A Model of Job Market Signaling with Discounting [1]

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ABSTRACT: The paper studies an extension of the traditional Spence [10] job market signaling game. Workers differ in two dimensions – discount factor as well as productivity and education cost – both of which are privately known only to the workers themselves. When the second dimension of idiosyncrasy is introduced, single crossing property between high and low productivities can be violated. In general, multiple sequential equilibria exist even after Cho and Kreps' [3] intuitive criterion is used for refinement. It is shown that if Grossman and Perry's [5] *perfect sequential equilibrium* concept is applied, an "essentially" *unique* equilibrium arises when it exists. Roughly, when discount factors do not differ too much, the unique equilibrium is still the least costly separating equilibrium for the high productivity types. But an "essentially" unique pooling equilibrium is selected by the refinement when discount factors are relatively far apart. The result shows the equivalence of outcomes of signaling and screening games in two dimensional information case.

[1] Microeconomics field paper, last updated Dec. 03, 2003. I am grateful to Larry Samuelson and William Sandholm for detailed comments and suggestions on the previous version. I also thank Theory Lunch participants at UW-Madison.

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1. Introduction

Interactions between privately informed and uninformed players have been long studied. The convention is to name those games where informed players move first "signal(l)ing" game and those with uninformed players moving first "screening" game. [3] Classical examples include the seminal job market signaling model by Spence [10] and the competitive insurance market screening model of Rothschild and Stiglitz [7] (henceforth, R-S). In the latter game with two risk types, it has been shown that if there is a (pure strategy) sequential equilibrium, it must be that competitive firms offer contracts which just separate the two risk types and break even. Equilibria do not exist when a pooling contract is preferred by both risk types and generates more profit than the above candidate equilibrium while the pooling contract itself cannot be an equilibrium. The necessary and sufficient condition for the equilibrium to exist is either the two types differ enough in risk or the prior probability of low risk types is not too high.

While in the R-S screening game there is usually a unique prediction of the outcome, the more complicated information structure of a corresponding signaling game dictates multiple sequential equilibria in general. However, if we have faith in the "intuitive criterion" refinement proposed by Cho and Kreps [3], the unique equilibrium selected, widely known as the "Riley outcome", coincides with the screening game equilibrium in a two type case. In a multiple type case, the selection power of intuitive criterion is rather weak, therefore Grossman and Perry's [5] (henceforth, G-P) "perfect sequential equilibrium" concept can be invoked to restore the equivalence of outcomes between the signaling game and the screening game. In both forms of the game then only the outcome that just separates all types can

[3] Although whether the first game form should be written as signaling or signalling has not yet been settled.

be a possible refined equilibrium. But non-existence problem rises with the stronger G-P refinement in the signaling game the same way there may not be an equilibrium in the screening game. [4]

The above results are built on two assumptions. First, the privately known information is of one dimension. In the job market signaling game, it is the workers' productivity and the corresponding education cost. And individuals differ only in their probability of loss in the insurance market model. Secondly, the "single-crossing property" is globally satisfied. In the education signaling model, this means more productive workers have lower marginal cost of education than the less productive workers everywhere, so the indifference curves of the two types can cross only once and in the right direction.

As illuminating and thought-provoking as Spence educational signaling game is, a richer model is needed because how smart (i.e. productive) a person can be is certainly not the only factor in the choice of education length. This paper studies a situation where workers may also differ in their patience. This can be interpreted as less patient workers come from poor families so they have the immediate need of supporting the family, and therefore are less capable of staying in school for a long time and delaying earning money even if they are of high productivity. In the paper, discounting is used to reflect this second dimension of idiosyncrasy among workers. Higher discount factors lower the cost of extending one's education. It is assumed that both the discount factor and the productivity are privately known only by the workers. Importantly, when the discount factor is added to the model, single-crossing property between high and low productivity types can no longer hold globally.

[4]For a detailed examination and a survey on the relationship between signaling and screening, see for example Stiglitz and Weiss [11] and Riley [6].

To begin, we derive the sequential equilibria satisfying the intuitive criterion of the above game. It is quite predictable that multiple equilibria will emerge, given the inability of the intuitive criterion to select a unique equilibrium even in multi-type one dimensional information signaling. Next, we apply the G-P concept which works very well in a multi-type environment of the basic signaling game. Our result is that the G-P concept can always pick up an "essentially" *unique* equilibrium if a refined equilibrium exists at all. Roughly, the selected equilibrium is still the least costly separation if the discount factors do not differ substantially, while pooling occurs when they are relatively far apart.

The paper that is most closely related to this one is Smart [9], which analyzed a screening model of competitive insurance markets with two unobservables. There risk aversion was introduced as the second dimension of private information and it relates to the tradition R-S model the same way the current paper relates to Spence job market signaling. The unique equilibrium in that model (when it exists) is either separating when the risk aversions differ mildly and pooling otherwise. Therefore, together with the current paper, the equivalence of the equilibrium outcomes in competitive screening and signaling games with two dimensions of private information is established if proper refinement is applied to the latter game form.

There have been some early studies on games with multidimensional private information. For example Quinzii and Rochet [8] and Engers [4] studied sufficient conditions for a break-even separating equilibrium to exist in a screening game. [5] Some of more recent papers on this topic are: Armstrong [1] studied a monopolistic screening problem with multidimensional

[5]Like the original Spence model, these early papers were kind of fuzzy on the information structure. Therefore it is more proper to characterize them as screening game models than signaling game models ,despite the word "signaling" in the titles of these papers.

preference of consumers; Chen [2] studied a signaling game with the privately informed party having two signals and two dimensions of private information and asked the question of which signal should be used to signal which information.

The paper is set out as follows: Section 2 describes the 2×2 private information job market signaling model, where workers possess different discount factors and productivity levels. Sequential equilibria surviving the intuitive criterion are characterized in section 3. Section 4 gives a numerical example. And in section 5 G-P's perfect sequentiality concept is applied to further refine the set of equilibria; we show that an "essentially" unique outcome is obtained when it exists. Section 6 briefly describes Smart's [9] competitive insurance market screening model and compares his equilibrium with the equilibrium of the current paper. Section 7 concludes the paper and all the figures are attached after the paper.

2. The Model

2.1. **Setup.**

Each worker is endowed with two dimensions of idiosyncrasies: a productivity level and a discount factor. We study the simplest case of 2×2 types. Denote education length by e, $e \in E = \mathbb{R}^+$. Production and cost are non-decreasing and increasing functions of education time. Let the two productivity functions be $p^1(e)$ and $p^2(e)$ and the corresponding education cost functions be $c^1(e)$ and $c^2(e)$, respectively. There are two discount factors $0 < \delta_1 < \delta_2 < 1$. Denote a worker's type by θ_{ij} with the first subscript being the indicator of her productivity and the second her discount factor, so a θ_{21} type denotes a type with productivity function $p^2(\cdot)$ and discount factor of δ_1. Let the type space be $\Theta = \{\theta_{11}, \theta_{12}, \theta_{21}, \theta_{22}\}$ and a generic element be

θ. Let the prior probability of type θ_{ij} be q_{ij} and $\sum_{i=1}^{2}\sum_{j=1}^{2} q_{ij} = 1$. Prior distribution of types is common knowledge among workers and firms.

Workers move first in the signaling game by choosing an education length, then competitive firms respond by offering a wage. This offer is influenced by their beliefs about the worker's type after observing her education choice. The worker's utility is the discounted highest wage offer w minus the education cost:

$$U_{ij}(w, e) = w\delta_j^e - c^i(e) \tag{1}$$

Here education cost is not discounted as it can be considered as how much a person suffers physically and mentally in the education duration, so it does not depend on how a person values future money.

At education level e, let a firm's posterior belief on worker's type be $\mu(e) = (\mu_{11}(e), \mu_{12}(e), \mu_{21}(e), \mu_{22}(e))$, $\sum_{i=1}^{2}\sum_{j=1}^{2} \mu_{ij}(e) = 1$. The profit of hiring a worker with production $p^i(e)$ by offering wage w at time e is:

$$\pi = p^i(e) - w \tag{2}$$

In addition, the following assumptions are made on the production and cost functions.

- A1. $p^i \in C^2$ and non-decreasing, concave, $i = 1, 2$. $0 < p^1(e) < p^2(e)$, and $p_e^1(e) \leq p_e^2(e), \forall\ e \geq 0$ (the subscript of a function denotes the derivative)

- A2. $c^i \in C^2$ and strictly increasing, convex quadratic functions, $i = 1, 2$. $c^1(0) = c^2(0)$ and $c^1(e) > c^2(e) > 0, \forall e > 0$. $c_e^1(e) > c_e^2(e)$ and $c_{ee}^1(e) = c_{ee}^2(e), \forall\ e \geq 0$

- A3. $p^i(0) > c^i(0)$, $p^i_e(0) + p^i(0)\ln(\delta_j) > c^i_e(0)$, $i = 1,2$ and $j = 1,2$

A1 is a fairly standard assumption on production; productions are non-decreasing functions of education length and type θ_2. are the high productivity types. A2 says high productivity types have lower education cost as well as lower marginal cost. Since time e joins the utility functions exponentially, a relatively restrictive assumption on the form of the cost functions is made to keep below exposition tractable, i.e. the cost functions are quadratic form and the two functions' second order derivatives are the same constant. Finally, A3 gives that at time $e = 0$, production is higher than cost for any productivity type and the discount adjusted marginal product is higher than marginal cost for any type θ_{ij}.

2.2. **Indifference Curves.**

On a diagram with time e on the horizontal axis, wage w on the vertical, worker's indifference curve $\bar{U}_{ij} = w\delta_j^e - c^i(e)$ has a slope of:

$$\frac{\partial w}{\partial e} = \frac{c^i_e(e)}{\delta_j^e} - w\ln\delta_j \tag{3}$$

Therefore, the slope is globally positive and increasing in e and w. We know that the slope of indifference curve measures the real cost of one additional unit of education; it depends on both the cost function $c^i(e)$ and the worker's patience for money δ_j. A flatter indifference curve would mean a relatively lower education cost. In the basic Spence model, we say that the single-crossing property holds if a high productivity worker's indifference curve is flatter than that of a low productivity worker at any point on the $e - w$ plane. Denote the slope of the indifference curves of type θ_{ij} at (e, w) by $S_{ij}(e, w)$. In the current model with two dimensional types, the rankings of the slopes of the indifference curves at given (e, w) can be

$$S_{11}(e,w) > S_{12}(e,w) \geq S_{21}(e,w) > S_{22}(e,w) \tag{4}$$

or

$$S_{11}(e,w) > S_{21}(e,w) \geq S_{12}(e,w) > S_{22}(e,w) \tag{5}$$

Therefore, single crossing property can be violated if the second expression is true. It is yet to determine where (4) and (5) hold on the plane respectively. Define:

$$S(e,w) \equiv S_{12}(e,w) - S_{21}(e,w), \quad e \geq 0, w \geq 0 \tag{6}$$

Lemma 1. *$S(e, w) = 0$ defines w as a continuously differentiable function of e, $\forall\ e \geq 0$. And there exist unique $e^o > 0$ and $w^o > 0$ such that:*

$$S(e^o, 0) = 0, \quad S(0, w^o) = 0$$

Proof. Since $0 < \delta_1 < \delta_2 < 1$, we have

$$\frac{\partial S}{\partial w} = -(\ln \delta_2 - \ln \delta_1) \neq 0$$

So by Implicit Function Theorem, $S(e, w) = 0$ defines wage w as a continuously differentiable function of education time e. Also solving $S(e, w) = 0$ gives

$$w = [\frac{c_e^1(e)}{\delta_2^e} - \frac{c_e^2(e)}{\delta_1^e}]/[\ln \delta_2 - \ln \delta_1] \tag{7}$$

It is obvious to see that

$$w^o = [c_e^1(0) - c_e^2(0)]/[\ln \delta_2 - \ln \delta_1]$$

For the existence and uniqueness of e^o, observe

$$S(e,0) = \frac{c_e^1(e)}{\delta_2^e} - \frac{c_e^2(e)}{\delta_1^e} = 0$$

$$\Leftrightarrow \quad \frac{c_e^1(e)}{c_e^2(e)} = (\frac{\delta_2}{\delta_1})^e$$

We have $c_e^1(0)/c_e^2(0) > 1 = (\delta_2/\delta_1)^0$; and $(c_e^1(e)/c_e^2(e))$ is strictly decreasing in e by assumption A2, while $(\delta_2/\delta_1)^e$ is strictly increasing in e. So there is a unique $e^o > 0$ such that $S(e^o, 0) = 0$. □

So the $S(e, w) = 0$ curve divides the plane into two regions and the following lemma tells us (4) and (5) each holds in one of them.

Lemma 2. *Let the implicit function $w(e)$ defined by $S(e, w) = 0$ be $w = f(e)$. Also define the following:*

$$GR \equiv \{\ (e, w) : w < f(e),\ e \geq 0, w \geq 0\ \}$$

$$BR \equiv \{\ (e, w) : w > f(e) \text{ or } e > e^o,\ e \geq 0, w \geq 0\ \}$$

then, we have (4) holds on the set GR and (5) on the set BR. And each indifference curve of any type θ_{ij} can cross $w = f(e)$ at most once, $i = 1, 2; j = 1, 2$.

Proof. Let (e, w) be a point on $S(e, w) = 0$, then for any $\tilde{w} < w$, we have:

$$\begin{aligned} S_{12}(e, \tilde{w}) - S_{21}(e, \tilde{w}) &= \frac{c_e^1(e)}{\delta_2^e} - \tilde{w}\ln\delta_2 - \frac{c_e^2(e)}{\delta_1^e} + \tilde{w}\ln\delta_1 \\ &= \frac{c_e^1(e)}{\delta_2^e} - \frac{c_e^2(e)}{\delta_1^e} - \tilde{w}(\ln\delta_2 - \ln\delta_1) \\ &> \frac{c_e^1(e)}{\delta_2^e} - \frac{c_e^2(e)}{\delta_1^e} - w(\ln\delta_2 - \ln\delta_1) \\ &= 0 \end{aligned}$$

So, (4) holds for $(e, \tilde{w})$. Similarly, it can be shown that (5) holds on BR.

To show each indifference curve can cross the curve $w = f(e)$ only once, first calculate the slope of $w = f(e)$:

$$\frac{\partial w}{\partial e} = -\frac{\partial S/\partial e}{\partial S/\partial w} = \frac{(\frac{c_{ee}^1(e)}{\delta_2^e} - \frac{c_{ee}^2(e)}{\delta_1^e}) + (\frac{c_e^2(e)}{\delta_1^e}\ln\delta_1 - \frac{c_e^1(e)}{\delta_e^2}\ln\delta_2)}{\ln\delta_2 - \ln\delta_1} \tag{8}$$

The sign of this slope is undetermined, but the following argument shows that when an indifference of any type crosses $w = f(e)$, it crosses $w = f(e)$

from below. Namely, indifference curves are steeper than $w = f(e)$ at any point where they cross. Since indifference curves are positive sloped everywhere, we just need to consider the case when (8) is positive. Moreover, it suffices to show the above claim for type θ_{22}, since θ_{22} has the smallest slope everywhere among all types. From equations (3), (7) and assumption A2, at any point on $w = f(e)$ we have

$$\begin{aligned} S_{22}(e,w) &= \frac{c_e^2(e)}{\delta_2^e} + \frac{\frac{c_e^2(e)}{\delta_1^e} - \frac{c_2^1(e)}{\delta_2^e}}{\ln\delta_2 - \ln\delta_1}\ln\delta_2 \\ &> \frac{\frac{c_e^2(e)}{\delta_1^e}\ln\delta_1 - \frac{c_e^1(e)}{\delta_2^e}\ln\delta_2}{\ln\delta_2 - \ln\delta_1} \\ &\geq \frac{(\frac{c_{ee}^1(e)}{\delta_2^e} - \frac{c_{ee}^2(e)}{\delta_1^e}) + (\frac{c_e^2(e)}{\delta_1^e}\ln\delta_1 - \frac{c_e^1(e)}{\delta_e^2}\ln\delta_2)}{\ln\delta_2 - \ln\delta_1} \end{aligned}$$

This proves the claim. Because indifference curves always cross $w = f(e)$ from below, each indifference curve can cross $w = f(e)$ at most once. $\square$

So Lemma 2 helps to separate the plane into GR (the "Good Region"), where the single crossing property between productivities holds and BR (the "Bad Region"), where the property collapses. Also, any point on $w = f(e)$ is a point of tangency of an indifference curve of type θ_{12} and one of type θ_{21}. Indifference curves of type θ_{12} and θ_{21} can potentially cross twice: once in the Good Region (in the "right" direction) and once in the Bad Region (in the "wrong" direction) (see Figure 1).

3. Sequential Equilibria with Cho-Kreps Refinement

In this section, we will try to characterize the equilibrium outcome of the game described in the previous section by appealing to the equilibrium concept of *sequentiality* and Cho-Kreps *intuitive* refinement. A worker's pure strategy is to choose a education length $e(\theta) \in E$ knowing privately his type

θ. Firms form beliefs $\mu(e)$ observing only the education choice of workers and their pure strategy is to offer wage $w(e,\mu) \in \mathbb{R}^+$.

3.1. **Sequentiality.**

Sequentiality requires that since firms have the same information, they should always have the same posterior belief on types on and off the equilibrium and competition dictates that all the firms simply bid the wage up to the expected productivity by the belief. So a (pure strategy) *Sequential Equilibrium* of the above game is a full characterization $\{\{e^*(\theta_{ij})\}_{i=1,2;j=1,2}, \mu^*(e), w^*(e,\mu^*(e))\}$ such that:

- Beliefs are consistent, i.e. beliefs are calculated from Bayes' Rule whenever possible [6].
- $w^*(e,\mu^*(e)) = p^1(e)(\mu^*_{11}(e)+\mu^*_{12}(e)) + p^2(e)(\mu^*_{21}(e)+\mu^*_{22}(e))$, $\forall e \geq 0$.
- $e^*(\theta_{ij}) \in \arg\max_{e\geq 0} \delta_j^e w^*(e,\mu^*(e)) - c^i(e)$, $i=1,2; j=1,2$.

In the current game, we say the sequential equilibrium is *separating* if $e^*(\theta_{1\cdot}) \neq e^*(\theta_{2\cdot})$, i.e. high productivity types do not pool with low productivity types. The equilibrium is *pooling* otherwise. To keep the exposition neat, we need the following notation:

- $\hat{p}(e) = q_{12}p^1(e) + q_{21}p^2(e)$
- $\tilde{p}(e) = (q_{11}+q_{12})p^1(e) + q_{21}p^2(e)$
- $\underline{e}_{ij} = \arg\max_{e\geq 0}\ p^1(e)\delta_j^e - c^i(e),\ i=1,2,\ j=1,2$
- $\underline{U}_{ij} = p^1(\underline{e}_{ij})\delta_j^{\underline{e}_{ij}} - c^i(\underline{e}_{ij}),\ i=1,2,\ j=1,2$
- $\bar{e}_{2j} = \arg\max_{e\geq 0}\ p^2(e)\delta_j^e - c^2(e),\ j=1,2$

In words, $\hat{p}(e)$ is the weighted sum of the productivities of type θ_{12} and θ_{21}; $\tilde{p}(e)$ is the weighted sum productivity of type θ_{11}, θ_{12} and θ_{21}. And $\underline{e}_{ij}$ is the time type θ_{ij} would choose if she was believed to be of low productivity and

[6]We know in signaling games, consistency does not put on further restrictions on beliefs off the equilibrium path.

the corresponding payoff $\underline{U}_{ij}$ is the minimal payoff for θ_{ij} in any sequential equilibrium. And $\bar{e}_{2j}$ are high productivity workers time choices if they successfully single themselves out from low productivity group. Notice the latter three are well defined by assumption A1-A3.

It is straightforward to show that $(\underline{e}_{12}, p^1(\underline{e}_{12})), (\underline{e}_{21}, p^1(\underline{e}_{21}))$ should both lie in GR or BR or on $S(e, w) = 0$.[7] Here we will concentrate on the case that they both belong to the Good Region:

- A4. $(\underline{e}_{12}, p^1(\underline{e}_{12})), (\underline{e}_{21}, p^1(\underline{e}_{21})) \in GR$

Essentially, I am assuming here the discount difference is not too substantial so the Good Region is big enough to include both $(\underline{e}_{12}, p^1(\underline{e}_{12}))$ and $(\underline{e}_{21}, p^1(\underline{e}_{21}))$. By this assumption, we have $(\underline{e}_{11}, p^1(\underline{e}_{11})) \in GR$ too and $\underline{e}_{11} < \underline{e}_{12} < \underline{e}_{21}$. [8] Moreover, to ensure the existence result in Proposition 7, we need to make one further assumption:

- A5. There exists e such that:

$$\delta_2^e \hat{p}(e) - c^1(e) > \underline{U}_{12}$$

$$\delta_1^e \hat{p}(e) - c^2(e) > \underline{U}_{21}$$

This assumption says there can potentially be pooling between θ_{12} and θ_{21}. As long as the ratio q_{21}/q_{12} is not too small, we can always find e such that point $(e, \hat{p}(e))$ gives both θ_{12} and θ_{21} higher payoffs than $\underline{U}_{12}$ and $\underline{U}_{21}$, respectively, i.e. A5 holds. All the subsequent results follow from

[7]To see this, suppose $(\underline{e}_{21}, p^1(\underline{e}_{21})) \in GR$, then at point $(\underline{e}_{21}, p^1(\underline{e}_{21}))$ there is an indifference curve of θ_{21} that is tangent with $w = p^1(e)$. Since $\underline{e}_{21} \in GR$, it must be that the indifference curve of θ_{12} which crosses the above point crosses $w = p^1(e)$ from below, therefore we must have $(\underline{e}_{12}, p^1(\underline{e}_{12})) \in GR$ and $\underline{e}_{12} < \underline{e}_{21}$. Similarly, it can be shown that if $(\underline{e}_{21}, p^1(\underline{e}_{21})) \in BR$, then $(\underline{e}_{12}, p^1(\underline{e}_{12}) \in BR$ and $\underline{e}_{21} < \underline{e}_{12}$. And if $(\underline{e}_{21}, p^1(\underline{e}_{21}))$ is on $S(e, w) = 0$, then $\underline{e}_{21} = \underline{e}_{12}$.

[8]Without A4, there can be one more type of equilibrium in Proposition 7: $\{\theta_{11}, \theta_{21}\}$ pooling.

assumptions A1-A5 and section 4 will give a numerical example to show all the assumptions can be satisfied.

Before appealing to the intuitive criterion, it is important to know the equilibrium prediction from sequentiality alone. There can be the following five types of equilibrium outcomes in terms of separation and pooling of workers' education choices:

- Separating equilibria
- $\{\theta_{12}, \theta_{21}\}$ pooling equilibria
- $\{\theta_{11}, \theta_{12}, \theta_{21}\}$ pooling equilibria
- $\{\theta_{12}, \theta_{21}, \theta_{22}\}$ pooling equilibria
- $\{\theta_{11}, \theta_{12}, \theta_{21}, \theta_{22}\}$ pooling equilibria

The remaining part of this section shows that the last two cases can never meet the intuitive criterion and the first three types of equilibrium outcomes can be further refined by the test. The key point is that any sequential equilibrium that involves type θ_{22} pooling with low productivity types fails the intuitive test, and the refinement also says that high productivity types that can separate themselves from low productivity types must separate by the least costly education time.

3.2. **Intuitive Criterion.**

For any sequential equilibrium, let U^*_{ij} be the equilibrium payoff of type θ_{ij}. To rule out "unbelievable" off-the-equilibrium beliefs, Cho-Kreps *intuitive* test catered to this game goes through the following procedure:

(i) For each unused message $e \neq e^*(\theta_{ij}),\ \forall i = 1, 2$ and $j = 1, 2$, let

$$D(e) = \{\theta_{ij} : U^*_{ij} > \delta^e_j p^2(e) - c^i(e)\}$$

(ii) Is it true that $\{\theta_{11}, \theta_{12}\} \subseteq D(e)$?

(iii) If so, is at least one of the following true? $U^*_{21} < \delta_1^e p^2(e) - c^2(e)$, $U^*_{22} < \delta_2^e p^2(e) - c^2(e)$

If there exists a e such that both (ii)[9] and (iii) are true, then the sequential equilibrium is said to violate the intuitive criterion. A series of lemmas are given below leading to the complete characterization of the sequential equilibria satisfying the intuitive criterion.

3.3. **Sequential Equilibria with Intuitive Criterion.**

The following lemma gives a necessary condition for a sequential equilibrium to satisfy the intuitive criterion.

Lemma 3. *A sequential equilibrium satisfies the intuitive criterion only if* $e^*(\theta_{22}) \neq e^*(\theta_{1j}),\ j = 1, 2.$

Proof. We will break down the proof to several cases:

(Case I) Suppose in a sequential equilibrium θ_{22} pools with types θ_{11} and θ_{12} at time e^*. Then the equilibrium payoffs of the three types are:

$$U^*_{11} = \delta_1^{e^*} p(e^*) - c^1(e^*)$$

$$U^*_{12} = \delta_2^{e^*} p(e^*) - c^1(e^*)$$

$$U^*_{22} = \delta_2^{e^*} p(e^*) - c^2(e^*)$$

where

$$p(e^*) = \begin{cases} (q_{11} + q_{12})p^1(e^*) + q_{22}p^2(e^*) & \text{if } e^*(\theta_{21}) \neq e^* \\ (q_{11} + q_{12})p^1(e^*) + (q_{21} + q_{22})p^2(e^*) & \text{if } e^*(\theta_{21}) = e^* \end{cases}$$

[9]If $\{\theta_{11}, \theta_{12}\} \not\subseteq D(e)$, intuitive criterion puts no restriction on the off-the-equilibrium belief at e, therefore the sequential equilibrium in question survives intuitive test. This is because if at least one of the low productivity types does not belong to $D(e)$, firms can offer $w = p^1(e)$ to this unsent message. Since in any sequential equilibrium type θ_{ij} can at least secure payoff $\underline{U}_{ij} = max_{\tilde{e}\geq 0}\ \theta_j^{\tilde{e}} p^1(\tilde{e}) - c^i(\tilde{e})$, no type θ_{ij} wants to deviate to e which gives a payoff of $\theta_j^e p^1(e) - c^i(e)$.

then in both cases we can find an unsent message $e' > e^*$ such that:

$$U_{11}^* > \delta_1^{e'} p^2(e') - c^1(e')$$

$$U_{12}^* > \delta_2^{e'} p^2(e') - c^1(e')$$

$$U_{22}^* < \delta_2^{e'} p^2(e') - c^2(e')$$

because the indifference curve of θ_{22} is globally flatter than those of θ_{11} and θ_{12}. Therefore, the sequential equilibrium cannot be intuitive.

(Case II) Suppose in a sequential equilibrium θ_{22} pools with θ_{12} at time e^*. Let $p^o(e^*) = q_{12}p^1(e^*) + q_{22}p^2(e^*)$, then there exists an unsent message $e' > e^*$ such that:

$$U_{11}^* \geq \delta_1^{e^*} p^o(e^*) - c^1(e^*) > \delta_1^{e'} p^2(e') - c^1(e')$$

$$U_{12}^* = \delta_2^{e^*} p^o(e^*) - c^1(e^*) > \delta_2^{e'} p^2(e') - c^1(e')$$

$$U_{22}^* = \delta_2^{e^*} p^o(e^*) - c^2(e^*) < \delta_2^{e'} p^2(e') - c^2(e')$$

(Case III) Suppose in a sequential equilibrium θ_{22} pools with θ_{11} at time e^*

(i)If $e^*(\theta_{12}) \neq e^*(\theta_{21})$.

Then $e^*(\theta_{12}) = \underline{e}_{12}$ and $U_{12}^* = \underline{U}_{12}$. And this cannot be a sequential equilibrium in the first place because if pooling with θ_{22} gives θ_{11} higher payoff than $\underline{U}_{11}$ then θ_{12} can mimic the pooling at e^* and earn a payoff higher than $\underline{U}_{12}$.

(ii)If $e^*(\theta_{12}) = e^*(\theta_{21}) = e^{**}$.

Let $p^o(e^*) = q_{11}p^1(e^*) + q_{22}p^2(e^*)$, then there exists an unsent message $e' > e^*$ such that:

$$U_{11}^* = \delta_1^{e^*} p^o(e^*) - c^1(e^*) > \delta_1^{e'} p^2(e') - c^1(e')$$

$$U_{12}^* > \delta_2^{e^*} p^o(e^*) - c^1(e^*) > \delta_2^{e'} p^2(e') - c^1(e')$$

$$U_{22}^* = \delta_2^{e^*} p^o(e^*) - c^2(e^*) < \delta_2^{e'} p^2(e') - c^2(e')$$

□

Hence it is shown that any sequential equilibrium involving θ_{22} pooling with low productivity types fails the intuitive criterion. This is equivalent to the result that in the basic Spence model, highest productivity type never pool with the other types by the intuitive test since she has a cost advantage over the other types. In the current model, type θ_{22} has the flattest indifference curve everywhere, so she should always be able to distinguish herself from the low productivity types.

The next lemma gives the condition when intuitive separating equilibrium exists, and also asserts that when it exists, it is unique.

Lemma 4. *A separating equilibrium satisfying the intuitive criterion exists if* $\tilde{e}_{12} \leq \tilde{e}_{21}$, *where* $\tilde{e}_{12}$ *and* $\tilde{e}_{21}$ *are defined implicitly by*

$$\underline{U}_{12} = \delta_2^{\tilde{e}_{12}} p^2(\tilde{e}_{12}) - c^1(\tilde{e}_{12})$$

$$\underline{U}_{21} = \delta_1^{\tilde{e}_{21}} p^2(\tilde{e}_{21}) - c^2(\tilde{e}_{21})$$

The unique *equilibrium outcome is:*

(1) $e^*(\theta_{1j}) = \underline{e}_{1j},\ j = 1, 2$

(2) $e^*(\theta_{2j}) = \begin{cases} \bar{e}_{2j} & \textit{if } \bar{e}_{2j} \geq \tilde{e}_{12} \\ \tilde{e}_{12} & \textit{otherwise} \end{cases} \qquad j = 1, 2$

(3) $w^*(e^*(\theta_{1j})) = p^1(e^*(\theta_{1j})),\ j = 1, 2$

(4) $w^*(e^*(\theta_{2j})) = p^2(e^*(\theta_{2j})),\ j = 1, 2$

Proof. (See Figure 2.)First of all, if $\tilde{e}_{12} > \tilde{e}_{21}$ there cannot be separating equilibrium because θ_{21} has to invest at least $\tilde{e}_{12}$ to separate from type θ_{12}, yet it gives θ_{21} payoff even lower than the minimal obtainable $\underline{U}_{21}$ by the definition of $\tilde{e}_{21}$.

Next, we will show when $\tilde{e}_{12} \leq \tilde{e}_{21}$ holds, the proposed outcome is indeed an equilibrium. By the condition and $e^*(\theta_{2j}) \geq \tilde{e}_{12}$, no type has the incentive to mimic any other type and all types θ_{ij} acquire a payoff at least as big as the minimal obtainable $\underline{U}_{ij}$, $\forall i = 1,2; j = 1,2$. And intuitive criterion requires that high productivity types choose the best education time among all those that can at least separate themselves from low productivity workers. Therefore, the separating equilibrium is unique. □

Roughly, $\tilde{e}_{12} \leq \tilde{e}_{21}$ requires that the discount difference is mild compared to the productivity difference, because when the discount difference is not substantial, the Good Region is relatively big and it is easier for the condition to hold. With sequentiality alone, there can be a continuum of separating equilibria with $e^*(\theta_{1j}) = \underline{e}_{1j}$, $j = 1,2$ and $\tilde{e}_{2j} \geq e^*(\theta_{2j}) \geq \tilde{e}_{12}$[10], $j = 1,2$. Namely, high productivity types choose education lengthes that are both enough to separate themselves from low productivity types and generate at least payoff $\underline{U}_{2j}$, $j = 1,2$. Adding intuitive requirement, this equilibrium prediction is further tightened to be the unique Pareto optimal separation. In the basic Spence model with two types, this would be the only sequential equilibrium meeting the intuitive criterion, while we need the restriction $\tilde{e}_{12} \leq \tilde{e}_{21}$ here to get the unique separation since single crossing no longer holds globally.

Lemma 5. *There exists an equilibrium satisfying the intuitive criterion where type θ_{12} and θ_{21} pool at e^* if the following two conditions hold*

(a) $\underline{U}_{12} \leq \delta_2^{e^*}\hat{p}(e^*) - c^1(e^*)$
$\underline{U}_{21} \leq \delta_1^{e^*}\hat{p}(e^*) - c^2(e^*)$
$\underline{U}_{11} \geq \delta_1^{e^*}\hat{p}(e^*) - c^1(e^*)$

(b) $\tilde{e}_{12} \geq \tilde{e}_{21}$ *where* $\tilde{e}_{12}$ *and* $\tilde{e}_{21}$ *are defined by*
$\delta_2^{e^*}\hat{p}(e^*) - c^1(e^*) = \delta_2^{\tilde{e}_{12}}p^2(\tilde{e}_{12}) - c^1(\tilde{e}_{12})$
$\delta_1^{e^*}\hat{p}(e^*) - c^2(e^*) = \delta_1^{\tilde{e}_{21}}p^2(\tilde{e}_{21}) - c^2(\tilde{e}_{21})$

The equilibrium outcome is:

[10] $\tilde{e}_{22}$ are defined implicitly by $\underline{U}_{22} = \delta_2^{\tilde{e}_{22}}p^2(\tilde{e}_{22}) - c^2(\tilde{e}_{22})$

(1) $e^*(\theta_{11}) = \underline{e}_{11}$

(2) $e^*(\theta_{12}) = e^*(\theta_{21}) = e^*$

(3) $e^*(\theta_{22}) = \begin{cases} \bar{e}_{22} & \textit{if } \bar{e}_{22} \geq \tilde{e}_{12} \\ \tilde{e}_{12} & \textit{otherwise} \end{cases}$

(4) $w^*(e^*(\theta_{11})) = p^1(e^*(\theta_{11}))$

(5) $w^*(e^*) = \hat{p}(e^*)$

(6) $w^*(e^*(\theta_{22})) = p^2(e^*(\theta_{22}))$

Proof. $e^*(\theta_{22}) \geq \tilde{e}_{12}$ and requirement (a) ensure that no type wants to mimic any other type in the proposed equilibrium outcome and each type at least gets payoff $\underline{U}_{ij}$ in the equilibrium, $i = 1, 2; j = 1, 2$. Types θ_{12} and θ_{21} can pool at e^* only if type θ_{21} cannot afford the cost of separating from type θ_{12}, which is indeed the case by restriction (b). So the proposed outcome is a pooling equilibrium. And intuitive criterion requires $e^*(\theta_{22})$ to be the least costly education choice for type θ_{22} to separate from the $\{\theta_{12}, \theta_{21}\}$ pool. □

Since all the functions are continuously differentiable, if there exist a e^* such that condition (a) and (b) hold, then there is a closed interval containing e^* such that (a) and (b) are satisfied. Therefore, in general, there can be a continuum of $\{\theta_{12}, \theta_{21}\}$ pooling equilibria. Again by the same argument after the previous lemma, what intuitive test does in addition to sequentiality here is that it restricts $e^*(\theta_{22})$ to the least costly education time for θ_{22} to separate from $\{\theta_{12}, \theta_{21}\}$.

Lemma 6. *There exists an intuitive equilibrium where type θ_{11}, θ_{12} and θ_{21} pool at e^* if the following two conditions holds*

(a) $\underline{U}_{11} \leq \delta_1^{e^*} \tilde{p}(e^*) - c^1(e^*)$
$\underline{U}_{12} \leq \delta_2^{e^*} \tilde{p}(e^*) - c^1(e^*)$
$\underline{U}_{21} \leq \delta_1^{e^*} \tilde{p}(e^*) - c^2(e^*)$

(b) $\tilde{e}_{12} \geq \tilde{e}_{21}$ *where* $\tilde{e}_{12}$ *and* $\tilde{e}_{21}$ *are defined by*

$$\delta_2^{e^*}\tilde{p}(e^*) - c^1(e^*) = \delta_2^{\tilde{e}_{12}}p^2(\tilde{e}_{12}) - c^1(\tilde{e}_{12})$$

$$\delta_1^{e^*}\tilde{p}(e^*) - c^2(e^*) = \delta_1^{\tilde{e}_{21}}p^2(\tilde{e}_{21}) - c^2(\tilde{e}_{21})$$

The equilibrium outcome is:

(1) $e^*(\theta_{11}) = e^*(\theta_{12}) = e^*(\theta_{21}) = e^*$

(2) $e^*(\theta_{22}) = \begin{cases} \bar{e}_{22} & \textit{if } \bar{e}_{22} \geq \tilde{e}_{12} \\ \tilde{e}_{12} & \textit{otherwise} \end{cases}$

(3) $w^*(e^*) = \tilde{p}(e^*)$

(4) $w^*(e^*(\theta_{22})) = p^2(e^*(\theta_{22}))$

Proof. Requirement (a) ensures that in equilibrium all types at least secure payoffs more than $\underline{U}_{ij}$, $i = 1, 2; j = 1, 2$. By $\bar{e}_{22} \geq \tilde{e}_{12} \geq \tilde{e}_{21}$, none of the three pooling types wants to mimic θ_{22} on the equilibrium path. $\tilde{e}_{12} \geq \tilde{e}_{21}$ says θ_{21} cannot distinguish herself from the pooling. Also intuitive criterion requires θ_{22} to choose the least costly education time to separate from the other three types. □

And by the same argument for Lemma 5, there can be a continuum of $\{\theta_{11}, \theta_{12}, \theta_{21}\}$ pooling equilibria. Now we are ready to combine the above results in the following proposition:

Proposition 7. *There exists at least one sequential equilibrium satisfying the intuitive criterion and all the equilibrium outcomes should be one of the three types described in* ***Lemma 4-6****.*

Proof. First of all, we need to show there exists no other types of equilibria different from the ones described in Lemma 4-6. This is because by Lemma 3 any equilibrium should not involve θ_{22} pooling with low productivity types; and A4 assumes away $\{\theta_{11}, \theta_{21}\}$ pooling.

To show the existence, it suffices to show that if separating equilibrium does not exist then $\{\theta_{12}, \theta_{22}\}$ pooling is sure to exist. If $\tilde{e}_{12} > \tilde{e}_{21}$ (defined in Lemma 4), then it must be the case that the two indifference curves of θ_{12}, θ_{21} which are tangent to $w = p^1(e)$ cross at a point $A \in BR$ below the curve $w = p^2(e)$. Suppose point A is above the curve $w = \hat{p}(e)$ which is depicted in Figure 3(a). (There indifference curves are labelled by the type of the worker.) Then θ_{12}, θ_{21} pooling at time e^* constitutes an equilibrium in Lemma 5 because by assumption A5 it is easy to check that all the conditions there are satisfied. When point A is below $w = \hat{p}(e)$, Figure 3(b) gives that θ_{12}, θ_{21} pooling at time $e^{*'}$ can be an equilibrium. □

It is straightforward to see that when a $\{\theta_{11}, \theta_{12}, \theta_{21}\}$ pooling equilibrium exists, a $\{\theta_{12}, \theta_{21}\}$ pooling equilibrium has to exist too. Therefore, combining this observation and the proof of the proposition we have the following corollary.

Corollary 8. *The equilibria satisfying the intuitive criterion described in* ***Lemma 4-6*** *can only (co)exist in the following ways:*

Case 1. *A unique separating equilibrium,*

Case 2. *A unique separating equilibrium and a continuum of $\{\theta_{12}, \theta_{21}\}$ pooling equilibria,*

Case 3. *A unique separating equilibrium, a continuum of $\{\theta_{12}, \theta_{21}\}$ pooling equilibria and a continuum of $\{\theta_{11}, \theta_{12}, \theta_{21}\}$ pooling equilibria,*

Case 4. *A continuum of $\{\theta_{12}, \theta_{21}\}$ pooling equilibria,*

Case 5. *A continuum of $\{\theta_{12}, \theta_{21}\}$ pooling equilibria and a continuum of $\{\theta_{11}, \theta_{12}, \theta_{21}\}$ pooling equilibria.*

Below we give a numerical example where Case 3 of the corollary stands. Thus intuitive criterion in general is not strong enough to select a unique equilibrium in the current game.

4. A numerical Example

In this section, a numerical example is given to show that all the assumptions made can be satisfied. In particular, we will give a case where all the three types of equilibria mentioned in the last section coexist. Let $\delta_1 = 0.7, \delta_2 = 0.8$ and the productivity and cost functions be:

$$p^1(e) = 8e,\ c^1(e) = e^2 + 4e$$

$$p^2(e) = 10e + 1,\ c^2(e) = e^2 + 6e$$

It is straightforward to show A1-A3 hold. Some calculation shows that the curve implicitly defined by $S(e, w) = 0$ has a vertical intercept w^o of about 15 and a horizontal intercept e^o of about 1.74. Also $\underline{e}_{11} = 0.274 < \underline{e}_{12} = 0.373 < \underline{e}_{21} = 0.58 \in GR$. So A4 holds. (See Figure 4.)

First, let us show that a separating equilibrium exists in this case. (Notice the existence of separating equilibrium does not depend on the prior distribution of the types.) Calculating $\tilde{e}_{ij}$ such that $\underline{U}_{ij} = \delta_j^{\tilde{e}_{ij}} p^2(\tilde{e}_{ij}) - c^i(\tilde{e}_{ij})$ we have:

$$\tilde{e}_{11} = 1.106 < \tilde{e}_{12} = 1.461 < \tilde{e}_{21} = 1.501$$

So $\tilde{e}_{12} < \tilde{e}_{21}$ ensures separating equilibrium exists by Lemma 4.

To get the other two types of equilibria to exist too, we just need to assume some extreme prior distribution of the types. For example, if

$$\mu = (\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}) = (1/n, 1/n, 1/2 - 1/n, 1/2 - 1/n)$$

then it is easy to see both $\hat{p}(e)$ and $\tilde{p}(e)$ should cross the three indifference curves in the Bad Region if n is big enough (Figure 4. shows $\hat{p}(e)$). And this satisfies the conditions in Lemma 5 and Lemma 6. Therefore, both the pooling types of equilibria exist too. And when n is big enough, we also have assumption A5 met.

5. Sequential Equilibrium with Grossman-Perry Refinement

In the hope of further strengthening our prediction in this 2×2 type signaling game, we invoke a stronger refinement concept of Grossman and Perry [5]. Again we just state the test adjusted to the current game. For any sequential equilibrium, the following procedure is applied to an unsent message e:

(i) Does there exist a subset of the types such that if they are offered a wage by the Bayesian belief of this subset of types at time e, they are better off than in the proposed equilibrium?

(ii) If so, is it true that no other type would find the above wage offer better than her equilibrium payoff?

If there exists an unsent message e such that the answers to (i) and (ii) are yes, then the sequential equilibrium is said to fail the G-P refinement. Basically the G-P test requires the off-the-equilibrium belief to be self-fulfilling, i.e. if a subset of the types is believed to be the deviators and offered wage by Bayesian update, then indeed they and only they would deviate from the equilibrium.

It is apparent that the G-P test is stronger than the intuitive criterion because if the deviators are believed to be one or both of the high productivity types, the above process is equivalent to the intuitive criterion. Therefore, to tighten up the prediction of the outcome of the game, we just need to examine those sequential equilibria that satisfy the intuitive criterion. The following three lemmas show which equilibria fail the G-P criterion.

Lemma 9. $\{\theta_{11}, \theta_{12}, \theta_{21}\}$ *pooling equilibria (if they exist) fail the G-P test.*

Proof. Suppose the three types pool at time e^* in an equilibrium. Since θ_{12}, θ_{21} both have flatter indifference curves than θ_{11} and $\hat{p}(e) > \tilde{p}(e)$, we can find an unsent message e such that:

$$U^*_{11} = \delta_1^{e*}\tilde{p}(e^*) - c^1(e^*) > \delta_1^e\hat{p}(e) - c^1(e)$$

$$U^*_{12} = \delta_2^{e*}\tilde{p}(e^*) - c^1(e^*) < \delta_2^e\hat{p}(e) - c^1(e)$$

$$U^*_{21} = \delta_1^{e*}\tilde{p}(e^*) - c^2(e^*) < \delta_1^e\hat{p}(e) - c^2(e)$$

So this violates the G-P test no matter how θ_{22} responds to the wage offer $w(e) = \hat{p}(e)$. □

Lemma 10. *When the unique separating equilibrium and* $\{\theta_{12}, \theta_{21}\}$ *pooling equilibria coexist, the separating equilibrium fails the G-P test.*

Proof. The fact that these two types of equilibria coexist determines that the separating equilibrium gives both θ_{12} and θ_{21} (weakly) lower payoffs than any $\{\theta_{12}, \theta_{21}\}$ pooling equilibrium. (See Figure 4.) Therefore, the separating equilibrium fails the G-P test because if θ_{12}, θ_{21} deviate to a time e that could have been a pooling choice for them, they are better off by being offered $\hat{p}(e)$ and the belief is self-fulfilling.

□

Lemma 11. *Among all the* $\{\theta_{12}, \theta_{21}\}$ *pooling equilibria (if they exist), let* e' *and* e'' *be the pooling choices of the most preferred equilibria by* θ_{12} *and* θ_{21}*, respectively, then:*

(1) *If* $e' = e''$*, then any* $\{\theta_{12}, \theta_{21}\}$ *pooling other than the most preferred fails G-P test.*
(2) *If* $e' \neq e''$*, then any* $\{\theta_{12}, \theta_{21}\}$ *pooling with* $e^*(\theta_{12}) = e^*(\theta_{21}) < min\{e', e''\}$ or $> max\{e', e''\}$ *fails the G-P test.*

Proof. If $e' = e''$, then any other pooling choice cannot be G-P perfect because θ_{12} and θ_{21} can deviate to $e' = e''$ and be better off when firms offer

$\hat{p}(e')$. And by the fact that $e' = e''$ is a $\{\theta_{12}, \theta_{21}\}$ pooling time in the first place, θ_{11} does not benefit from deviating to $e' = e''$, so belief is self-fulfilling.

If $e' \neq e''$, any equilibrium with θ_{12}, θ_{21} choosing a time not in between e' and e'' cannot be G-P perfect by the same argument above. □

In words, among all the $\{\theta_{12}, \theta_{21}\}$ pooling time choices, only the ones between the most preferred by the two types can potentially pass the G-P test. e' and e'' can be different because we assume in the model that $p^i(\cdot)$ are non-decreasing functions. If $p^i(\cdot)$ are constants as in the basic Spence model, both θ_{12} and θ_{21} should have the same most preferred pooling outcome, in which they choose the least costly education time that can jointly separate themselves from type θ_{11}. Hence when it happens that $e' = e''$, this G-P perfect equilibrium is the unique Pareto optimal one among all $\{\theta_{12}, \theta_{21}\}$ pooling outcomes. We will refer to these perfect $\{\theta_{12}, \theta_{21}\}$ pooling equilibria as "the best $\{\theta_{12}, \theta_{21}\}$ pooling equilibria". Now we can combine the previous three lemmas into the final result:

Proposition 12. *If a G-P perfect sequential equilibrium exists, then it is either:*

(1) *the separating equilibrium if it is the unique equilibrium satisfying the intuitive criterion (Case 1 of* ***Corollary 8****); or*

(2) *one of the best $\{\theta_{12}, \theta_{21}\}$ pooling equilibria if there are multiple equilibria meeting the intuitive requirement (Case 2-5 of* ***Corollary 8****).*

Proof. By Corollary 8 and Lemma 9-11. □

From the discussion of Lemma 11, we consider the prefect equilibria (when they exist) to be "essentially" unique by the spirit of the original Spence model where $p^i(\cdot)$ are constants, $i = 1, 2$. The "two" equilibria selected in the two cases by the G-P perfect sequentiality concept (if they exist) are exactly the same equilibrium outcomes as in Smart's [9] two dimensional information screening game. Nonexistence arises for the same reason too. Take the unique separating equilibrium for example. If the equilibrium outcome involves high productivity types overeducating than $\bar{e}_{2j}, j = 1, 2$, and the prior distribution of types is $\mu = (\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}) = (0.001, 0.001, 0.001, 0.997)$, then it is easy to see that unless the productivity difference is extreme, overeducation is not worth the cost of separating for type θ_{22}. But pooling by this type with low productivity types can never be an equilibrium by Lemma 3. Thus, there fails to exist any perfect sequential equilibrium.

The expanded model incorporates the original Spence model by taking $\delta_1 = \delta_2$. Moreover, we can roughly say that when the discount factors are close, the unique equilibrium picked is the separating equilibrium because in this case the Good Region is relatively big, making it easier to achieve separation. Therefore, adding the additional private information does not change the equilibrium outcome of the Spence model unless it substantially changes the costs structures of different productivity types. In that case, the "best $\{\theta_{12}, \theta_{21}\}$ pooling equilibria" are selected.

6. Smart's Competitive Insurance Model with Two Unobservables

In this section, we briefly describe Smart's [9] competitive insurance model with two dimensional private information and compare its equilibrium with the perfect sequential equilibrium in the current paper. An insurance market consists of a large finite number of agents facing idiosyncratic risks of loss in money income and degrees of risk aversion. Both are privately known only by the individual. Assume there are two possible probabilities of loss, high and low; there are high and low risk aversions. Competitive insurance firms offer individuals contracts that describe the premium and the insured income.

Adding risk aversion in this model interferes with the single-crossing property as discounting did in the job market signaling game. Subgame perfection is used as the equilibrium concept in the following two-stage screening game. In the first stage, firms choose whether to enter the market and, if so, offer a single insurance policy to potential customers. In the second stage, customers choose a single policy among those on offer. An equilibrium is separating if all low risk types are offered and accept different contracts from high risk types and an equilibrium is pooling otherwise.

The main result (Proposition 2 of Smart [9]) is that: *If an equilibrium exists, it is unique. Equilibrium is separating if a "certain condition" holds and is "high loss probability, low risk version" type and "low loss probability, high risk aversion" type pooling otherwise.* To show how the equilibrium changes as the difference in risk aversion between types increases or how the "certain condition" works, Proposition 3. of the paper gives the following result: *Keeping everything else fixed and varying the high loss probability, then there exists a cutoff probability such that the equilibrium (if one exists) is separating if the high loss probability is below this cutoff point and above the low loss probability, and pooling if it is above the cutoff.*

Hence the nature of the unique equilibrium (when it exists) of the above competitive screening is similar to the one we have derived in the last section for the job market signaling. Both models introduced an additional unobservable into the basic screening and signaling games and acquired the same main result: when the new unobservables matter little, separating equilibria of the basic models are maintained; while when the new dimension of information substantially changes the type structure, both extended models exhibit pooling.

7. Conclusion

Private information about discount factors is introduced in the traditional Spence job market signaling model. Single crossing is violated in the extended model, so the intuitive criterion is not strong enough to pick out a unique sequential equilibrium. However, if the stronger G-P test is applied, an "essentially" unique equilibrium is selected when a refined equilibrium exists. This result and the result of Smart [9] demonstrate the equivalence of the equilibrium outcomes of screening games and signaling games with two dimensional private information.

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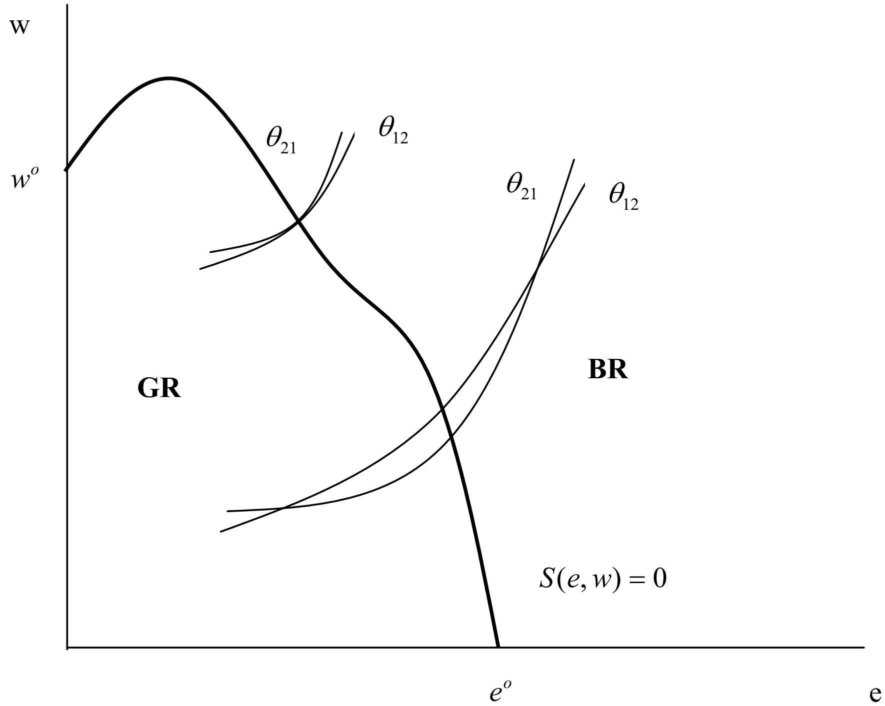

Figure 1 (Good Region and Bad Region)

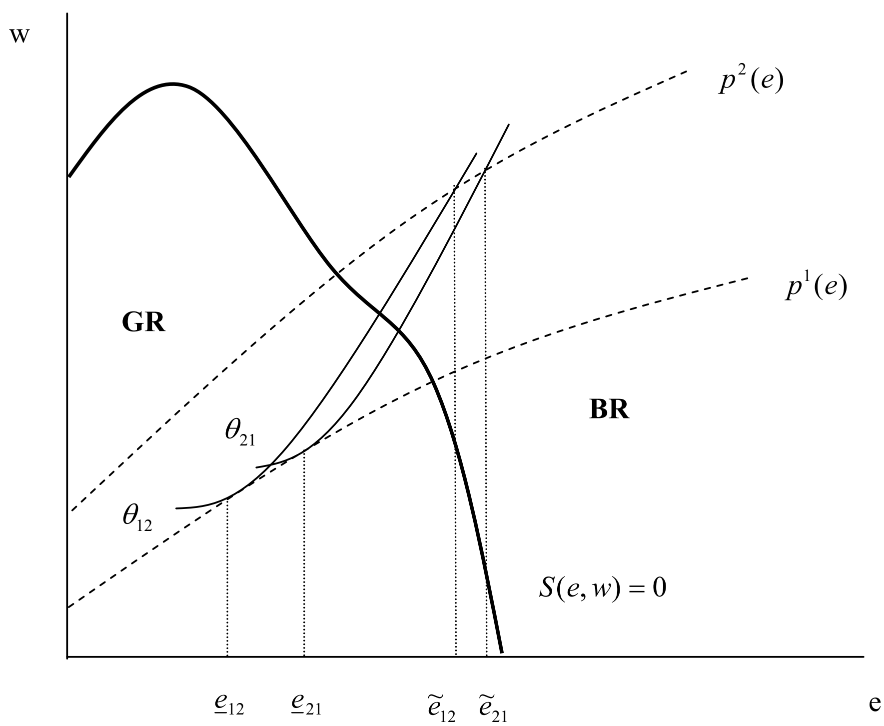

Figure 2 (Separating Equilibrium)

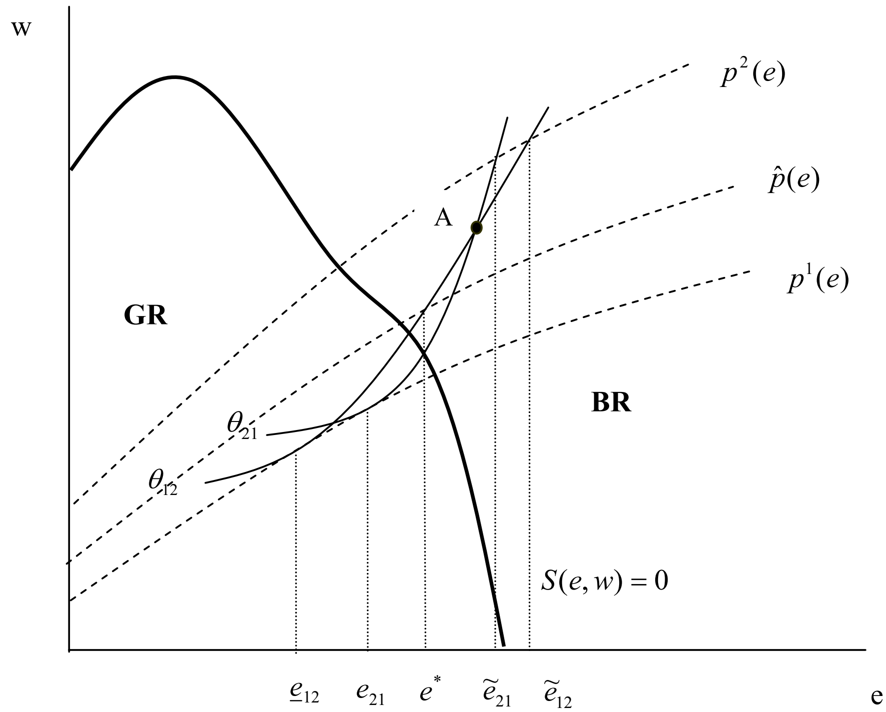

Figure 3 (a) (Existence)

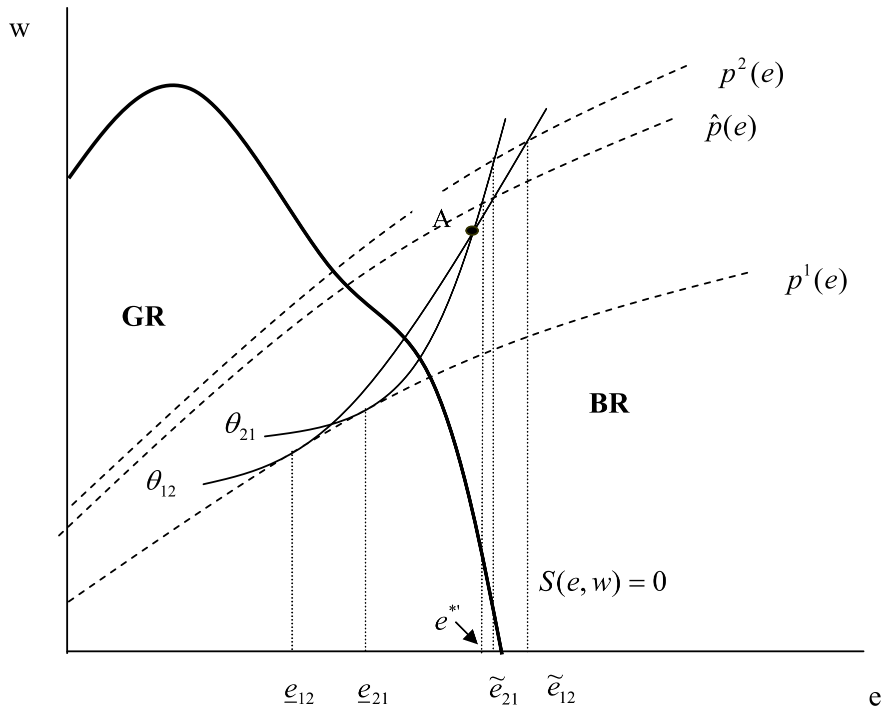

Figure 3 (b) (Existence)

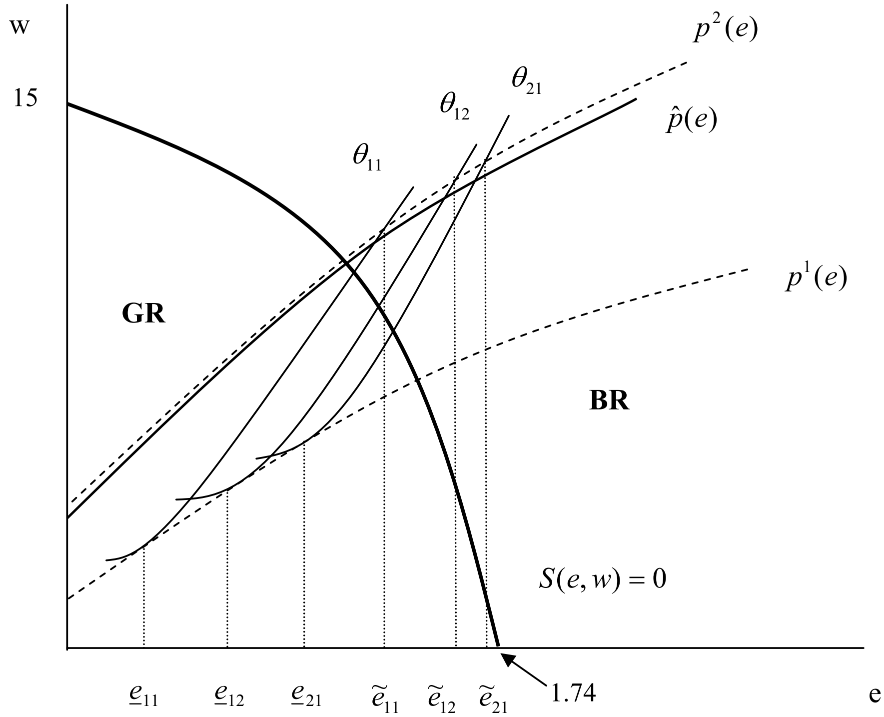

Figure 4 (Coexistence of the three types of equilibria)