Econometrics 710
Final Exam, Spring 2011

The exam consists of one question, broken in several parts.

The model is

\[ y_i = x_i' \beta + e_i \]  
\[ E(z_i e_i) = 0 \] (1)

The dimensions are: \( x_i, z_i, \) and \( \beta \) are \( k \times 1, k > 1, \) and \( y_i \) and \( e_i \) are \( 1 \times 1. \) Let

\[ Q = \begin{bmatrix} Q_{xx} & Q_{xz} \\ Q_{zx} & Q_{zz} \end{bmatrix} = \begin{bmatrix} E(x_i x_i') & E(x_i z_i') \\ E(z_i x_i') & E(z_i z_i') \end{bmatrix} \]

Assume both \( Q_{xx} \) and \( Q_{xz} \) have full rank \( k. \)

Let \( \hat{\beta} \) be the least-squares estimate obtained by regressing \( y_i \) on \( x_i, \) and let \( \tilde{\beta} \) be the 2SLS estimator obtained by estimation of (1) using the instrument \( z_i. \)

1. Find

\[ \delta = \lim_{n \to \infty} (\hat{\beta} - \tilde{\beta}) \]

2. Suppose that in addition to (1) and (2),

\[ E(x_i e_i) = 0 \] (3)

Quite simply, what does this condition mean? What is \( \delta \) under this assumption?

3. Write the difference \( \hat{\beta} - \tilde{\beta} \) as a function of sample moments of \( x_i, z_i, \) and \( e_i. \)

4. Under (1)-(3), find the asymptotic distribution of

\[ \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \begin{pmatrix} x_i \\ z_i \end{pmatrix} e_i \]

as \( n \to \infty. \)

5. Under (1)-(3) find the asymptotic distribution of \( \sqrt{n} (\hat{\beta} - \tilde{\beta}) \) as \( n \to \infty. \)

6. Suppose that

\[ E(e_i^2|x_i, z_i) = \sigma^2 \] (4)

How does the asymptotic variance from question 5 simplify under (4)?

7. Propose an estimator of the asymptotic variance under (4).

8. Propose a test statistic for (3) under (4) and find its asymptotic distribution under the assumption that \( Q > 0 \)

9. Describe how to use this statistic to test the hypothesis that \( x_i \) is exogenous.

10. Extra Credit. Show where \( Q > 0 \) is used in the answer to question 8.