Economics 448: Human Capital and Growth Models

September 20, 2012
Need to augment Solow Model

Thus we will enrich model, by questioning and weakening the exogeneity assumptions.

On to endogenous growth models. Endogenous because the rate of growth of driving variables (e.g., technical change) are internal to the model (endogenous).
T. W. Schultz pioneered the idea of “human capital” investment in human beings.

- Interestingly, the importance of human capital (late 1940s) came to him as he realized that models of economic growth didn’t explain differences in per capita income (across countries). Contemporary view (following Marshall) labor was a homogenous lump, only the amount mattered.

- Schultz recognized investment opportunities to increase skills and capabilities. Thus investment in people another form of capital, human capital.
Human Capital

Human capital any form of investment in people. Important that the capital is embodied in the person. Thus, the owner of capital cares about working conditions.

- Schooling
- Training programs
- Experience (on the job training)
- Health
- Migration – an investment to leave a poor labor market and move to a good labor market. Pay fixed cost today for higher wages, earnings “tomorrow”.
- Premarket and pre schooling investments by parents (e.g., child care, Head Start). “Early life investments” all the rage today.
Consequences of Embodiment

Should note that human capital being embodied in people has consequences.

- Can not use human capital as a form of collateral. Slavery outlawed.
- Gives rise to incomplete markets. Can not write a contract to indenture self for educational/training loans.
- May have market failure. Provides justification for government intervention.
Modeling Human Capital

Will extend the Solow growth model to include human capital. D. Ray makes a number of simplifying assumptions to keep the model tractable.

Assumes population growth and depreciation are zero ($n = \delta = 0$). Importantly, there is only skilled labor, measured by the human capital per capita.

Common to think of two kinds of labor, skilled and unskilled. To reduce model to only skilled labor highlights the importance of human capital, but comes at a price.
Human and Physical Capital

Can think of there being two types of capital, physical and human capital.

Human capital is deliberately accumulated, not just the outcome of population growth (which is zero) or exogenously specified technological progress.
Retaining notation as before let per capita output (income) be

\[ y = k^\alpha h^{1-\alpha} \]

\( y \) and \( k \) are per capita output and physical capital, \( h \) is per capita human capital.

As before some of output is consumed and the remainder can be used to create new physical capital \( sy \) and human capital \( qy \). So consumption is \( c = (1 - s - q)y \).
Physical Capital: \( k(t + 1) - k(t) = sy(t) \)

Human Capital: \( h(t + 1) - h(t) = qy(t) \)

Think of \( qy(t) \) as the quantity of physical resources spent on education and training.

In long run all variables \((y, k, h)\) growing at common rate. A rate determined by savings rate \(s\) and propensity to invest in human capital, \(q\).
Common rate

Let \( r = h(t)/k(t) \) then

\[
\frac{k(t + 1) - k(t)}{k(t)} = \frac{sy}{k} = \frac{sk^{\alpha}h^{1-\alpha}}{k} = sr^{1-\alpha}
\]

\[
\frac{h(t + 1) - h(t)}{h(t)} = \frac{qy}{k} = \frac{qk^{\alpha}h^{1-\alpha}}{k} = qr^{\alpha}
\]

Solve for \( r \) yields

\[
r = q/s
\]
Closing the Model

$r$ makes perfect sense the larger the ratio of savings in human capital is relative to that of physical capital the larger is the long–run ratio of $h$ to $k$.

Now use the value of $r$ to compute the long–run growth rate.

$$\frac{h(t+1) - h(t)}{h(t)} = sr^{1-\alpha} = s^\alpha q^{1-\alpha}$$

Hence, long–run growth rate of per capita income, per capita physical capital and per capita human capital is $s^\alpha q^{1-\alpha}$. 
Implications of Human Capital

There are five implications.

1. Physical capital may exhibit diminishing returns yet there may be no convergence in per capita income.
2. Constancy of returns. Now $s$ and $q$ now have growth rate effects, and not just level effects as in the Solow model.
3. Growth effects in item (2) related constancy of physical and human capital combined.
4. Introduction of human capital helps to explain why rates of return to physical capital may not be as high in poor countries as the simple Solow model predicts.
5. The model predicts no tendency toward unconditional convergence (even if parameters all the same).
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Empirical Predictions of HC Growth Model

The model has two predictions

1. Conditional convergence after controlling for human capital. By conditioning on the level of human capital, poor countries have a tendency to grow faster.

2. Conditional divergence after controlling for initial level of per capita income. By conditioning on the level of per capita income, countries with more human capital grow faster.
Barro (1991) paper in the *QJE.* (famous)

Discussion by Ray concludes that the model with human capital provides a better fit than do the models with exogenous factors. Specifically, does a better job of predicting the growth of some of sub–Saharan countries (with very low levels of human capital in the sample period (1960–1985).

However the model with human capital still fails to account satisfactorily for the magnitudes of growth displayed by Korea and Taiwan.
Growth & Development Accounting

A way to measure technical progress. Same basic idea used in growth and development accounting.

Growth Accounting used with time series data (e.g., annual information on a single country).

Development Accounting used to compare two countries at the same point in time. Typically use cross-sectional data (on countries, geographical regions).

Basic Idea: Think of production as composed of two parts:

\[
\text{Output} = \text{Productivity} \times \text{Factors of Production}
\]
Use Cobb Douglas (per worker) production function:

\[ y(t) = A(t)k(t)^\alpha h(t)^{1-\alpha} \]

where \( A(t) \) is a general productivity term
\( k(t)^\alpha h^{1-\alpha} \) composite term of two factors (physical & human capital)
Growth Accounting (Cont)

Take logs

\[ \ln y(t) = \ln A(t) + \alpha \ln k(t) + (1 - \alpha) \ln h(t) \]

Recall that the time derivative of \( \ln(z(t)) \) is \( \frac{d \ln(z(t))}{dt} = \frac{1}{z} \frac{dz}{dt} \)
Growth Accounting (Cont)

Take derivative w.r.t. time $t$

$$
\frac{1}{y} \frac{dy}{dt} = \frac{1}{A} \frac{dA}{dt} + \alpha \frac{1}{k} \frac{dk}{dt} + (1 - \alpha) \frac{1}{h} \frac{dh}{dt}
$$

Represent time derivative by a dot above the variable, $\dot{z} = \frac{dz}{dt}$. Use “carrot” to denote a percent change $\hat{z} = \frac{\ddot{z}}{z}$.

$$
\dot{y} = \dot{A} + \alpha \dot{k} + (1 - \alpha) \dot{h}
$$

Recall that $\alpha$ is the income share of capital while $1 - \alpha$ is the income share of human capital.
Thus the rate of growth of output is the sum of productivity growth and the share weight sum the growth of factors of production.

We observe: $y, k, h$. Requires effort and much attention to detail. Calculation where the devil is in the details.

Direct measurement of the rate of growth of productivity is not credible. (You could try, but no matter the estimate, no one would believe it.)

Hence, “measure” growth rate of productivity as residual

$$\hat{A} = \hat{y} - \alpha \hat{k} - (1 - \alpha) \hat{h}$$
Growth Accounting

The above formulation assumes data on education (to measure HC) is available.

Show for yourself that if the production function is:

$$Y(t) = A(t)K(t)^\alpha P(t)^{1-\alpha}$$

then the growth accounting equation is:

$$\hat{y} = \alpha \hat{k} + (1 - \alpha)\hat{P} + \hat{A}$$
Comparison with Textbook

\[ \hat{y} = \alpha \hat{k} + (1 - \alpha) \hat{P} + \hat{A} \]

Textbook:

\[ \frac{\Delta Y(t)}{Y(t)} = \sigma_k(t) \frac{\Delta K(t)}{K(t)} + \sigma_P(t) \frac{\Delta P(t)}{P(t)} + TFPG(t) \]

\[ TFPG = \hat{A} \]

Ray’s formulation allows income shares of capital and labor to vary over time.
Comments on TFP

Important $P(t)$ should be the working population. Sometimes well approximated by total population, sometimes times not.

Total population not accurate for labor force if major changes in labor force composition (entry by women, or longer schooling period or declining retirement age).
TFP Growth

- Units of $A$ are arbitrary so level of $A$ is meaningless. What’s important is the rate of change of TFP.

- Assumed production function exhibits constant returns to scale. *where assumed?*

- If production function exhibits increasing return to scale the observed factor shares *underestimate* the true productivity of factors. Which implies we *overestimate* the rate of technical progress.
Development Accounting

Start with same basic idea:

\[ \text{Output} = \text{Productivity} \times \text{Factors of production} \]

Assume each country \( i = 1, 2 \) has Cobb–Douglas production function

drop time subscript as doing calculation at the same \( t \)

\[ Y_i = A_i K_i^\alpha N_i^{(1-\alpha)} \]

where \( N_i \) is working population or human capital in country \( i \)

\( A_i \) measure of productivity

\( K_i^\alpha N_i^{1-\alpha} \) composition factor of production
Divide p.f of country 1 by p.f. country 2:

\[
\frac{y_1}{y_2} = \frac{A_1 K_1^\alpha N_1^{1-\alpha}}{A_2 K_2^\alpha N_2^{1-\alpha}}
\]

\[
\frac{y_1}{y_2} = \left[ \frac{A_1}{A_2} \right] \left( \frac{K_1^\alpha N_1^{1-\alpha}}{K_2^\alpha N_2^{1-\alpha}} \right)
\]

\[
Q = \mathcal{P} \times \mathcal{F}
\]

or

\[
\mathcal{P} = \frac{Q}{\mathcal{F}} = \frac{y_1/y_2}{K_1^{\alpha_1} N_1^{1-\alpha} / K_2^{\alpha_2} N_2^{1-\alpha_2}}
\]
Example:

<table>
<thead>
<tr>
<th>Country</th>
<th>$y$</th>
<th>$k$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table: Data to Compare Productivity
Example - Calculation

Assume that countries have same technology with income share of capital \( \alpha = 1/3 \) and \( 1 - \alpha = 2/3 \) the income share of human capital.

\[
\frac{A_1}{A_2} = \frac{\frac{24}{1}}{\frac{27^{1/3} \times 8^{2/3}}{1^{1/3} \times 1^{2/3}}}
\]

\[
= \frac{24}{3 \times 4} = 2.
\]

Hence, Country 1 has twice the productivity of Country 2.
Table 7.2 (ed 2)
Table 1: Development Accounting (2006)

<table>
<thead>
<tr>
<th>Country</th>
<th>$y$</th>
<th>$k$</th>
<th>$h$</th>
<th>$\text{FoP} = k^{1/3} h^{2/3}$</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Norway</td>
<td>0.92</td>
<td>1.08</td>
<td>0.97</td>
<td>1.01</td>
<td>0.92</td>
</tr>
<tr>
<td>UK</td>
<td>0.76</td>
<td>0.69</td>
<td>0.97</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Canada</td>
<td>0.75</td>
<td>0.86</td>
<td>1.01</td>
<td>0.96</td>
<td>0.79</td>
</tr>
<tr>
<td>Japan</td>
<td>0.69</td>
<td>1.10</td>
<td>0.99</td>
<td>1.02</td>
<td>0.67</td>
</tr>
<tr>
<td>S.Korea</td>
<td>0.54</td>
<td>0.73</td>
<td>0.93</td>
<td>0.86</td>
<td>0.63</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.29</td>
<td>0.27</td>
<td>0.79</td>
<td>0.56</td>
<td>0.52</td>
</tr>
<tr>
<td>Peru</td>
<td>0.14</td>
<td>0.12</td>
<td>0.82</td>
<td>0.44</td>
<td>0.32</td>
</tr>
<tr>
<td>India</td>
<td>0.13</td>
<td>0.10</td>
<td>0.74</td>
<td>0.38</td>
<td>0.35</td>
</tr>
<tr>
<td>Cameroon</td>
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<td>0.58</td>
<td>0.23</td>
<td>0.44</td>
</tr>
<tr>
<td>Zambia</td>
<td>0.034</td>
<td>0.032</td>
<td>0.65</td>
<td>0.24</td>
<td>0.14</td>
</tr>
</tbody>
</table>