Lecture 29
Insurance Markets

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1. Online course evaluations available until Friday December 13, 2013.

Insurance

- Insurance is a mechanism by which to smooth consumption over uncertain states of nature.
- In U.S. formal insurance market exists. We are required by state law to have liability insurance to drive a car.
- Dane country (and elsewhere in WI) experienced a severe drought during the summer of 2012.
- Many farmers lost their crop. Approximately, one–third had crop insurance, typically the largest farms.
- In this example, The “uncertain state” is weather and its influence on the corn crop. Assume for simplicity that weather is the only source of variability of the corn crop. If the weather is Good production is high, if Bad production is low.
- Critical. States of Nature makes reference to events beyond the control of the farmer.
Moral Hazard

- Recognize the challenge of moral hazard to creation of insurance. Hope by this time, the threat of moral hazard to insurance market.
- Underwriter (of insurance) must know the risk probabilities.
- Events insured must be verifiable.
- Insure risks beyond the control or influence of the insured party.
Structure of Insurance

- Many different types of insurance. Property, casualty (liability), health, and life insurance.
- Life insurance. Purchase policy, pay fee, $F$. Beneficiary of policy receives payment upon death of the insured.
- Insured event must be verifiable (death certificate).
- Payment excludes suicides (the extreme form of moral hazard).
Casualty Insurance

- Liability insurance. Automobile insurance, pays off if owner found “liable” for accident.
- Pays medical care of self and injured parties.
- Pays for repair (replacement) of vehicle(s).
- Pay insurance premium for coverage over period of time (year).
- Premiums depend on characteristics of insured.
(Idealized) Life Insurance.

Let $T$ represent person’s age (lifetime).

The mortality density function is $f(t)$ which represents the probability of death, at exact age $t$.

Then the probability that an individual survives to age $x$ is $Pr(T > x) = 1 - F(x)$. This is frequently called the survivor function (the probability of surviving to age $x$).

Let the $Pr(T > \tau) = S(\tau)$ is the probability that the person survives to at least age $\tau$. 
Mortality: Exponential Random Variable

- Let \( f(t) = \lambda e^{-\lambda t} \), pdf, for exponential r.v..
- Memoryless process, probability of death is constant.
- Used because of simple properties.
- What is the value of a life insurance that pays \( B \) upon death, given that mortality process is characterized by exponential distribution with parameters \( \lambda \)?
Life Insurance Example

Let interest rate equal $r$:

$$E[V] = \int_0^\infty Bf(t) \exp(-rt) \, dt$$

$$= B \int_0^\infty \lambda \exp(-\lambda t) \exp(-rt) \, dt$$

$$= B\lambda \int_0^\infty \exp(-[\lambda + r]t) \, dt$$

$$= \frac{\lambda}{r + \lambda} B$$
Numerical Example

- If $\lambda = 0.0125$, life expectancy at birth = 80 years.
- $r = 0.03$
- $V = B \frac{0.0125}{0.0125 + 0.03} = \frac{0.0125}{0.04125}$
- $V = 0.303B$
Markets as we know are impersonal (though especially hard to remember in the job market).

The insurance company writes policies for a large number of customers.

The insurance company works off the law of large numbers: in “large” samples, the sample mean is “close” to the population mean.

Pooling Insurance company engages in pooling (or averaging) which is a form of smoothing.

Ideally, the customers should be dispersed over a large geographical region. Why?
As a mathematical theorem, the Law of Large Numbers has conditions on the nature of the population and the sampling process.

These conditions relate to similarity (homogeneity) of the population units, and possible dependence of sampled observations.

Think of a large of population of similar farms. Same size, same farming skills, same quality of land. Homogenous farms all very much a like.

Sampling process such that individual elements are independent of one another. Knowing that production on farm $i$ is $Y_i$ provides no information about production on farm $j$, $Y_j$. 
Homogeneity and Independence

- Simplest (and first) LLN developed for homogenous and independent samples. \{ Y_1, Y_2, \ldots, Y_n \}

- Yet LLN exist for heterogeneous populations with elements that exhibit some dependence. Yet, there is a limit on the amount of dependence allowable.

- In the application of insurance, dependence, and particularly positive dependence is what matters most.

- If farms are from the same community (e.g., Waunakee, Verona) farm production likely to be dependent, because farms will experience much the same weather.
Let the probability of crop damage in the population be $\theta$. And for $n$ policy holders, denote the proportion of policy holders experiencing crop damage by $P_n$.

Then for sufficiently large $n$, $P_n \approx \theta$.

For any $c > 0$, and $\epsilon > 0$, there exists an $n$, such that

$$\Pr (|P_n - \theta| > c) < \epsilon.$$ 

That is, with enough (similar) policy holders the insurance can diversify sufficiently to effectively eliminate its risk.
A caveat

True in the perfect world of mathematics, where the LLN is truly a statement on infinitely large samples.

So the LLN does not say that in every large sample (say $n = 10,000$) $P_n$ and $\theta$ will be close. Rather LLN says that eventually (some $n$) the sample quantity will converge to the population value.
Think Hurricane Sandy.

Many extreme weather events affect everyone within region. Positive Dependence.

To protect capital reserves, the insurance company must limit exposure to risks. Hence, company may limit the number of policies written in a region. (Risk management)

Crop–Hail Insurance one of the first types of crop insurance available (in 1820s in France) because hail concentrated in narrow isolated bands.

Summer 2011 — Waunakee Hail Storm. Our neighborhood (1–2 mile radius). Other side of town not affected.
See that insurance spreads risk over group.

Want the insured event to be beyond the individual’s control.

Must be able to verify outcome.

Insurance company needs to limit the amount of positive association of outcomes among policy holders. (Insurers and Reinsurers)
Mutual Insurance

Same principles of insurance apply in developing country context. May not have institutions established that will support market solution.

Ray’s example at beginning of chapter of two farmers Asaf and Sharif. Described as identical identical (same crop, same amount of land \(\implies\) homogenous) but each subject to uncertainty of harvest.

Can establish insurance scheme between themselves. (Mutual insurance, no share holders, only policy holders.)
Asaf and Sharif

Insurance will be an agreement that if one harvest is bad and the other is good then the one with the good harvest will give income to the other.

Insurance is pooling to lower risk.

Standard way to measure risk is by the variance or dispersion in outcomes.

Denote Asaf’s harvest by $Q_a$ and Sharif’s harvest by $Q_s$. These are random variables, and yield the bountiful harvest with probability $p$ and the low harvest with probability $1 - p$.

The two point probability distribution is the second simplest random variable. I want to make a basic point, and it is more direct to do that using the notation of a general or generic random variable.
Let $Q_a$ and $Q_s$ be random variables with pdf $f$ and cdf $F$. The mean of the distribution is $E[Q_a] = E[Q_s] = \mu$, and the variance is $\text{Var}(Q_a) = \text{Var}(Q_s) = \sigma^2$.

$Q_a, Q_s$ follow the same probability distribution but are distinct realizations of this random (i.e., uncertain) agricultural process.

Let $Z = Q_a + Q_b$ be the pooled agricultural output. Basically the insurance agreement is one of sharing output.

Asaf and Sharif are identical so natural outcome is to divide total output equally.

Thus to see the potential advantage of sharing want to compare $Q_a$ or $Q_s$ versus $Z/2$. 
Potential Advantage of Pooling

Need to calculate the mean and variance of $Z/2$ and compare to $Q$ (drop subscript).

Mean of $Z$ is

$$E[Z/2] = \frac{1}{2}(E[Q_a] + E[Q_s]) = \frac{1}{2}(\mu + \mu) = \mu$$

The variance of $Z/2$ is

$$Var(Z/2) = Var\left[\frac{1}{2}(Q_a + Q_s)\right]$$

$$= \frac{1}{4}(Var(Q_a) + Var(Q_s) + 2Cov(Q_a, Q_s))$$

$$= \frac{2\sigma^2 + 2Cov(Q_a, Q_s)}{4}$$

$$= \frac{\sigma^2 + Cov(Q_a, Q_s)}{2}$$
Covariance

- The last term $\text{Cov}$ is covariance, which is how $Q_a, Q_s$ co–vary.
- Covariance measures the association of two random variables.
- If the variables move together (one large the other likely is large) then Covariance is positive.
- If the variables move opposite one another (one large the other likely small) then Covariance is negative.
- If the variables are independent, have no association, then Covariance is zero.
Pooling if . . .

- When does pooling reduce the variance of the harvest?
- Tantamount to asking: $\text{Var}(Z/2) < \sigma^2$
- Substituting the expression for $\text{Var}(Z/2)$ in the inequality

\[
\text{Var}(Z/2) < \sigma^2 \\
\frac{\sigma^2 + \text{Cov}(Q_a, Q_s)}{2} < \sigma^2 \]

\[
\sigma^2 + \text{Cov}(Q_a, Q_s) < 2\sigma^2 \\
\text{Cov}(Q_a, Q_s) < \sigma^2.
\]
Pooling

If $Q_a$ and $Q_a$ are independent, $\text{Cov}(Q_a, Q_s) = 0$.

$Q_a, Q_b$ can have some positive association. Indeed, as long as $Q_a, Q_b$ are not perfectly correlated ($=1$), the variation of the shared variance will be less than the individual harvest.

Express condition in terms of correlation coefficient $\rho$,

For arbitrary random variables, $X_1, X_2$,

$$\text{Cov}(X_1, X_2) = \rho \sigma_1 \sigma_2$$

$$\rho = \frac{\text{Cov}(X_1, X_2)}{\sigma_1 \sigma_2}$$
Now use the fact that $Q_a$, $Q_s$ come from the same agricultural process so their means and variances are the same so

$$\text{Cov}(Q_a, Q_s) = \rho \sigma \sigma = \rho \sigma^2$$

So our condition becomes

$$\text{Cov}(Q_a, Q_s) < \sigma^2$$

$$\rho \sigma^2 < \sigma^2$$

$$\rho < 1.$$
Review:

- Insurance is a mechanism by which to smooth consumption over uncertain states of nature.
- Provision of insurance increases utility for risk averse agents.
- Provision of insurance provide incentive to shirk (less effort).
- Insurance must balance risk aversion and work incentives.

- **Key component:** information.
Voluntary Behavior

All contracts, whether formal or informal are voluntarily entered. Can not force someone to accept insurance.

Adverse Selection — healthy individuals (e.g., young) may want not want to enroll.

If the healthy opt out and only those who most need insurance participate the insurance plan may not be financially viable.
For the given structure of information (who knows what and when), must make sure that all parties of the insurance contract are no worse off and some are better off.

This means:

1. Insurance companies break even; premiums and benefits are financial viable.

2. Risk averse individual is better off with insurance than without.

3. Insurance contract written so that risk averse individual does not shirk.

Illustrate these ideas through an example developed from one presented on page 604 of the textbook.
Example in the text introduces utility cost of high effort $C = 2$ (utils). The class example sets $C = 0$ but introduces non–farm income $\Delta$ if work only part–time on farm.

Individual faces two choices:


2. Work part–time as farmer, $Pr(Q = H) = q = 1/4$, non farm income $= $300.

Investigate the provision of insurance with and without complete information.
Parameters

Will use numerical quantities as in example p. 604.

Preferences $U(z) = \sqrt{z}$
Payoffs: $H = 2,000$, $L = 1,000$
Nonfarm income (option 2) $\Delta = 300$
Option 1: $\Pr(H) = p = 0.75$
Option 2: $\Pr(H) = q = 0.25$
Enforcement

- Developing countries, insurance informal contracts, social norms encourage reciprocity and punish deviations through social sanctions.
- Once again must investigate incentives of individuals to reciprocate and to deviate.
- Assumption: rational individuals take option that yields highest payoff. (Payoff in utility)
- Enforcement: through social sanctions.
- Equation (15.6) summary of enforcement forces to maintain insurance contract.
Insurance and Credit Blurred Boundary

- Key feature of insurance: lack of memory on history.
- Basing insurance payouts to history of past claims, a scheme ceases to be one of pure insurance, and acquires some characteristics of credit.
- Such history dependent insurance contracts permits a greater degree of consumption smoothing.
- Such arrangements: part insurance, part credit are common in developing countries.
- Social norms and family members perform many of the functions done by markets in developed countries.