In this problem set you are asked to solve dynamic programming problems similar to that discussed in class. The first investigates migration decision making with more than two periods and more than two locations. The second studies the effect of exogeneous mortality (i.e., outside the control of the household) on fertility.

(1) Suppose a newly graduated UW–Madison grad (Jodie) is considering whether to remain in Wisconsin or move to one of the four states listed in Table 1. She partitions her lifetime in half with the first–half comprised of three working periods, while the last half is assumed to represent retirement. In the first the periods Jodie makes a location choice decision. There is no decision made in the the second half of the life; all decisions are made at or immediately before retirement. There is no discounting, however, utility flow of the retirement (last) period is three times the value of utility in each of the three decision periods. To place the model in a real world context, each of the first three periods represent a 10 year segment of the worker’s career, say 25 to 55 while retirement is from age 55 to 85.

The decision time line is shown in Figure 1. Today corresponds to time 0. Thus, Jodie makes a decision as to where to move before starting her working career. At time zero, Jodie is deciding where to live and work for the first period of adulthood (ages 25–34). Since she has yet to start work, Jodie’s parents will loan her moving costs at no interest, but must be paid back out of first period’s earnings. Jodie gets to make three subsequent migration decisions. Any moves made in these time periods will be self–financed and paid at the time of the move. Period 3 is the last decision and its purpose is to allow Jodie to move to a location for retirement.

Jodie’s preferences are such that during a work period utility is \( u = \sqrt{c} + a \), where \( c \) is consumption, and \( a \) is climate amenity. Consumption is earnings less any moving costs. Utility in the retirement period is \( \sqrt{3 \cdot c + 3 \cdot a} \) reflecting its longer duration.

Jodie realizes her retirement income will depend on her savings, social security (should it continue to exist) and perhaps a pension plan from her employer. But to keep these initial calculations somewhat straightforward Jodie assumes that her retirement income is $1,000,000, independent of her retirement location.\(^\text{1}\) Jodie recognizes that places may offer different “marriage market” opportunities, but she decides to defer consideration that complication to another (future) time.

The payoffs (earnings and amenity) for each location are described in Table 1.

In constructing the table, Jodie decides that income growth potential is the same across location so she only needs to be concerned with baseline amounts. She has a job offer that may let her remain in Wisconsin. Consequently she has no uncertainty about her earnings.

\(^{1}\)Recall the length of the last period equals the sum of the three work periods. Consequently, at $1M, her annual income is assumed to be about $33,000.
potential in Wisconsin. Jodie however, is uncertain as to how well she will “match” with the other locations. After some reflection, she decides again to keep things simple, and to represent her uncertainty as a binary outcome “low” or “high” with the outcomes having equal probability. Moreover, Jodie considers these outcomes as permanent and represents her earnings whenever she is in a state.  

Jodie determines that the first move to a state costs $15,000 while a return move to a previously visited state is less at $5,000.  

(a) Describe the optimal life time sequence of migration.  

(b) What if Ohio were dropped from consideration, how would it affect Jodie’s plan?  

(c) Assume Jodie decides to represent her retirement pension as proportional to her average lifetime earnings; pension \( \propto W_1 + W_2 + W_3 \). How do you expect her retirement decision will change. I AM ASKING FOR THOUGHTFUL EVALUATION, not that you RECALCULATE THE OPTIMAL SEQUENCE although doing so would be a direct check on your intuition.  

(d) How important do you think marriage payoffs would have to be to change Jodie’s migration plans?  

(2) Assume that a woman at marriage lives for four periods. Suppose that births are biologically feasible in the first two periods after marriage, but that the woman is infertile in the third and fourth periods. Offspring survive infancy (their first period of life) with certainty, but may die in their second period of life, as a “child” (\( 0 \leq \pi < 1 \)). Within periods, deaths occur subsequent to decisions about births. Thus, an offspring born in the first period of the woman’s life and survives infancy (its first period of life) may die in its second period of life before the second period fertility decision is made. Such a death can not be “replaced” by a birth in the third period, because the woman is infertile. For the same reason, a birth in period 2 is not replaceable.  

It is assumed that conditional on surviving the second period,

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2Just to be clear a visit to a state removes all uncertainty. Say Jodie moves to New York on the first move and receives a “low” match. In any period Jodie’s earnings in NY is 120,000. And specifically, should she move away from NY any return will yield earnings of 120,000 per (work) period.  

3Just to be clear, say Jodie moves from Wisconsin to the California in the first period. That move will cost $15,000. If in the second period she moves from California to Florida, New York or Ohio will cost $15,000 while a return move to Wisconsin costs $5,000. If in the third period Jodie were to move back to California that move would cost $5,000. This is to highlight that moving to a previous location is cheaper, not just returning to Wisconsin as “home”.  

4Moreover, a birth in the second period must survive child mortality in period 3.
survival into adulthood (outliving the mother) equals 1. The family is assumed to derive utility only from those offspring who survive to adulthood.\(^5\)

Let \(n_j = 1\) indicate a birth at the beginning of period \(j = 1, 2\) of the woman’s life cycle and zero otherwise. Likewise, let \(d_k^j = 1\) indicate a death of an offspring of age \(k\), \(k = 0, 1\) at the beginning of period \(j\), zero otherwise, given that a birth occurred at the beginning of period \(j - k\). By convention an “infant” is age zero (in its first period of life), and a “child” of age one (in its second period of life). Thus, letting \(N_j\) be the number of surviving offspring at the beginning of period \(j = 1, 2, 3\) of the woman’s life cycle.

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\begin{align*}
N_0 &= 0, \\
N_1 &= n_1(1 - d_0^1)) = M_1^0, \\
N_2 &= M_1^0(1 - d_1^2) + n_2(1 - d_0^2) = M_2^1 + M_2^0, \\
N_3 &= M_2^1 + M_2^0(1 - d_3^3) = M_3^2 + M_3^1.
\end{align*}
\]

where \(M_k^j = \{0, 1\}\) indicates the existence of an offspring of age \(k = 0, 1, 2\) at the end of period \(j\).

Further, let \(c\) be the fixed exogenous cost of a birth and \(Y\) income per period. Finally, the utility in periods 1, 2, 3 is that period’s consumption, \(Y_j - cn_j\), \(j = 1, 2\), \(Y_j\), \(j = 3\) and utility in period 4 is that period’s consumption plus the utility from the number of surviving children \(Y_4 + U(N_4)\). Lifetime utility is not discounted and income is set to zero for convenience.

**Question:** Assuming \(\pi = 0.3\), \(c = 0.71\), \(U(1) - U(0) = 1.8\), \(U(2) - U(1) = 0.8\). Find the optimal policy for this environment.

(3) Now assume that everything is the same as above, except that the mortality regime is such that the mortality risk is constant; the probability that an offspring dies in a period equals \(\pi\). Assume that children must survive three periods to each adulthood.

**Question:** Assuming \(\pi = 0.3\), \(c = 0.71\), \(U(1) - U(0) = 1.8\), \(U(2) - U(1) = 0.8\). Find the optimal policy for this environment.

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\(^5\)One can think of the fourth period of the couple’s life as longer than a single period of life so that a birth in the second period that survives infancy (in the couple’s second period) and its childhood (the couple’s third period), i.e., \(d_0^2, d_3^1 = 0\), will survive to adulthood while the couple is still alive.