Again, in both these last two cases, when it is a government auction, certain details in how it’s run may be mandated by law. However, a private seller is free to do what they want. Not randomizing among tied bids costs nothing; awarding the object to the second-highest bidder costs little if the bids are close together, and might only have to be done once in a while to disrupt collusion.)

• Also, if collusion is suspected, a higher reserve price may be optimal, since it is much more likely the sale will conclude at the reserve price.

One of these observations was that ascending auctions are worse (from a collusion standpoint) than sealed-bid auctions. There’s another way in which this turns out to be true: in settings where bidders are free to name their own bids, bids can be used to communicate with other bidders, enabling collusion. Two cool examples of this come from spectrum auctions, and are discussed in the Klemperer book:

• In 1999, Germany used a simultaneous ascending auction to auction off 10 blocks of spectrum, with the rule that a new bid had to exceed the previous high bid by 10%. Mannesman opened the bidding by bidding 18.18 million deutschmarks on blocks 1-5 and 20 million on blocks 6-10. The only other credible bidder – T-Mobile – correctly took this as a signal that if they bid 10% more (20 million) on blocks 1-5 and didn’t bid on 6-10, the two would divide the market in that way and capture everything at the same low price; which was exactly the result.
In a 1996-97 US spectrum auction, U.S. West was competing with McLeod for lot number 378 – a license in Rochester MN. Most bids were in thousands of dollars, but at one point, US West bid $313,378 and $62,378 for licenses in Iowa where McLeod had been the only bidder. McLeod got the message that these bids in Iowa were “punishment” for competing in Rochester, outbid US West for the Iowa licenses and stopped bidding in Rochester.

In both cases, there was no explicit agreement to collude, but the auction structure allowed bidders to communicate with each other and reach an understanding in the middle of the auction.

Collusion using the strategy of all bidding the same amount is, of course, easy to detect. (In fact, a large number of sealed-bid government procurement auctions have indeed resulted in all bidders bidding the same amount to the dollar, which is very suspicious.) However, a more clever collusive arrangement – say, all bidders shading their bids downwards relative to equilibrium levels – is not necessarily detectable. A paper by Pat Bajari and Lixin Ye, “Detecting Collusion in Procurement Auctions,” shows that collusion among bidders is impossible to detect if information is not available about the competing firms’ costs. However, if data about the firms’ costs is available, they give an empirical method to test for collusion.

(In fact, McAfee and McMillan claim that most of the DOJ’s convictions for bid-rigging have come after one of the cartel members, unhappy with his share of the profits, turns in his co-conspirators.)

McAfee and McMillan point out that a successful cartel among bidders must overcome four challenges:

- They need a way to divide the profits – just like truthful revelation must be incentive-compatible in the auction itself, bidders will still have private information that must be shared within the cartel.
- Contracts to fix prices are generally illegal, and therefore some other way to enforce the agreement must be found.
- Collusion leads to higher profits, which makes an industry look more attractive to new entrants, which may destroy the collusive arrangement.
- And of course, the seller can take actions to disrupt the cartel.

M and M focus on the first problem, dividing the spoils, considering two separate cases: “weak cartels,” which can coordinate bids but cannot transfer money between members; and “strong cartels,” which can. They assume the auction is a first-price auction with a reserve price, and for the most part, assume that all bidders are a part of the cartel. They assume away the problem of enforcement – assuming either a repeated-game setting, or some other enforcement mechanism – but focus on a one-shot auction and consider the optimal mechanism for each type of cartel to choose a winner in each auction.

In a common-values setting, choosing a winner is unimportant, since the object is worth the same to anyone – the bidders can use any randomizing device to select who should bid high in each auction. (A bid-rigging arrangement in the electrical equipment market in the 1950s used the phases of the moon to coordinate on collusive bids!) So McAfee and McMillan focus on the private-values case. Specifically, they assume a symmetric IPV setting, where the bidders are all a part of a cartel, and the seller is planning a first-price auction with a reserve price and a randomized tie-breaking rule.

(They point out that the reserve price can be thought of either as an actual reserve price, or as “the lowest bid the bidders think they can get away with,” that is, the lowest winning bid the bidders think will not attract antitrust scrutiny.)

Weak Cartels

McAfee and McMillan first consider the case of weak cartels, a group of bidders who can coordinate their bids but cannot reallocate the object after the auction or transfer money between them. (These constraints may be due to antitrust regulators, or other reasons.) Once again, they assume a single auction, but assume that enforcement of a collusive agreement is not a problem. (This may seem strange, since repeated auctions seem like the most likely enforcement mechanism.) They assume that the identity of the winner and the
amount of the winning bid will be made public after the auction, and require that this be sufficient to know whether anyone has deviated from the collusive agreement.

The environment is symmetric IPV. The results on weak cartels are as follows:

- With weak cartels, the only way the cartel can allocate the object efficiently is to earn the profits they could get noncooperatively

- With weak cartels, if \( \frac{f(v)}{1-F(v)} \) is increasing, expected profits are maximized if bidders with values below \( r \) bid 0 and bidders with values above \( r \) all bid \( r \)

The proof of the first result is basically the envelope theorem. If the collusive arrangement is efficient, then a bidder with type \( t_i \) must win with probability \( F_{n-1}(t_i) \); the envelope theorem then specifies his expected profits, which are the same as under noncooperative bidding.

The proof of the second result assumes that the bidders use a direct-revelation mechanism among themselves to designate a winner. Without transfers, a bidder with type \( t_i < r \) can’t earn positive profits, so any mechanism where he gets the object is dominated by one where the object is not won and he gets 0. Bidder \( i \)’s expected profits, then, can be written as

\[
E\pi_i(v_i) = \int_r^v \pi_i(v_i)f(v_i)dv_i
\]

Integrating by parts with \( u = \pi_i \) and \( v = -(1 - F(s)) \) gives

\[
E\pi_i(v_i) = -\pi_i(v)(1 - F(v))\bigg|_{v=r}^{v=v} + \int_r^v (1 - F(v_i))E_{-i}h_i(v_i, v-i)dv_i
\]

where \( h_i(v_i, v-i) \) is the probability \( i \) gets the object given the type profile. (The fact that \( \pi_i' = E_{-i}h_i \) is exactly the envelope theorem.) Since \( \pi_i(r) = 0 \), the first term vanishes, and we’re left with

\[
E\pi_i(v_i) = \int_r^v \frac{1 - F(v_i)}{f(v_i)}E_{-i}h_i(v_i, v-i)f(v_i)dv_i
\]

Defining \( H(v_i) = \frac{1-F(v_i)}{f(v_i)} \), rewriting \( E_{-i}h_i \) as an integral over the other types, and then summing over all \( i \) ends up giving us

\[
E \sum_i \pi_i(v) = E \sum_i H(v_i)h_i(v)
\]

So the optimal collusive mechanism maximizes this function, subject to feasibility and incentive-compatibility, which requires that \( E_{-i}h_i(v_i, v-i) \) be nondecreasing in \( v_i \). (That this is necessary is pretty obvious; that it’s sufficient will follow from the fact that the optimal mechanism ends up being incentive-compatible.)

If we assume that \( \frac{f(v)}{1-F(v)} \) is increasing, as happens with most “normal-looking” distributions, then \( H \) is decreasing. So maximizing \( E \sum_i H(v_i)h_i(v) \) should require giving the object to the bidder with the lowest \( v_i \). But since incentive-compatibility requires the chance
of getting the object to be nondecreasing in the type you report, the best the group can do is to allocate the object randomly among the bidders who report \( v_i \geq r \). (McAfee and McMillan take a page of integrals to make this argument.)

So the best a weak cartel can do is to allow one of the bidders, chosen at random among those who report \( v_i \geq r \), to be the only serious bidder and win the object for \( r \). One way to accomplish this is for every bidder with \( v_i \geq r \) to bid \( r \), counting on the seller to randomize. There’s empirical evidence this happens sometimes – multiple papers (cited by M and M) document government auctions receiving lots of identical winning bids. Another way to accomplish this is with rotating bidding – as in the phases-of-the-moon arrangement, one bidder is chosen by rotation to be the “winning” bidder at price \( r \), if he so chooses; if he does not want to bid \( r \), the next bidder is chosen, and so on.

(Of course, if the cartel wanted to divide profits in some asymmetric way; or if the seller did not “cooperate” by randomizing; or if antitrust authorities got suspicious about all the identical bids, such a rotating arrangement would still work.)

McAfee and McMillan also point out that even for cartels that can transfer money between members, this type of collusive arrangement is optimal among mechanisms that give 0 profits to bidder with type \( v_i = r \). They discuss a setting with unlimited entry by low-value bidders whose values are known to be no higher than \( r \) – perhaps explaining why \( r \) was chosen as a reserve price in the first place. In order to keep these bidders from trying to get in on the cartel’s profits, the cartel would need to ensure that bidders with \( v_i \leq r \) get zero profits; with that constraint, the mechanism above is the best they can do.

**Strong Cartels**

The other case McAfee and McMillan consider is a “strong cartel,” a cartel that can exclude nonserious bidders and can transfer wealth between its members.

(The ability to exclude nonserious bidders is important, because the optimal mechanism for a strong cartel will have \( \pi_i(r) > 0 \), that is, all cartel members earn positive expected profits.)

The result on strong cartels is that, when transfers are allowed, there exists a mechanism the bidders can use to truthfully reveal who values the object the most; he is then assigned to be the only bidder, wins the object for \( r \), and in addition makes transfers to the other bidders.

Specifically, there exists an increasing function \( T \) such that the bidder with the highest type \( v \) pays \( T(v) \) in total – \( r \) for the object and \( \frac{1}{n-1}(T(v) - r) \) to each of the other bidders – and this turns out to be a valid direct revelation mechanism.

This can be accomplished by holding a first-price knockout auction among the bidders ahead of time, where each of them, in equilibrium, bids \( T(v_i) \). Then the “winner” wins the right to bid \( r \) in the real auction, and divides up \( T(v_i) - r \) among the other bidders. M and M calculate this function \( T \) and show it is an equilibrium.
Note that $T(v_i)$ is not the same as the bidder’s equilibrium noncooperative bid in a first-price auction: since losing bidders now make money, winning the object is less valuable, so bids are depressed. However, this game still has a strictly increasing equilibrium bidding function, it’s just a different one.

(As usual, incentive compatibility is equivalent to the envelope theorem, which can be used to calculate the function $T$.)

Of course, the “knockout” auction can occur either before or after the actual auction; any bidder can be randomly assigned to bid $r$ in the real auction and then resell the object to the highest-value cartel member using this mechanism after the fact. This type of arrangement has been documented in a number of different auction settings, including antiques, fish, and timber.

As it happens, the average transfer to each “loser” is exactly

$$\frac{1}{n} E(v^{(2)} - r | v^{(1)} \geq r)$$

which is also the amount by which the winner’s profits exceeds what he would have gotten (in expectation) in the noncooperative equilibrium. By revenue equivalence, the winner would expect to pay $E(v^{(2)})$ without collusion; by colluding, he instead pays $r$; this says that the additional surplus to the bidders is divided equally.

(M and M also point out that empirically, knockout auctions are sometimes held with sealed bids and sometimes held orally; but that unlike with usual second-price auctions, bidding truthfully is not a dominant strategy here. Since losers get paid more when the second-highest bid is higher, the second-highest bidder has an incentive to inflate his bid.)

**Incomplete Cartels**

McAfee and McMillan also consider the possibility of cartels that include some, but not all, of the bidders. Let $G$ denote the cumulative distribution of the highest bid among the bidders outside the cartel. Then the cartel will still use a knockout auction to ascertain who has the highest value, and then bid to maximize $(v - b)G(b)$ in the actual auction. They point out that this is the same problem the cartel would face if the reserve price was unknown and drawn from the distribution $G$.

When the nonmembers are aware of the cartel, what follows is basically an asymmetric auction, with one bidder (the cartel) having a value drawn from the distribution $F^k$, and $n - k$ bidders having values drawn from $F$. M and M abandon this problem as too hard, and instead characterize equilibrium in a special case, where each bidder’s value is either 0 or 1.

**The Seller’s Response**

Going back to complete cartels, in either setting – strong or weak cartel – the result is that nobody bids above $r$. (M and M cite a paper showing that in one particular region, in the
early 1980s, the winning bid in 30% of timber auctions was within 1% of the reserve price. Collusion works well among local firms, facing familiar competition with the same bidders; and the costs of transporting timber limited competition to local firms. Not surprisingly, in auctions in which “outsiders” bid as well, the winning bid was higher, on average 2 to 3 times the reserve price.

McAfee and McMillan discuss three ways the seller can respond when he suspects collusion:

- Raise the reserve price
- Keep the reserve price secret (in essence, making it stochastic)
- Interfering with the cartel’s enforcement mechanism

They show how to calculate the reserve price when operating against a cartel, which is higher than the optimal reserve price when bidders are not colluding. They give an example to show that with relatively few bidders, bidding noncooperatively against the optimal noncooperative reserve price gives higher bidder profits than colluding against the optimal collusive reserve price. (For $F$ uniform on $[0, 1]$, equilibrium bidding is better than detectable collusion when there are fewer than 9 bidders.)

(This may somewhat solve the problem, since we’d expect collusion to be harder to sustain for $n$ large; for $n$ small, it’s not profitable if the seller can respond by adjusting the reserve price!)

In terms of making the reserve price stochastic, they don’t analyze the problem, they just point out that if the seller’s value is not common knowledge or it’s unclear whether or not he expects collusion, making the optimal reserve price unclear, the bidders would at least have to communicate prior to the auction in order to coordinate their bids, making collusion riskier or more fragile.

Finally, as we discussed before, the seller can try to interfere with the cartel’s monitoring of its members. This might involve choosing a sealed-bid auction over an ascending auction, so that deviations by bidders are harder to detect; refusing to randomize among tying bids; or occasionally selling to the “wrong” bidder. Also, since in a repeated setting, the incentive to deviate is based on the magnitude of the short-term gains, a project should be sold as a single large contract, rather than broken up into several smaller contracts, when collusion is suspected.