The dimensions are: $x_i$, $u_i$, and $\beta$ are $k \times 1$, $z_i$ is $\ell \times 1$ where $\ell \geq k > 1$, $\Pi$ is $\ell \times k$ and $y_i$ and $e_i$ are $1 \times 1$.

The difficulty in the problem is that $(y_i, x_i, z_i)$ are not jointly observed. Instead, we have two independent samples from the marginal distributions of $(y, z)$ and $(x, z)$:

- Sample 1: iid observations of $(y_i, z_i)$, $i = 1, \ldots, n$
- Sample 2: iid observations of $(x_j, z_j)$, $j = 1, \ldots, J$

You can imagine that you have two independent samples from the same joint distribution, but in the first sample $x_i$ is missing, and in the second sample $y_j$ is missing.

1. Write out the reduced form equations:
   (a) Write the reduced form equation for $y_i$ as a function of $z_i$, $\beta$, and $\Pi$.
   (b) Explicitly write the error in this reduced form as a function of the errors $e_i$ and $u_i$ and parameters.
   (c) Write the population parameter $\beta$ as a function of population moments of $(y_i, x_i, z_i, \Pi)$
   (d) Write the population parameter $\Pi$ as a function of population moments of $(y_i, x_i, z_i)$
   (e) What is the condition for identification of $\beta$?

2. Define $Q = E(z_i z_i')$.
   (a) Write out estimators $\hat{Q}$ and $\tilde{Q}$ for $Q$ using Sample 1 and Sample 2
   (b) Find the probability limit of $\hat{Q}$ as $n \to \infty$
   (c) Find the probability limit of $\tilde{Q}$ as $J \to \infty$
   (d) Are the probability limits in (b) and (c) the same?
   (e) Which estimator is more efficient?

3. Suppose you know $\Pi$. Find an estimator $\tilde{\beta}$ for $\beta$.
   Hint: Use the reduced form equation for $y_i$
   (a) Write out this estimator.
   (b) Which sample is used?
   (c) Show that $\tilde{\beta} \to p \beta$. Which sample size ($n$ or $J$) goes to infinity for this convergence?

4. Find an estimator $\hat{\Pi}$ for $\Pi$
   (a) Write out the estimator.
   (b) Which sample is used?
   (c) Show that $\hat{\Pi} \to p \Pi$. Which sample size ($n$ or $J$) goes to infinity for this convergence?

5. Put your answers to 2 and 3 together to find an estimator $\tilde{\beta}$ for $\beta$ when $\Pi$ is unknown.
   (a) Write down the estimator.
   (b) Show that $\tilde{\beta} \to p \beta$. What assumptions on $n$ and $J$ are required?