Model Averaging, Asymptotic Risk, and Regressor Groups
Supplemental Appendix

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Simulation Details

The simulation programs were written in R and run under Windows Vista. The programs are available at http://www.ssc.wisc.edu/~bhansen/progs/progs.htm.

The results are presented graphically, with MSE displayed as a function of $R^2$. The value of $R^2$ was varied on the 19-point grid \{0.00, 0.05, 0.10, 0.15, ..., 0.90\}. For a fixed $\alpha$ and $R^2$ the value of $c$ was then determined as

$$c = \sqrt{\frac{R^2}{\sum_{j=1}^{M} j^{-2\alpha} (1 - R^2)}}$$

Given $c$, we then set $\beta_j = cj^{-\alpha}$ and

$$y_i = \beta_0 + \sum_{j=1}^{M} \beta_j x_{ji} + e_i$$

with $\beta_0 = 0$.

We varied $\alpha \in \{0, 1, 2, 3\}$ and $n \in \{50, 150, 400, 1000\}$.

The default model (Model 1) set the errors $e_i$ and regressors $x_{ji}$ as iid $N(0, 1)$ and set $M = 12$. The remaining models explored the deviations from these default settings.

We explored non-normal errors, heteroskedastic errors, correlated regressors, and $M = 24$.

All models were designed so that the error is conditionally mean zero and has an unconditional variance of one.

The results for model 1 and model 6 are calculated using 10,000 simulation replications. For models 2 through 5, the calculations used 2000 simulation replications.

1. Model 1: Normal Regression
   - $e_i \sim N(0, 1)$
   - uncorrelated regressors
   - $M = 12$

2. Model 2: Non-Normal Error
   - $e_i \sim \frac{4}{5}N\left(-\frac{1}{3}, \frac{5}{9}\right) + \frac{1}{5}N\left(\frac{4}{3}, \frac{5}{9}\right)$

3. Model 3: Heteroskedastic Error
   - $e_i \sim N\left(0, \frac{1}{2}\left(1 + x_{ji}^2\right)\right)$

4. Model 4: Correlated Regressors
   - $e_i \sim N(0, 1)$
   - $E\left(x_{ji}^2\right) = 1$, $E(x_{ji}x_{ki}) = 0.5$ for $j \neq k$
5. Model 5: Increased Number of Regressors

- $e_i \sim N(0, 1)$
- $M = 24$

6. Model 6: Autoregression

In the paper, the figures display the normalized MSE for the estimators MMA$_4$, MMA, Stein, Lasso, and BMA. Here, we also display the normalized MSE for the estimator SAIC and the MMA$_4$ estimator with the regressors ordered in reverse (from smallest to largest coefficients) and is labeled as “Reversed”.
Figure 1: Model 1: $\alpha = 0$

Figure 2: Model 1: $\alpha = 1$
Figure 3: Model 1: $\alpha = 2$

Figure 4: Model 1: $\alpha = 3$
Figure 5: Model 2: $\alpha = 0$

Figure 6: Model 2: $\alpha = 1$
Figure 7: Model 2: $\alpha = 2$

Figure 8: Model 2: $\alpha = 3$
Figure 9: Model 3: $\alpha = 0$

Figure 10: Model 3: $\alpha = 1$
Figure 11: Model 3: $\alpha = 2$

Figure 12: Model 3: $\alpha = 3$
Figure 13: Model 4: $\alpha = 0$

Figure 14: Model 4: $\alpha = 1$
Figure 15: Model 4: $\alpha = 2$

Figure 16: Model 4: $\alpha = 3$
Figure 17: Model 5: $\alpha = 0$

Figure 18: Model 5: $\alpha = 1$
Figure 19: Model 5: $\alpha = 2$

Figure 20: Model 4: $\alpha = 3$
Figure 21: Model 6: Autoregression