

## Syllabus – Economics 806 Population Games and Evolutionary Dynamics

### Course Description

This course will survey the theory of population games and evolutionary dynamics. A significant portion of class time will be spent discussing the mathematical techniques in multivariate analysis, probability theory, and dynamical systems central to these areas of research. The main readings will be the chapter drafts from my book-in-progress on these topics (*Population Games and Evolutionary Dynamics*, forthcoming, MIT Press), as well as a few recent papers from the literature.

From time to time, I will assign exercises from the book, as well as computer exercises in *Dynamo*. The latter is a computer program for drawing and analyzing evolutionary dynamics created by me and Emin Dokumaci and posted on my website. *Dynamo* runs within *Mathematica*. Copies of *Mathematica* are installed on three computers in the third floor lab, and a student version of *Mathematica* can be purchased from DoIT for the price of an expensive textbook.

### Course Outline

- I. Population games
- II. Potential games, stable games, and supermodular games
- III. Revision protocols and evolutionary dynamics
- IV. Deterministic dynamics: families and properties
- V. Best response and projection dynamics
- VI. Global convergence of evolutionary dynamics
- VII. Local stability under evolutionary dynamics
- VIII. Nonconvergence of evolutionary dynamics
- IX. Stochastic evolution and deterministic approximation
- X. Infinite horizon behavior and equilibrium selection

### References

*General references on evolutionary dynamics*

Fudenberg, D., and D. K. Levine (1998). *Theory of Learning in Games*. MIT.

Hofbauer, J., and K. Sigmund (1998). *Evolutionary Games and Population Dynamics*. Cambridge.

Hofbauer, J., and K. Sigmund (2003). "Evolutionary Game Dynamics," *Bulletin of the American Mathematical Society (New Series)* 40, 479-519.

Sandholm, W. H. (2008). *Population Games and Evolutionary Dynamics*. Forthcoming, MIT.  
Weibull, J. W. (1995). *Evolutionary Game Theory*. MIT.

*Some particularly germane readings*

- Björnerstedt, J., and J. W. Weibull (1996). "Nash Equilibrium and Evolution by Imitation," in K. J. Arrow *et. al.* (eds.), *The Rational Foundations of Economic Behavior*. St. Martin's Press. (for parts III and IV)
- Sandholm, W. H. (2007). "Pairwise Comparison Dynamics and Evolutionary Foundations for Nash Equilibrium", unpublished manuscript, University of Wisconsin. (for parts III and IV)
- Hofbauer, J. (2000). "From Nash and Brown to Maynard Smith: Equilibria, Dynamics, and ESS," *Selection* 1, 81-88. (for part VI)
- Hofbauer, J., and W. H. Sandholm (2007). "Survival of Dominated Strategies under Evolutionary Dynamics," unpublished manuscript, University of Vienna and University of Wisconsin. (for part XIII)
- Benaim, M., and J. W. Weibull (2003). "Deterministic Approximation of Stochastic Evolution in Games," *Econometrica* 71, 873-903. (for parts IX and X)
- Sandholm, W. H. (2006). "Simple Formulas for Stationary Distributions and Stochastically Stable States," *Games and Economic Behavior* 59 (2007), 154-162. (for part X)

*Early contributions and historical background*

- von Neumann, J., and O. Morgenstern (1944). *Theory of Games and Economic Behavior*. Princeton.
- Nash, J. F. (1950). *Non-Cooperative Games*. Ph.D. dissertation, Princeton University. Reprinted in *The Essential John Nash*, H. W. Kuhn and S. Nasar, Eds. (2002). Princeton.
- Brown, G. W. and J. von Neumann (1950). "Solutions of Games by Differential Equations." In H. W. Kuhn and A. W. Tucker, Eds., *Contributions to the Theory of Games I*. Annals of Mathematics Studies 24. Princeton.
- Brown, G. W. (1951). "Iterative Solutions of Games by Fictitious Play," in *Activity Analysis of Production and Allocation*, T. C. Koopmans, Ed. Wiley.
- Maynard Smith, J., and G. R. Price (1973). "The Logic of Animal Conflict," *Nature* 246, 15-18.
- Maynard Smith, J. (1982). *Evolution and the Theory of Games*. Cambridge.
- Schuster, P., and K. Sigmund (1983). "Replicator Dynamics," *Journal of Theoretical Biology* 100, 533-538.
- Beckmann, M., C. B. McGuire, and C. B. Winsten (1956). *Studies in the Economics of Transportation*. Yale.
- Smith, M. J. (1984). "The Stability of a Dynamic Model of Traffic Assignment—An Application of a Method of Lyapunov," *Transportation Science* 18, 245-252.

*Some math references*

*Analysis and linear algebra:*

Lang, S. (1997). *Undergraduate Analysis*, 2<sup>nd</sup> ed. Springer.

Hiriart-Urruty, J.-B., and C. Lemaréchal (2001). *Fundamentals of Convex Analysis*. Springer.

Friedberg, S. H., A. J. Insel, and L. E. Spence (1989). *Linear Algebra*, 2<sup>nd</sup> ed. Prentice Hall.

Horn, R. A., and C. R. Johnson (1985). *Matrix Analysis*. Cambridge.

*Dynamical systems:*

Hirsch, M. W., and S. Smale (1974). *Differential Equations, Dynamical Systems, and Linear Algebra*. Academic Press.

Robinson, C. (1995). *Dynamical Systems: Stability, Symbolic Dynamics, and Chaos*. CRC Press.

Smith, H. L. (1995). *Monotone Dynamical Systems: An Introduction to the Theory of Competitive and Cooperative Systems*. American Mathematical Society.

Smirnov, G. V. (2002). *Introduction to the Theory of Differential Inclusions*. American Mathematical Society.

*Probability theory and Markov processes:*

Durrett, R. (2005). *Probability: Theory and Examples*, 3<sup>rd</sup> ed. Brooks/Cole.

Billingsley, P. (1995). *Probability and Measure*, 3<sup>rd</sup> ed. Wiley.

Norris, J. R. (1997). *Markov Chains*. Cambridge.

Brémaud, P. (1999). *Markov Chains: Gibbs Fields, Monte Carlo Simulation, and Queues*. Springer.

Stroock, D. W. (2005). *An Introduction to Markov Processes*. Springer.

Dembo, A., and O. Zeitouni (1998). *Large Deviations Techniques and Applications*, 2<sup>nd</sup> ed. Springer.

Benaim, M. (1999). "Dynamics of Stochastic Approximation Algorithms," In *Séminaire de Probabilités XXXIII*, J. Azéma et. al., Eds., 1-68. Lecture Notes in Mathematics 1709. Springer.