1. For the data on “month,” “leaded gas,” and “cord lead,” the model of interest is

\[ Y_i = \alpha + \beta X_i + \epsilon_i \]  

(1)

where \( Y \) is cord lead, \( X \) is leaded gas, and the disturbance \( \epsilon_i \) has mean 0 and variance \( \sigma^2(Y|X) = \sigma^2(\epsilon|x) \).

a. Write the population regression function that expresses the mean of \( Y \) as a function of \( X \).

b. Explain whether the following equation is correct: \( E(Y|X) = \alpha + \beta X + \epsilon \).

c. Estimate the conditional variance \( \sigma^2(Y|X) \). Is your estimator unbiased? Explain.

d. Find and interpret the “standard error of estimate.” What is the metric (i.e., measurement units/scale) of the standard error of estimate?

f. Suppose \( \beta = 0 \): what is the probability of observing \( \hat{\beta} > 1 \)?

g. Construct the 95% confidence interval for the regression coefficient \( \beta \).

h. At the .05 level of significance, do a t-test of the null hypothesis \( \beta = 0 \) against the two-sided alternative. Also test \( \beta = 1.6 \) against the alternative \( \beta < 1.6 \).

i. Carry out the appropriate F-test (\( \alpha \)-level .05) of the null model

\[ Y_i = \alpha + \epsilon_i \]  

(2)

against the alternative

\[ Y_i = \alpha + \beta X_i + \epsilon_i \]  

(3)

j. What is the relationship between the F-test in “i” and the t-test of \( \beta = 0 \) in problem “g”?

k. Give the point estimate and the 95% confidence interval estimate of the mean \( \mu(Y|X = 140) \).