A random sample of 12 chemists yielded data on three variables: 1) \( y \equiv \text{salary} \); 2) \( x \equiv \text{number of publications} \); and 3) \( z \equiv \text{years since Ph.D.} \). The analysis of the data yielded the following quantities:

\[
\begin{align*}
\sum y_i^2 &= 431 \\
\sum x_i^2 &= 17 \\
\sum z_i^2 &= 1588 \\
\sum x_i y_i &= 61 \\
\sum x_i z_i &= 107 \\
\sum z_i y_i &= 783
\end{align*}
\]

where all variables are measured in deviation units. On the basis of these quantities the following regression models were fitted:

Model 1: \( \hat{y}_i = 0.877 x_i + 0.434 z_i \)

Model 2: \( \hat{y}_i = 3.59 x_i \)  
\( \text{SSREG} = 219 \quad \text{SSRESID} = 212 \)

Model 3: \( \hat{y}_i = 0.493 z_i \)  
\( \text{SSREG} = 386 \quad \text{SSRESID} = 45 \)

Model 4: \( \hat{x}_i = 0.67 z_i \)  
\( \text{SSREG} = 7.2 \quad \text{SSRESID} = 9.8 \)

Model 5: \( \hat{z}_i = 6.29 x_i \)  
\( \text{SSREG} = 673 \quad \text{SSRESID} = 915 \)

For the purpose of computing degrees of freedom, all models have intercept terms (I just brought them over to the left-side of the equation when I put \( Y \) in deviation units.)

The following questions pertain to Model 3.

1. For Model 3, find the standard error of \( \hat{\beta}_{yz} \).

2. For Model 3, find the value of the test statistic for testing the null hypothesis \( \beta_{yz} = 0.51 \).

3. Construct the 95 percent confidence interval for \( \beta_{yz} \).

The following questions pertain to Model 1:

4. For Model 1, estimate the disturbance variance \( \sigma^2[y|x, z] \).

5. For Model 1, what proportion of the variation in \( y \) is “explained” by the regression?

6. For Model 1, test the null hypothesis \( \beta_{yx,z} = \beta_{yz,x} = 0 \) against the alternative that not both are zero. Fix \( \alpha \) at 0.05.

7. For Model 1, estimate the standard error \( \sigma_{\hat{\beta}_{yz,x}} \) of \( \hat{\beta}_{yz,x} \).

8. For Model 1, test the hypothesis \( \beta_{yz,x} = 0.40 \) against the two-sided alternative (\( \alpha = 0.05 \)).

9. For Model 1, do an F-test of the null hypothesis \( \beta_{yx,z} = 0 \) against the two-sided alternative (\( \alpha = 0.05 \)).
10. Explain and numerically account for the difference in the simple regression coefficient of \( z \) in Model 3 and the partial regression coefficient of \( z \) in Model 1.

This is the end of questions pertaining to the data on chemists.

11. A researcher with \( n=20 \) observations on \( y, x, z, r \) and \( v \) fits the following models with the specified results:

\[
\begin{align*}
\text{Model} & \quad \text{SS} & \\
1 & y=f(x, z, v, r) & \text{SSreg}=200 \quad \text{SSresid}=75 \\
2 & y = f(x, z) & \text{SSreg}=170 \quad \text{SSresid}=105 \\
3 & y = f(x) & \text{SSreg}=120 \quad \text{SSresid}=155 \\
4 & y = f(z,v) & \text{SSreg}=105 \quad \text{SSresid}=170 \\
5 & y=f(r, (x+z)) & \text{SSresid}=104 \\
6 & y= f(x,z,r) & \text{SSreg}=175 \quad \text{SSresid}=100 \\
7 & (y+3v)= f(x, z, r) & \text{SSreg}=320 \quad \text{SSresid}=90
\end{align*}
\]

Compute the relevant test statistic (you don’t actually have to arrive at a conclusion about rejecting or not) for each of the following hypotheses:

a. In the model \( y = f(x, z, v, r) \), the coefficients of \( v \) and \( r \) are simultaneously zero against the alternative “not both zero.”

b. \( H_0 : \beta_{yz.xr} = \beta_{yr.xz} = 0 \) vs. “not both zero”.

c. \( \beta_{yz.xr} = \beta_{yz.zr} \) vs. not equal.

d. \( \beta_{yx.zxr} = -3 \)

12. Suppose the true linear regression model for \( y \) is \( y=f(x,v) \), but the researcher, who does not have data on \( v \), fits \( y=f(x) \). If you have data on \( y, x, \) and \( v \), what would you look at to determine if the coefficient of \( x \) in the researcher’s model is biased?

13. Indicate whether the following statements are true or false.

a. Multicollinearity among regressors in a model is an important source of bias in least-squares estimators of the partial regression coefficients.

b. The gauss-markov theorem states that under the assumptions of the classical linear regression model, the OLS estimator of the parameters has minimum variance among all unbiased estimators.