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Midterm Exam – Economics 713

1. (10 points)

Let G be a two player zero sum game. Show that if $\sigma_1 \in \Delta S_1$ is a strategy of player 1 which minmaxes player 2, then σ_1 is not strictly dominated.

2. (10 points)

You and a friend are computing the subgame perfect equilibria of an infinitely repeated game $G^\infty(\delta)$. Your friend suggests that to perform the computation efficiently, you should begin by eliminating all dominated actions from the stage game. Evaluate your friend's suggestion.

3. (15 points (5, 10))

A (finite) normal form game G is a *potential game* if there exists a function $P: S \to \mathbf{R}$ (known as a *potential function*) such that

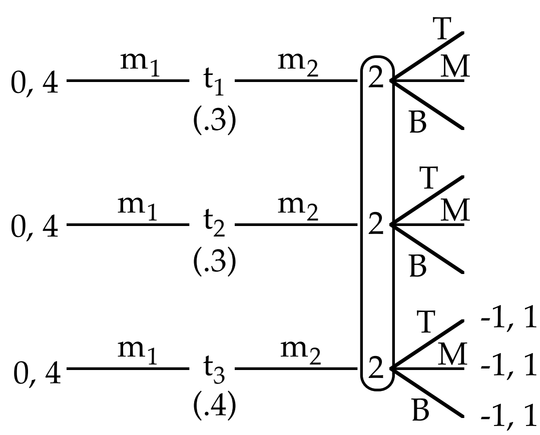
$$u_i(s_i', s_{-i}) - u_i(s_i, s_{-i}) = P(s_i', s_{-i}) - P(s_i, s_{-i}) \text{ for all } s_i,\ s_i' \in S_i,\ s_{-i} \in S_{-i}, \text{ and } i \in N.$$

(*i*) Explain in words what this condition means.

(*ii*) Prove that every potential game has at least one pure strategy Nash equilibrium.

4. (15 points)

Enter payoffs into the game diagram below in such a way that the resulting game has an equilibrium outcome which satisfies the intuitive criterion but which fails equilibrium dominance. Prove that the game you construct has these properties.

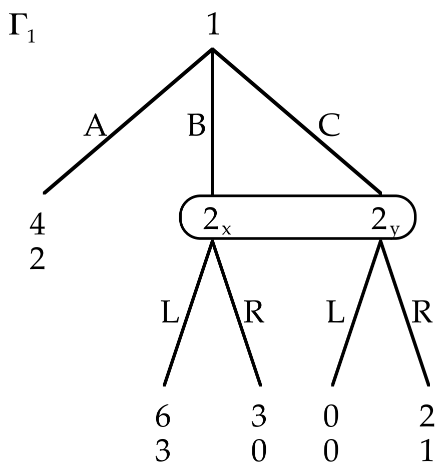


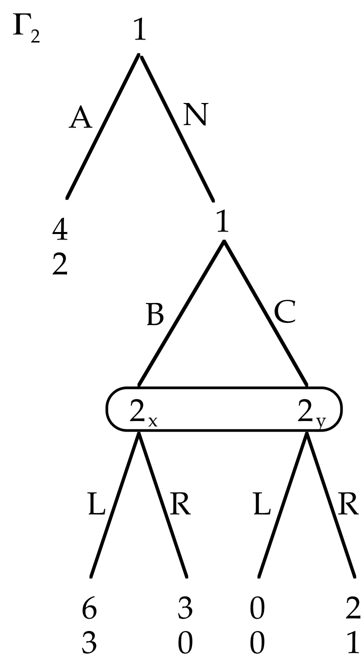
5. (25 points (15, 5, 5))

(i) Compute all sequential equilibria of games Γ_1 and Γ_2 below.

(ii) What are the reduced normal forms of these two games?

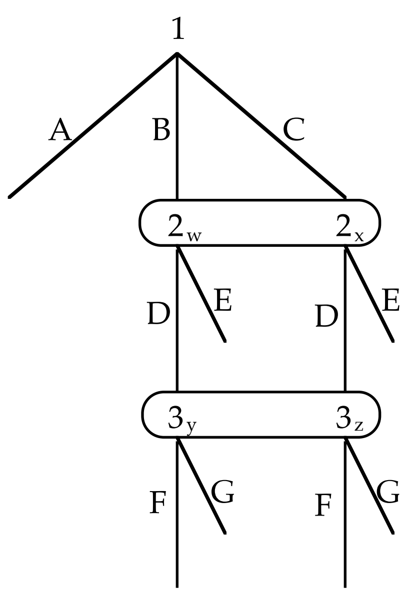
(iii) What does this example tell us about the notion of sequential equilibrium?





6. (10 points)

If player 1 chooses strategy A in the game tree pictured below, which beliefs are consistent for players 2 and 3?



7. (15 points)

In any two player normal form game, the rationalizable strategies are precisely those which survive iterated strict dominance. The proof of this result rests on the following fact: that in a two player game, a mixed strategy of player 1 which is not strictly dominated must be a best response to some mixed strategy of player 2.

To establish this fact, let $S_1 = \{s_1^1, \dots, s_1^l\}$ and $S_2 = \{s_2^1, \dots, s_2^m\}$. Define a vector payoff function for player 1 by $v_1(\sigma_1) = (u_1(\sigma_1, s_2^1), \dots, u_1(\sigma_1, s_2^m))$, and let $V_1 = \{w_1\text{: } w_1 = v_1(\sigma_1)$ for some $\sigma_1 \in \Delta S_1\}$ be the set of player 1's feasible vector payoffs. Now suppose that $\sigma_1 \in \Delta S_1$ is not strictly dominated. Then using the separating hyperplane theorem, one can show that there is a nonzero vector $p \in \mathbf{R}_+^m$ such that $p \cdot v_1(\sigma_1) \geq p \cdot w_1$ for all $w_1 \in V_1$. It follows that a normalized version of p is a mixed strategy for player 2 against which σ_1 is a best response for player 1.

Now suppose we have an n player normal form game with $n \geq 3$, and consider the following statement: "A mixed strategy of player 1 which is not strictly dominated is a best response to some mixed strategy profile of players 2 through n." Explain why the argument stated above for the two player case fails to establish this statement, and provide a weaker version of the statement which the argument is sufficient to establish.