Optimal Unemployment Insurance in an Estimated Job Search Model with Savings*

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Abstract

This paper estimates a job search model with savings on Danish microdata that include observations on wealth and wages. Controlling for extensive observed and unobserved worker characteristics heterogeneity, the estimation relates observed unemployment spells to the model implied hazard rate for each worker. The model estimates are sensible and fit the data well. Optimal UI policy is determined in the estimated model as a trade-off between insurance provision and distortion of search incentives. The analysis emphasizes an important policy sensitivity to the interest rate and the importance of including transitional dynamics in the analysis.

JEL Classification Numbers: D1, D9, E2, E6, J4, J6.

1 Introduction

The paper estimates a job search model with savings on Danish unemployment spell data and determines optimal unemployment benefit levels for the estimated model. In the model, an unemployed worker decides on how intensely to search for a new job based on a comparison of the marginal search cost and the net gain of moving from unemployment into employment. The worker can insure against income fluctuations through savings but the insurance will necessarily be imperfect due to the nature of the income process, imperfect asset markets, and the presence of borrowing constraints. Unemployment benefits provide additional insurance but adversely affect the worker's incentives to search for employment. The optimal policy study is focused on the trade-off between providing insurance beyond the worker's ability to self-insure through savings and the adverse impact on the worker's job search incentives. The core of the identification of the model is based exactly on these relationships by relating the worker's observed savings, unemployment benefits, and earnings when employed to observed unemployment durations.

The model estimation successfully captures the key relationships in the data. Notably, wealthier individuals are observed to experience longer unemployment durations which is explained by the model through a negative relationship between the choice of search intensity and savings.[1] Furthermore, for a given benefit level data show a U-shaped relationship between unemployment duration and the wage level of the worker. While not widely recognized, this type of relationship is quite natural in a sequential job search model with savings where shifts of the wage offer distribution imply both substitution and income effects with opposing impacts on the search decision.[2] Both of these relationships are robust to conditioning on observed and unobserved worker characteristics. They are both important identifiers of the curvature of the utility of consumption function. The estimate implies a constant relative risk aversion coefficient of 2.21.

The role of unemployment benefits is in the paper purely one of providing insurance against

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[1] This relationship has previously been established on Danish, Dutch and French data in Lentz and Tranæs (2004), Bloemen and Stancanelli (2001) and Algan, Chéron, Hairault, and Langot (2001), respectively.

[2] The intuition for the non-monotonocity is similar to the interpretation of income and substitution effects associated with wage changes in the labor supply literature. Sequential search and directed search models without savings imply monotone relationships between unemployment duration and the wage.

consumption fluctuations at the cost of distorting search incentives.[3] Papers such as Baily (1978), Flemming (1978), Hansen and Imrohoroglu (1992), and Wang and Williamson (2002) have studied this question in models with savings. However, it is a common feature of these papers as well as the broader literature on optimal unemployment insurance that the use of savings as a self insurance instrument has been seriously curtailed. In studies where savings are allowed, the return to savings is often set at such a low rate that holding savings is costly and consequently the option to use savings as a self-insurance instrument has low value.

I find that the optimal benefit policy is highly sensitive to the relationship between the interest rate and the subjective discount rate. Specifically, the optimal benefit level ranges from a 43% replacement rate for an interest rate almost equal to the subjective discount rate to an 82% replacement rate for a zero interest rate. The sensitivity of savings to the level of the interest rate is a well known result in the consumption-savings literature. However since savings and unemployment insurance are not often studied together, the high sensitivity of unemployment insurance design to the interest rate and subjective discount rate is rarely emphasized.

As also argued in Joseph and Weitzenblum (2003), rather than simply comparing steady states, I find that once savings are included in the optimal policy analysis, one must include transitional dynamics in order to avoid a serious downward bias in the optimal unemployment benefit results. I quantify the downward bias which can be as large as 10 percentage points in the optimal replacement rate.

The analysis of the job search model with savings is complicated by the inability to establish global concavity of the value functions. Danforth (1979) shows that in the special case where employment is an absorbing state, one can characterize the reservation wage choice in relation to the degree of absolute risk aversion of the utility function. In the case of decreasing absolute risk aversion, the reservation wage choice will be increasing in wealth. Flemming (1978) and Acemoglu and Shimer (1999) are examples of the constant absolute risk aversion case combined with the assumption that search costs are monetary, and that there is no lower bound on wealth.[4] In this

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[3] Studies such as Acemoglu and Shimer (1999) emphasize efficiency issues associated with providing unemployment benefits.

[4] Acemoglu and Shimer (1999) assume a directed search technology. Jobs differ with respect to their wage and the worker can choose which job to apply for. Higher-wage jobs will have longer queues and a choice of a higher

case, the search choice is unaffected by wealth and the results for this special case do generalize to the case in which employment is not an absorbing state. However in general, once employment is no longer an absorbing state, construction of characterization theorems of the worker's search and savings choices is made difficult because the value functions may not be globally concave. Lentz and Tranæs (2005) establish characterization theorems for the search intensity model for the more general case where search costs can be non-monetary.

The paper is structured as follows. In section 2 the model and its key characteristics are presented. Section 3 presents the estimation strategy, data, and estimation results. Based on the results of the estimation, the paper proceeds by determining optimal benefit levels at an individual level in section 4.1 and section 4.2 considers optimal group wide insurance schemes. Finally, section 5 concludes.

2 The Model

Consider an infinitely lived, utility maximizing worker who faces risk of job loss. When employed he receives a fixed wage w and during unemployment he receives unemployment benefits b, where $b < w$. The worker can smooth consumption over income states by use of savings that carry a return of r. There are no other insurance instruments available to the worker.

Generally, the worker faces two decision problems; how much to consume and how much to search. The objective is to maximize the discounted stream of future utility which is assumed to be separable in both time as well as consumption and search. During employment, the decision problem facing the worker is simply how much to save for the next period. Since the wage distribution is degenerate, on-the-job search is ruled out and issues such as effort choices on the job will also be ignored. It will simply be assumed that the worker faces an exogenous job separation rate δ. During unemployment, the decision problem is more complex. Like in the state of employment, the worker will have to decide how much to save. Furthermore, the worker decides how much effort to put into job search. A higher search intensity will raise the probability of receiving a job offer but also implies a greater utility loss.

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probability of moving into employment (a choice of a lower wage job) is associated with a future income loss.

The model ignores life-cycle savings motives. While obviously a strong abstraction, the life cycle savings motive is less pronounced in Denmark than in for example the United States because the Danish welfare system to a greater extent smooths out income and expenditure over the life cycle through generous pensions, high income taxation, and free education and health care.

It is assumed that each worker faces a wage distribution in which the second and higher moments are zero. Different workers face different wages and as such the overall economy will display a full wage distribution. But each worker faces a single wage only.[5] I will be employing an estimation strategy that relates observed unemployed spell duration to the model implied unemployment hazard rate for given worker characteristics. As such, the maintained assumption of the model is that unemployment duration variation is a result of search intensity variation only. The assumption is consistent with empirical evidence in Devine and Kiefer (1991) where it is argued that the acceptance/rejection decision of wage offers plays a small role in observed unemployment hazard rate variation.[6] The focus on the search intensity decision also has the advantage of simplifying certain aspects of the subsequent unemployment insurance policy analysis.

It is assumed that all workers are eligible to receive unemployment benefits b and that benefits are of infinite duration. This is a good approximation to the actual Danish unemployment system in which benefit duration in the 1980's and early 1990's was indeed at times infinite and at no point shorter than 5 years. In the uncommon event where a worker is ineligible to receive unemployment benefits, he moves into the welfare system where he receives payments roughly equal to 2/3 of the unemployment benefits. During the benefit period, the UI system would force the worker to accept brief employment spells or education offers. But the first of these 'harassments' was not forced on the worker until after more than two years of unemployment. The average unemployment spell in the data is 16 weeks. While there are restrictions on eligibility, they are generally quite easy to satisfy and the question of qualifying for benefits is not of great concern for the Danish worker. The system is voluntary and if the worker decides to participate he must pay an insurance premium.

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[5] The assumption rules out reservation wage considerations and is a simplification relative to the fairly large literature on estimation of wage search models. See Devine and Kiefer (1991), Wolpin (1995), Van den Berg (1999), and Eckstein and Van den Berg (2007) for a survey of the literature.

[6] This is, of course, not evidence that the wage offer distribution is degenerate but may simply reflect the existence of on-the-job search. But it nevertheless points to the search intensity decision as a prime source of worker behavior driven unemployment hazard variation.

The system is heavily subsidized and the worker pays only about 1/3 of the actual premium. More than 80% of the labor force participates in the system. Non-participants are generally very low wage workers and very high wage workers. For the very low wage workers, the welfare system will provide comparable insurance to the UI system. For the very high wage workers, the UI system provides very little insurance due to an upper bound on benefit payments.

In effect, the upper bound on benefit payments is so restrictive that the Danish UI system can be characterized by a wage independent benefit scheme. In the data in this paper, everybody faces the same constant level of benefits except for the lowest 4 percent of the wage earners. The lowest 4 percent of wage earners receive benefits equal to 90% of their previous wage. As a result, the data displays great variation in replacement rates and as such the degree of insurance.

The entire decision problem can be written as:

$$\begin{aligned}
\max_{\{c_t,s_t\}_{t=0}^{\infty}} \quad & E\sum_{t=0}^{\infty}(1+\rho)^{-t}\left[u\left(c_t\right)-e\left(s_t\right)\right] \\
st \quad : \quad & k_{t+1}=(1+r)\,k_t+n_t w+\left(1-n_t\right)b-c_t \\
& c_t\geq 0 \\
& k_t\in\left[\underline{k},\bar{k}\right] \\
& s_t\geq 0 \\
& \Pr\left(n_{t+1}=1|n_t=1\right)=1-\delta,\ \Pr\left(n_{t+1}=0|n_t=1\right)=\delta \\
& \Pr\left(n_{t+1}=1|n_t=0\right)=\mu\left(\lambda s_t\right),\ \Pr\left(n_{t+1}=0|n_t=0\right)=1-\mu\left(\lambda s_t\right),
\end{aligned}$$

for all t. k_t is the worker's wealth at time t. r is the interest rate and ρ is the subjective discount rate. $n_t\in\{0,1\}$ denotes the state of employment at time t, where $n=1$ denotes employment and $n=0$ denotes unemployment. It is assumed that $u(\cdot)$ is strictly increasing and strictly concave, $e(\cdot)$ is strictly increasing and weakly convex with $e(0)=0$, and $\mu(\cdot)$ is strictly increasing and strictly concave with $\mu(0)=0$ and $\lim_{x\rightarrow\infty}\mu(x)=1$.

λ will be referred to as a base offer arrival rate. In the structural estimation λ will be allowed to vary across workers according to observed worker characteristics as well as an unobserved worker fixed effect.

The lower bound on wealth $\underline{k}$ can be interpreted as a capital market imperfection. However, it can also simply be a lower bound that naturally arises from the assumption of non-negative consumption combined with asymptotic budget balance. See Aiyagari (1994) for more on this type of argument. The upper bound will be set so that it is not restrictive in equilibrium. This can be done as long as $r < \rho$.

The model can be formulated recursively. Let $V_e(k)$ be the maximal present value of being employed with wealth k. Similarly, $V_u(k)$ is the value function associated with unemployment at wealth k. Let the choice of next period's wealth level be denoted by k'. The Bellman-equations of the model can then be stated as:

$$\begin{aligned} V_e(k) &= \max_{k' \in \Gamma_w(k)} \left\{ u\left((1+r)k + w - k'\right) + \frac{(1-\delta)V_e(k') + \delta V_u(k')}{1+\rho} \right\} && (1) \\ V_u(k) &= \max_{k' \in \Gamma_b(k), s \geq 0} \left\{ u\left((1+r)k + b - k'\right) - e(s) + \frac{\mu(\lambda s)V_e(k') + (1-\mu(\lambda s))V_u(k')}{1+\rho} \right\}, && (2) \end{aligned}$$

where $\Gamma_y(k) = \{k' \in \mathbb{R} | \underline{k} \leq k' \leq (1+r)k + y\}$. It will right away be assumed that:

$$\mu(\lambda s) = 1 - \exp(-\lambda s). \tag{3}$$

The policy functions of the model, savings when employed and unemployed as well as the choice of search intensity are denoted by $k_e(k)$, $k_u(k)$ and $s(k)$, respectively.

The optimal search choice satisfies the first order condition,

$$e'\big(s(k)\big) = \frac{\lambda}{1+\rho} \exp(-\lambda s(k)) \big[V_e\big(k_u(k)\big) - V_u\big(k_u(k)\big)\big]. \tag{4}$$

Under a set of sufficient conditions, most notably the existence of a lottery in wealth, Lentz and Tranæs (2005) show that separability between consumption and search in the utility function will result in a decreasing search intensity choice in wealth. The sufficient conditions ensure concavity of the value functions. While the sufficient conditions have not been made in the model at hand, numerical model solutions in this paper always yield globally concave value functions and consequently given the separable utility function assumption that the search choice is decreasing in wealth. Intuitively, search intensity is decreasing in wealth because the gains to search $V_e(k) - V_u(k)$ can be shown to diminish as wealth increases. Due to the separability in the utility function, search costs are unaffected by wealth holdings and the result follows directly.

Given $r < \rho$, the worker will always reduce wealth holdings during unemployment spells. There exists an upper wealth bound below which the worker will increase wealth holdings while employed. For wealth holdings above the upper wealth bound, the worker dissaves in both employment states but the worker always dissaves by more in the unemployed state.[7]

In a regular search model without savings, it is well known that the search intensity choice varies monotonically with shifts of the wage offer distribution.[8] In the model represented by equations (1) and (2), an increase in the wage represents a right-shift of the offer distribution. Once savings are included in the model the monotonicity result no longer holds. When the worker can smooth consumption via savings, a right-shift in the wage offer distribution is associated with two opposing effects analogous to the income and substitution effects associated with a wage increase in labor supply theory. A right-shift of the wage distribution implies a greater pay-off to search activity and the substitution effect dictates that the worker substitute into more search. However, when the worker can transfer income from the employed to the unemployed state via savings, a right-shift in the wage distribution raises consumption also in the unemployed state, which implies an income effect on unemployed search intensity. In isolation, the income effect dictates that the unemployed worker search less. In the case where the worker cannot transfer income from the employed state to the unemployed state either because she is at the lower wealth bound or because wealth cannot be stored, there is only a substitution effect.[9]

In the case where workers can save, model simulations suggest that the substitution effect dominates at low wage levels where the search choice is consequently increasing in the wage. If the utility of consumption has a sufficient degree of curvature, the income effect will eventually begin to dominate resulting in a decreasing relationship between the search intensity choice and the wage.

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[7] These results are formally established in Lentz and Tranæs (2005) subject to the existence of a wealth lottery. In the model at hand, these results always obtain in the numerical solutions of the model.

[8] See among others Mortensen (1986) where it is shown that a right-shift of the wage offer distribution increases search intensity.

[9] The argument generalizes directly to the employed search intensity choice in an on-the-job search model with savings.

3 Estimation of the Model

I estimate the model by relating observed unemployment spell durations to the model implied unemployment hazard rate. Let $h_i(\tau)$ be the model implied unemployment hazard rate in the τ'th period of the unemployment spell. The probability of observing a spell length t for worker i is then simply $\Pr(T_i = t) = h_i(t)\,\Pi_{\tau=1}^{t-1}(1 - h_i(\tau))$ which is the basis of the likelihood function for the data. The estimation strategy requires unemployment spell duration data where observed unemployment spell durations are linked with observed worker characteristics.

Denote by t_{ij} the length of worker i's j'th spell. Let $z_{ij} = 0$ denote that the spell is right censored, and $z_{ij} = 1$ otherwise. κ_{ij} is worker i's observed wealth level at the outset of the j'th spell and w_{ij} is the wage that the worker is expecting to receive in the new job. Finally, denote by X_{ij} a set of other worker characteristics which includes education, age, gender, spouse's income, number of children, and occupation.

The search intensity choice will be characterized by positive duration dependence because wealth is being gradually reduced throughout the spell. Given an observed initial amount of wealth at the outset of the spell κ, one can infer the search intensity choice at any point during the spell. The wealth inference is made by iterating on $k_u(\cdot)$ (i.e. the inference about the worker's wealth holdings one period into an unemployment spell is $k_u(\kappa)$). Naturally, in the case where one directly observes wealth throughout the spell, this procedure is moot. Such wealth data is not available, though. Wealth is only observed at a yearly frequency. Therefore, one cannot observe the change in wealth holdings during the spell which is observed at a weekly frequency. [10]

In the estimation of the model, worker heterogeneity can enter via wealth at the outset of the spell κ, the wage level w and the offer arrival rate λ. While the offer arrival rate is not itself observable, all of the remaining observed worker heterogeneity will enter via this parameter. Furthermore, the estimation will also allow unobserved worker heterogeneity to enter via the offer arrival rate. The

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[10] In the estimation, it is simply assumed that the wealth level at the outset of the unemployment spell is equal to the wealth level at the beginning of the year of the spell. Ideally, based on the model one can attempt to adjust the wealth holding according to the week of the year the spell actually starts in order to more precisely capture the wealth holdings at the beginning of the spell. But simulations of the model suggest that this will result in very minor adjustments to the wealth holdings and consequently the procedure is not likely to affect the results in any significant way.

unobserved heterogeneity is assumed to be uncorrelated with the observed worker characteristics and is analyzed in much the same way as in Heckman and Singer (1984). In particular, it is assumed that:

$$\lambda_{ij}\left(\beta,\sigma_i\right)=\exp(X_{ij}'\beta+\sigma_i),\ \sigma_i\sim G\left(\cdot\right),$$

where β will be a set of parameters to be estimated.[11] The unobserved heterogeneity term σ_i will be assumed to be drawn from a common probability distribution with L support points. The distribution and the support points will all be estimated.

It is a valid concern that possible correlation between unobserved and observed characteristics may bias the estimates. However because of the common occurrence of multiple spells for the same individual (on average the data have 7.05 spells per worker), identification will use observed characteristics variation within an individual's spells which reduces the importance of the issue of potential correlation between the unobserved heterogeneity term and observed characteristics.

The unemployment hazard rate in the model is given by $\mu\left(\lambda s\right)$ and is directly determined by the worker's search intensity choice. The search intensity choice will be a function of the worker's initial wealth, her wage, her offer arrival rate, and via the reduction in wealth over the unemployment spell, how long she has been unemployed. Finally, the search decision will also depend on all of the structural model parameters denoted by θ. The search choice is given by $s\left(\kappa,w,\lambda,t,\theta\right)$. All in all, for a given unobserved heterogeneity term σ_i, the probability of observing the tuple $\{t_{ij},z_{ij},\kappa_{ij},w_{ij},X_{ij}\}$ is simply the probability of an arrival of an offer in week t_{ij} and no arrivals prior to this:

$$\begin{aligned}L_{ij}\left(\theta,\beta,\sigma_i\right) &= \mu\left(\lambda_{ij}\left(\beta,\sigma_i\right)\cdot s\left(\kappa_{ij},w_{ij},\lambda_{ij}\left(\beta,\sigma_i\right),t_{ij},\theta\right)\right)^{z_{ij}}\times\\ &\quad \prod_{t=1}^{t_{ij}-1}\left[1-\mu\left(\lambda_{ij}\left(\beta,\sigma_i\right)\cdot s\left(\kappa_{ij},w_{ij},\lambda_{ij}\left(\beta,\sigma_i\right),t,\theta\right)\right)\right].\end{aligned}$$

The likelihood of observing all of worker i's J_i spells is then simply,

$$L_i\left(\theta,\beta,G\right)=\sum_{l=1}^{L}\Pr\left(\sigma_i=\sigma_l\right)\prod_{j=1}^{J_i}L_{ij}\left(\theta,\beta,\sigma_l\right).$$

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[11] Both spouse's income as well as the number of children are included in X. Ideally, these characteristics should enter via the budget constraint but this will add another dimension of heterogeneity to the estimation. While this is not conceptually hard, it pushes the computational requirements beyond what is currently feasible. The characteristics are included in the offer arrival rate in an attempt to also control for this dimension of observed heterogeneity.

The full likelihood for all the observations is given by:

$$L(\theta, \beta, G) = \prod_{i=1}^{N} L_i(\theta, \beta, G) . \tag{5}$$

The estimates of (θ, β) are found via basic maximum likelihood estimation, that is:

$$\left(\hat{\theta}, \hat{\beta}, \hat{G}\right) \in \arg\max_{\theta,\beta,G} L(\theta, \beta, G) . \tag{6}$$

One cannot obtain a closed form solution for $s(\cdot)$. Thus, for each parameter choice one must numerically solve the model and evaluate the likelihood function based on the new policy functions. The details of the numerical issues are described in the appendix.

3.1 Data

The data used in the estimation below were made available by Centre for Labour Market and Social Research in Århus, Denmark. The dataset follows 0.5% of the Danish population on a weekly basis from 1980 through 1994 recording basic information on each individual's employment status. The data are then merged with individual specific information which includes financial data from the Danish tax authorities. In particular, this includes information on individual earnings and wealth holdings where the wealth measure includes all assets as well as liabilities except pension savings.[12] The tax filings also provide information on any income stemming from other family members. Other databases provide information on the level of education of the worker, the amount of work experience, age and gender of the worker, the number of children the worker cares for, etc.

A data set is then constructed where the basic observation is an unemployment spell. For each spell, the length and the worker ID of the spell is recorded, whether the length is right censored, the level of wealth at the beginning of the spell, the year in which the spell started, the wage of the worker and other worker characteristics. In general, all worker characteristics are observed only on a yearly basis while the employment status is observed weekly. Multiple worker spells are quite common which will be very useful in dealing with issues of unobserved heterogeneity. The data are summarized in table 1.

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[12] Wealth data were collected as a basis for the Danish wealth tax that was in effect through 1996. The wealth tax only taxed wealth holdings above a very large threshold which happens to be well above the upper censoring bound on wealth. Thus, one need not worry that the wealth measure underestimates wealth holdings due to tax avoidance incentives. Gross asset and liability observations are censored above 1.1 mio DKK (inflation adjusted indexed to 1990 DKK) by Statistics Denmark due to anonymity considerations.

All spell lengths greater than 52 weeks have been censored at 52 weeks and all spells with durations below 3 weeks have been ignored in that they provide only noise. Most of these spells are either vacations or spells that do not represent unemployed search but rather a brief spell of non-employment associated with a job-to-job switch. Estimations were run in which the short spells were included. The point estimates did not change appreciably but variance increased. Seasonal and other types of temporary layoffs where the worker returns to her previous employer after a relatively brief period of time have also been eliminated from the data. The final data set consists of a total of $12,865$ spells.

As discussed above, the Danish unemployment insurance system is well approximated by a constant benefit level policy. The model allows for a full wage distribution for the economy but assumes that each worker faces a single wage. All higher moments of the wage distribution are assumed to be zero. The first moment of the individual's wage offer distribution can be identified in various ways. In the estimation below, the wage measure is simply the realized wage of the year prior to the year of the unemployment spell. Observationally, it is clear that there is a noise component to the realized wage. As such, one can attempt to reduce the noise by taking an average over several wage observations around the year of the unemployment spell in question. Alternatively, one can adopt the approach that the worker's expectation about the future wage is best approximated by the realized wage in the new job. Furthermore, one can attempt to predict the wage via a wage regression. This approach has the disadvantage that wage regressions typically only explain roughly 25-30% of the observed wage variation. All of these choices are consistent with the model in that it is assumed that the wage of the particular worker is fixed. Reduced form estimations were performed on all of the different wage measures mentioned and they all yield the same qualitative results.

3.2 Estimation Results

The estimation is performed given a constant relative risk aversion specification of consumption, $u(c) = c^{1-\alpha}/(1-\alpha)$. Search cost are assumed to be exponential, $e(s) = As^{\gamma}$, where A is not separately identified from the level of the offer arrival rate λ and is set to be numerically convenient, $A = 100$. The period length in the simulation is set at one week. This is in part driven by the weekly observation frequency in data. But also, it allows a less restricted use of the search intensity decision because it is not assumed to be fixed for long periods of time. The interest rate r is set an annual rate of 5%, that is $r = (1.05)^{1/52} - 1$. The subjective discount rate is set at the slightly higher annual rate of 5.1%, which ensures the existence of an upper bound on the ergodic wealth distribution. The job destruction rate is set at $\delta = 1/243$ which fits the average employment spell in the data of 243 weeks. The estimate is taken from Rosholm and Svarer (2000) who estimate the parameter on the same basic data as in this paper. The benefit level is normalized at $b = .1$. All income and wealth observations are adjusted according to this normalization. Finally, the lower wealth bound is set at the minimum observed wealth level $\underline{k} = -41.3$. The upper bound is set above the upper bound on the ergodic wealth distribution which is also well above the maximum observed wealth distribution.

The identification strategy does not identify the difference between r and ρ. This parameter relationship primarily affects savings behavior and has little impact on search behavior. Theoretically, one would expect that the the (r, ρ) relationship will affect the duration dependence of the search choice because the rate of dissaving during unemployment is sensitive to changes in (r, ρ). But given that most workers are not liquidity constrained, the changes in rates of dissaving are not large enough to generate noticeable effects on duration dependence.

Identification of a common lower wealth bound has not been feasible. The wealth bound must lie at or below the minimum wealth observation in the data. Thus, identification will be driven by the very few observations with very low wealth levels. The vast majority of the spell observations are characterized by wealth levels such that a change in the lower wealth bound below the minimum wealth observation has no effect on worker behavior. Consequently, the sensitivity of the search decision with respect to wealth is purely driven by the utility function parameters.

All in all, the structural estimation determines $(\alpha, \gamma, \beta, G)$ where β includes 26 parameters. The estimates of all parameters except the yearly dummy parameters are given in table 2 . The yearly dummy estimates are displayed in figure 1. The number of support points in G is set at $L = 4$. Ideally, one would prefer to continue to add support points until additional points no longer improve the estimation. However, estimating the model is computationally very expensive and experimentation with the number of support points in G is not currently feasible.[13]

Comparing the model estimates to the more flexible form analysis of the same data in Lentz and Tranæs (2004), it is seen that the model fits the data well. Generally, the signs and statistical significance of the β coefficients are similar to the corresponding estimates in Lentz and Tranæs (2004). It is seen that the signs of the age and education effects are negative. Similar results are found in the literature such as Meyer (1990).

While the risk aversion estimate is well within the range of previous such estimates, it is worth noting that the estimate does imply significantly more risk averse workers than the typical assumption of log-utility in the optimal unemployment insurance literature. Furthermore, the search cost function estimate implies that the search behavior of Danish workers is relatively insensitive to changes in incentives. This is consistent with the findings on Danish labor supply elasticities in Frederiksen, Graversen, and Smith (2001).

Turning to the effect of occupation, it is seen none of these estimates are significantly different from zero. The residual occupational category is unskilled workers. Thus, no occupational category has significantly different offer arrival rates relative to unskilled workers.

It is interesting to note the gender difference associated with spousal income. A woman seems to experience longer unemployment spells the higher her spouse's income. This is consistent with the argument that the household insures the worker and further emphasizes the point that one should ideally include spouse's income in the budget constraint rather than in the offer arrival rate. However, the effect on men is directly the opposite. Furthermore, the effects are statistically very strong.

A dummy for whether the worker owns real estate has also been included in the analysis. One might suspect that if a significant portion of the worker's wealth is tied up in real estate and if the credit markets are imperfect in the sense that workers cannot borrow against real estate holdings, then this should reduce the insurance value of the wealth holdings. If this is the case, one should expect a positive sign on the dummy variable. Alternatively, one might suspect that moving costs

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[13] Lentz and Tranæs (2005) estimate a proportional hazard model with unobserved heterogeneity on the same data as in this paper. In this estimation, G was estimated to have 4 support points. Adding more points beyond this did not improve the estimation.

are significantly higher if the worker is a homeowner. As such, homeowners may face lower offer arrival rates due to lower geographical mobility. As it turns out, the sign is negative suggesting that the latter effect dominates. The effect is not significantly different from zero, though.[14]

The yearly dummy effects in figure 1 very clearly capture the effect of the business cycle on the offer arrival rate. In times of recession, the offer arrival rate is low and the unemployment rate is high and vice versa. Furthermore, one can clearly detect a lead effect in the offer arrival rate of about one year.[15]

The utility function estimates primarily affect the wealth and wage effects on the hazard rate. In figure 2, the estimated unemployment hazard is shown for an average offer arrival rate ($\lambda = .09826$). The wealth and wage effects are shown in isolation in figure 3 where reduced form estimates of the wealth and wage effects are also included with dashed lines. The reduced form estimates are based on a standard proportional hazard model and are discussed in detail in Lentz and Tranæs (2005).

The model imposes a negative wealth effect on the hazard. Thus, if a positive relationship exists in the data the best the model can do is to eliminate the wealth effect altogether. However, from the reduced form estimation in Lentz and Tranæs (2005), it is known that there is an overall positive relationship between duration and wealth in the data and the structural model successfully captures this relationship. The fundamental positive relationship between wealth and unemployment duration is also found on Dutch and French data in Bloemen and Stancanelli (2001) and Algan, Chéron, Hairault, and Langot (2001), respectively.

Reduced form estimations in Lentz and Tranæs (2005) confirm that the relationship between unemployment duration and wages is non-monotone. This is done under a wide variety of wage measures. It is seen that the structural estimation captures this relationship as well: For workers with sufficiently high wealth levels, the income effect associated with an increase in the permanent wage

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[14] Again, like in the case of spousal income and the number of children variables, one may have reservations about including the real estate dummy in the offer arrival rate given that the effect on the hazard rate should be a behavior driven phenomenon and consequently it should be included elsewhere in the model. However, this has been computationally infeasible and so a compromise has been made in order to allow for some control of the observed worker heterogeneity.

[15] The model does not include a stochastic aggregate shock process. As such, the agents are assumed to believe that the aggregate offer arrival shocks are permanent. The estimated yearly dummy effect obviously contradicts such beliefs. The model can very well be expanded to include a stochastic aggregate shock process in the offer arrival rate. However, it is a reasonable conjecture that it will at most have a secondary impact because the average unemployment spell is only about 16 weeks and the relevant planning horizon is consequently quite short relative to the permanence of an aggregate business cycle frequency shock.

eventually drives down the search choice. At low wealth levels, the estimated unemployment hazard rate is monotonically increasing, though. This is driven by the model that dictates a monotone wage effect for the very low wealth levels.

It is important to note that the estimated wage effect need not be non-monotone. This is a result of the basic relationships in the data. In order to get this result, one must have a sufficient degree of risk aversion. For a sufficiently low degree of risk aversion the wage effect is monotone for all wealth levels. Consequently the wage relationship is an important identifier of the degree of risk aversion in the model.

In general, α and γ are identified by both the wage and the wealth effects. A higher α will tend to introduce a non-monotone wage relationship but also a stronger wealth effect. The estimate in the analysis is determined by both a positive relationship between spell duration and wealth and the non-monotone relationship between spell duration and the wage of the worker. γ primarily affects the magnitude of the response to changes in wealth and wages. Data show that the magnitude of the change in the observed unemployment hazard rate over the wealth and wage dimensions is relatively small which consequently yields a high γ estimate.

Turning to the duration dependence of the search decision, it is seen in figure 4, that the estimated effect is small. For an average individual who holds wealth corresponding to the median of the wealth distribution, the hazard rate changes only from 6.70% to 6.98% over a 10 year long unemployment spell. In the first year, the hazard rate rises only to 6.73% implying a change of only 0.03 percentage points. This reflects an estimated wealth change that is not very big as well as the high γ estimate. Had one assumed a lower interest rate, the dissaving would be stronger and consequently the duration dependence would be stronger. However, the high γ estimate does limit how large the effect can be.

One can of course choose (k, w, λ) combinations where the duration dependence is more pronounced but the basic message here is that one should not expect the wealth effect to play a dominant role in empirical duration dependence studies. And in fact, the general result in these studies is that the overall duration dependence (as seen in the baseline hazard) is either zero or negative suggesting the impact of other stronger effects such as loss of skills, discouragement, exhausting the pool of potential jobs and the like. A similar result is shown in Lentz and Tranæs (2005), where the wealth effect on spell duration is shown to be statistically significantly negative but small in absolute terms.

The magnitude of the moral hazard effect is estimated to be quite low because of the relatively high estimate of the search cost function curvature, γ. One should interpret this result with some care. It is a result of the fact that the observed unemployment duration responses to changes in economic incentives are small in the data. However, this is not to say that the analysis is incompatible with results in for example Meyer (1990) where the unemployment hazard reacts quite strongly to the expiration of benefits. In contrast to Meyer's (1990) study of benefit exhaustion in the United States, it is unlikely that any agent in the Danish data is actually faced with close to zero consumption in any state. Therefore, one should be careful not to extrapolate the curvature of the search cost function into the region that is relevant for the extremely low consumption cases. The optimal unemployment benefit policy analysis in the following sections is effectively not constrained by this limitation. Since the data do in fact identify that unemployment durations are somewhat inelastic even at relatively high replacement rates, the relevant optimal policy region becomes one of providing insurance levels within the existing range of replacement rates in the Danish data.

4 Optimal Unemployment Benefit Insurance

This section will study the optimal provision of unemployment benefit insurance in the estimated model. The benefit policy is constrained to a fixed level of benefits b and a fixed proportional income tax τ. Benefits are assumed to last indefinitely and the worker is always eligible to receive them.

Thus, I will be disregarding the issue of the optimal design of a benefit profile over unemployment duration and simply impose the constant profile over duration constraint. The isolated question of optimal benefit design over duration has been studied in job search models without savings in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). Here, it is shown that the cost minimizing profile is decreasing in unemployment duration. Pavoni (2006) modifies these insights with the introduction of utility bounds to show that the optimal benefit profile, while initially decreasing, eventually reaches a lower bound in finite duration and remains there. This is combined with the result that the optimal employment tax is constant and independent of unemployment history. Kocherlakota (2004) and Shimer and Werning (2007) study the optimal design problem with hidden savings and establish that once the direct link between income and consumption is broken, the optimal income path during unemployment may very well be constant. If it is not, it is shown that the welfare loss of constraining the benefit path to be constant is small. Coles (2006) introduces severance payments to the optimal mechanism design problem with hidden savings and shows that a constant benefit path combined with a lump sum payment yields welfare outcomes very close to the optimal.

Throughout the optimal policy analysis in this section, it will be assumed that $\underline{k}$ is independent of the unemployment insurance system. The lower bound has to be consistent with non-negativity of consumption for all UI schemes. The strictest condition on the lower wealth bound is therefore imposed by the $b = 0$ UI scheme which implies that $\underline{k} = 0$. Thus, the policy analysis focuses on the case with no borrowing. The independence condition on the lower wealth bound allows a focus on the effects of the UI system in isolation from any changes in the credit conditions.[16]

4.1 Individual Unemployment Benefit Insurance Schemes

In this section, I will study optimal unemployment benefit provision from an individual level taking the current state as well as the type of the worker as an initial condition. The income tax rate is set such that the worker's expected future stream of discounted tax payments exactly balance her expected future stream of discounted benefit receipts. In this sense, the system can be said to be actuarially fair. It is assumed that the worker can commit to this scheme and that it will remain fixed even as the state of the worker changes. This, in spite of the fact that there are cases where both parties of the contract would happily dissolve it to sign a new one. However, the design of a more sophisticated contract is beyond the scope of this paper.

The individually optimal unemployment benefit system is found by taking the state of an individual worker as given and determine the preferred replacement rate subject to the constraint that the discounted tax and benefit streams balance each other. The design problem can be stated as

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[16] The impact of unemployment insurance on credit constraints is a very interesting question but is likely better studied in a model that allows for default.

follows:

$$
\begin{aligned}
\max_{b,\tau} V_i(k)\,,\ i &\in \{e,u\} \\
st: V_e(k) &= \max_{k'\in\Gamma_w(k)} \left\{ u\left((1+r)k + (1-\tau)w - k'\right) + \frac{(1-\delta)V_e(k') + \delta V_u(k')}{1+\rho} \right\} \qquad (7) \\
V_u(k) &= \max_{k'\in\Gamma_b(k), s\geq 0} \left\{ u\left((1+r)k + (1-\tau)b - k'\right) - e(s) + \right. \\
&\qquad \left. \frac{\mu(\lambda s)V_e(k') + (1-\mu(\lambda s))V_u(k')}{1+\rho} \right\} \qquad (8) \\
B_i(k) &= 0 \\
B_e(k) &= \tau w + \frac{(1-\delta)B_e(k_e(k)) + \delta B_u(k_e(k))}{1+r} \\
B_u(k) &= -(1-\tau)b + \frac{\mu(\lambda s(k))B_e(k_u(k)) + (1-\mu(\lambda s(k)))B_u(k_u(k))}{1+r},
\end{aligned}
$$

where $k_e(k)$, $k_u(k)$ and $s(k)$ are the policy functions associated with the maximization problems. The budget constraint, $B_i(k) = 0$, associated with the unemployment benefit scheme is non-trivial. It takes into account the worker's behavior under the given insurance scheme and therefore makes the worker internalize the moral hazard problem.

The optimal unemployment benefit schemes for the estimated model are shown in table 3 as a function of state and type. The scheme is represented by a replacement rate b/w and the associated income tax rate that balances the budget. The preferred replacement rate is a negative function of wealth. This reflects the fact that a wealthier worker is already well insured against income fluctuations via her wealth. Therefore, a wealthy worker can avoid the distortive effects of unemployment benefits without giving up much insurance.

An unemployed worker generally prefers more insurance than an employed worker. But for a given wealth level and replacement rate an unemployed worker faces a higher tax rate than an employed worker because benefits are being paid immediately. Thus, once the importance of the insurance aspect fades for higher wealth levels, the cost effect may actually drive the unemployed worker to demand slightly less insurance than the employed. It is important to keep in mind that a given wealth level corresponds to fewer weeks worth of consumption for a high wage type than a low wage type. Thus, a given wealth level has less insurance value for a higher wage. This explains why high wage types want more insurance than low wage types for a given wealth level. In the data, the average worker holds a net wealth level of roughly half a year's worth of wages. In table 3, this corresponds to a value of $w = 0.2$ and $k = 5.2$.

Table 4 presents a sensitivity study that varies the curvature of the utility of consumption function and the curvature of the search cost function. The replacement rates are calculated for a wage $w = 0.2$ which is the median wage. The results in table 4 are found by setting the utility function parameters at a particular set of values and then re-estimate the model subject to the utility function parametrization in question. The optimal replacement rates are then calculated for the average offer arrival rate of the new estimation. This way the unemployment risk is held constant over changes in the utility function parameters and the changes in the optimal replacement rates consequently only reflect the changes in the utility function parametrization.

The middle row of table 4 displays the sensitivity of the results in table 3 to a change in the curvature of the search cost function. In this case, the search cost function has been assumed to be quadratic and as such, the moral hazard problem is now significantly more pronounced. And indeed, the optimal replacement rates are significantly lower because an increase in the unemployment benefits has a greater impact on the worker's search choice. The sensitivity of optimal UI to the moral hazard dimension of the problem is also analyzed in Hansen and Imrohoroglu (1992) where a similar type of sensitivity is found. The top and bottom rows then vary the degree of risk aversion. Generally, one finds the natural result that less risk averse workers prefer less insurance.

Overall, one finds high optimal replacement rates for the estimated model. In fact, the optimal replacement rates are within the range of the high actual replacement rates in the Danish unemployment benefit system. This is primarily due to the high estimate of the curvature of the search cost function. The high degree of curvature implies that the worker's search choice is quite insensitive to changes in incentives. Thus, one can offer a high level of insurance without worrying that search choices are being distorted. As was seen in table 4, the optimal replacement rates drop dramatically for higher levels of moral hazard.

It is also seen that the preferred replacement rates are insensitive to the wage of the worker. This result suggests that an optimal benefit system should offer a constant replacement rate to all wage types. This is in stark contrast to the current Danish system in which the replacement rate is highly wage sensitive. Danish low wage workers have replacement rates at about 90% and consequently seem to be over-insured whereas high wage workers would seem to be under-insured.

4.2 Group Wide Unemployment Benefit Insurance Schemes

After the exploration of optimal individual unemployment benefit insurance schemes in the previous section, the analysis turns to optimal group wide unemployment benefit insurance. I determine optimal replacement rates for groups of identical type workers according to a standard utilitarian social welfare criterion. Thus, the wage and offer arrival rate are identical for all workers in the insurance scheme but the state varies according to each individual's employment history. The benefit scheme is constrained to satisfy inter-temporal budget balance.

The analysis includes full transitional dynamics from the initial state of the group to the steady state of the new benefit scheme. Once wealth is part of the analysis, transitional dynamics must be included so as to avoid a serious downward bias in the optimal replacement rate results that would otherwise occur if one were to simply maximize social welfare across steady states. It is assumed that the social planner can commit to the benefit scheme which is assumed to be constant even as the group's state distribution is changing.

Consider a group of identical type workers who face an unemployment insurance scheme characterized by a constant benefit level b and a proportional income tax τ. The worker's problem is represented by the Bellman equations (7)-(8). Denote by $\varphi(\cdot) : \{e,u\} \times [\underline{k}, \bar{k}] \rightarrow \mathbb{R}_+$ the state distribution for the group. (7) and (8) together induce a mapping $\Psi_{b,\tau}$ that maps a given state distribution into next period's state distribution according to the policy functions and parameters of the model. Given an initial state distribution φ, the state distribution density for state $y \in \{e,u\} \times [\underline{k}, \bar{k}]$ in the next period can then be written as,

$$(\Psi_{b,\tau}\varphi)(y) = \int_{x \in \{e,u\} \times [\underline{k}, \bar{k}]} P(x, y | b, \tau)\, \varphi(x)\, dx,$$

where $P(x, y | b, \tau)$ is the transition function probability of moving from state x into state y given the unemployment benefit scheme (b, τ). The fact that $\Psi_{b,\tau}$ depends on other parameters than (b, τ) has been suppressed for notational convenience.

Denote by $\varphi^0(\cdot)$ the initial state distribution of the group. Employing a simple utilitarian welfare criterion, the basic design problem can then be stated as,

$$\begin{aligned} \max_{b,\tau} \quad & \int_k \left[V_e\left(k|b,\tau\right)\varphi^0\left(e,k\right) + V_u\left(k|b,\tau\right)\varphi^0\left(u,k\right)\right]dk \\ & s.t.:\left(b,\tau\right)\in\Theta, \end{aligned} \tag{9}$$

where Θ is a set that restricts the choice of benefit scheme. By stating the welfare criterion in terms of value functions, one correctly takes into account the dynamic adjustment that each individual will face given the current state of the individual and subject to the new benefit-tax scheme.

In the following, Θ will be defined as the set of inter-temporally balanced benefit-tax schemes,

$$\Theta = \left\{\left(b,\tau\right)\in[0,w)\times[0,1]:\sum_{i=0}^{\infty}\left(1+r\right)^{-i}\int_k\left(\tau w\varphi^i_{b,\tau}\left(e,k\right)dk-\left(1-\tau\right)b\varphi^i_{b,\tau}\left(u,k\right)\right)dk=0\right\}, \tag{10}$$

where

$$\begin{aligned} \varphi^{i+1}_{b,\tau} &= \Psi_{b,\tau}\varphi^i_{b,\tau},\ i=1,2,\ldots \\ \varphi^1_{b,\tau} &= \Psi_{b,\tau}\varphi^0. \end{aligned}$$

The set of balanced budgets is found by Monte Carlo methods. Clearly, for a given benefit level there may be multiple tax rates that solve the budget problem, but the welfare maximizing one is always the minimum tax rate.

Unlike in the previous section, the individual preference for insurance under this scheme will be affected by cross-subsidization. It is clear that conditional on the current state of the worker, the individual worker's discounted stream of benefits may not exactly balance her discounted stream of taxes. In particular, currently employed workers will expect to be subsidizing the currently unemployed and perhaps more surprising, the currently poor workers will be subsidizing the currently wealthy because the wealthy workers have lower unemployment hazard rates.

Figure 5 shows a set of optimal replacement rates calculated for the group of $w = 0.2$ type workers. The horizontal axis represents the initial state of the economy stated in terms of a replacement rate. Specifically, the initial asset distribution is the steady state distribution for the given replacement rate and the tax rate that balances the steady state budget for the given replacement rate. Generally, in a high replacement rate initial state, workers hold lower savings levels relative to a low replacement rate initial state. If the interest rate is low, the steady state savings distribution is less sensitive to the choice of replacement rate. The optimal replacement rate conditional on the initial state is shown on the vertical axis. The optimal replacement rate is determined according to the program in equations (9) and (10). Not surprisingly, a right-shift of the initial savings distribution across workers lowers the optimal replacement rate which is why the optimal replacement rate curves in Figure 5 are increasing.

As mentioned in the introduction to the policy analysis, the lower wealth bound is set at $\underline{k} = 0$ which provides a clean setting for the optimal policy study. Given the steady state initialization, the results are not sensitive in a first order sense to the choice of the lower wealth bound as is shown in Appendix A.2 where the analysis is performed subject to the estimated lower wealth bound of $\underline{k} = -41.3$.

Replacement rates are found for the estimated curvature of the search cost function as well as the case of quadratic search costs ($\gamma = 2$). The offer arrival rate λ is set at the average estimated level for each given search cost specification. Thus, for the case where $\gamma = 13.36$ the base offer arrival rate is set at $\lambda = .098$ whereas the case where $\gamma = 2$, the base offer arrival rate is set at $\lambda = .45$. Furthermore, the sensitivity of the replacement rates to the interest rate is also highlighted by calculating optimal replacement rates for annual interest rates of five and zero percent. The subjective discount rate is constant at an annual rate of 5.1%.

Generally, the optimal replacement rate is increasing in the initial replacement rate which is due to the fact that workers hold less wealth in higher replacement rate initial states. Thus, given the lower initial wealth holdings, workers have a higher preference for unemployment benefit insurance.

The gap between the interest rate and the subjective discount rate is effectively the cost of using savings as a self-insurance instrument. While previous studies of optimal unemployment insurance have noted the importance of access to savings as a self-insurance instrument, the cost of using savings as insurance has not been noted. It is seen that the optimal replacement rates are highly sensitive to the interest rate. At zero percent, self-insurance is expensive and workers substitute into unemployment benefits. For interest rates almost equal to the subjective discount rate, optimal replacement rates drop dramatically.

The analysis in this section also provides a good indication of the optimal UI policy for the full Danish economy. The $(w, \lambda) = (.2, .098)$ worker type is the predominant type in the economy. The existing Danish benefit policy as it pertains to this worker type corresponds to a 50% replacement rate. As is seen from Figure 5, the optimal UI policy is estimated to be 45%. The results in section 4.1 suggest modest variation in the demand for insurance across wage types. Therefore, absent issues of cross worker type subsidization, a replacement rate of 45% is a good indication of the optimal UI policy for the entire Danish economy given initialization in the estimated steady state.

The intersect between the replacement rate curve and the diagonal has special significance. Here, the social planner does not wish to change the initial state. Based on this point, the optimal replacement rate for the estimated model is 43%. If the interest rate were zero, the estimated replacement rate would be 82%. It is worth noting that these replacement rates are in fact not too different from the observed replacement rates in Denmark. For a quadratic search cost function, the estimated replacement rate is as low as 14%. In this particular case, the use of self-insurance is sufficiently cheap and there is enough moral hazard associated with the use of unemployment benefits that workers choose to rely almost exclusively on self-insurance.

In the interest of providing a reference point to the existing UI literature where a log utility with a quadratic cost function is a common specification, following the same methodology as in Figure 5, I determine the optimal replacement rate given this specification and an initial replacement rate of 50%. The interest rate is set at an annual rate of 5%. To provide some fit to data, I set $\lambda = 1.2$. This ensures that the specification matches the estimated steady state unemployment rate. The optimal replacement rate is in this case 15%.

Previous literature on optimal unemployment insurance typically determines optimal benefit policy by maximizing the social welfare criterion across steady states associated with different balanced benefit schemes. The problem can be written as:

$$\begin{aligned}
&\max_{b} \int_{k} \left[V_e\left(k|b, \tau^*\left(b\right)\right) \varphi^*_{b,\tau^*(b)}\left(e,k\right) + V_u\left(k|b,\tau^*\left(b\right)\right) \varphi^*_{b,\tau^*(b)}\left(u,k\right) \right] dk \qquad (11)\\
&s.t. : \tau^*\left(b\right) = \min_{\tau} \left\{ \tau \in [0,1] : \int_k \left(\tau w \varphi^*_{b,\tau}\left(e,k\right) dk - \left(1-\tau\right) b \varphi^*_{b,\tau}\left(u,k\right)\right) dk = 0 \right\}\\
&\varphi^*_{b,\tau} = \Psi_{b,\tau} \varphi^*_{b,\tau},
\end{aligned}$$

where $\varphi^*_{b,\tau}$ is the ergodic state distribution for the benefit scheme characterized by (b, τ). $\tau^*(b)$ is the minimum tax rate that balances the steady state budget.

In the case where savings are included in the analysis, simple maximization across steady states will introduce a significant downward bias in the results. To illustrate the bias, the replacement rate that maximizes (11) is found to be 45% in the quadratic search cost case and given a zero interest rate. As seen in figure 5, this compares to a 57% optimal replacement rate when transitional dynamics are included. In the 5% interest rate case, the replacement rate that maximizes (11) was found to be less than 0.5%. Including transitional dynamics for this case yields an optimal replacement rate of 14%.

The bias arises because the steady state wealth distribution is a function of the benefit scheme. Lower benefits result in higher wealth holdings. For interest rates close to the subjective discount rate, the sensitivity of the wealth distribution to the benefit level can be so high that lowering benefits while holding taxes fixed can even increase the steady state social welfare criterion! While any worker is obviously worse off for a given wealth level, the steady state wealth distribution can increase so much that the evaluation of worker welfare at the higher wealth levels more than offsets the poorer insurance. The erroneous conclusion that overall worker welfare has increased as a result of the lower benefit level is based on the simple fact that the consumption cost of building up the higher wealth levels in the new steady state has been ignored. These results emphasize the importance of including transitional dynamics in the analysis. A similar point is emphasized in Joseph and Weitzenblum (2003).

5 Conclusion

This paper has estimated a job search model with savings and subsequently determined optimal benefit policy for the estimated model. Depending on the cost of self-insuring via savings, the optimal replacement rate ranges between 43% and 82%. If savings carry a low return relative to the subjective discount rate, self-insurance via savings will be expensive and consequently workers will want to rely more on unemployment benefit insurance to guard against income fluctuations. As a comparison, the actual replacement rate of the median income worker is around 50%. Thus, the level of the actual policy is within the range of optimal policy. However, the analysis does show, that the optimal replacement rate is almost constant across income levels which is in stark contrast with the current Danish system where replacement rates vary significantly across income levels. The observed 90% replacement rates for low income workers and the very low replacement rates for high income workers are not supported by the optimal policy study. The analysis also studied sensitivity to counter factual levels of moral hazard. It was shown that at higher levels of moral hazard, the optimal replacement rate drops to as low as 14% as workers switch into savings as their main insurance vehicle.

The estimation strategy relates observed unemployment durations to the model implied unemployment hazard rate which the worker can affect by the choice of search intensity. Data show a positive relationship between wealth holdings and unemployment duration which the model successfully captures as a result of worker search behavior; wealthier workers search less. The U-shaped relationship between unemployment duration and the worker's wage is also successfully captured as a straightforward search choice response to variation in wage expectations in a search model with savings. The savings aspect is important in this respect since search models without savings cannot generate a non-monotone relationship between the search choice and the wage. The relationships between unemployment duration and other worker characteristics are explained as a combination of a market effect captured in the individual offer arrival function parameter λ and the worker's search response to the market effect.

The estimated utility of consumption function implies a constant relative risk aversion coefficient of 2.21 and the estimated search cost function implies moderate hazard rate responses to changes in incentives. While Danish workers respond to changes in economic incentives in ways that are consistent with the model, the observed unemployment hazard rate response is modest in magnitude for the observed changes in incentives. As a consequence, while the model does imply positive duration dependence of the unemployment hazard rate, it will likely be dominated by other effects. Indeed, most duration dependence analyses show a slightly negative or zero trend in the baseline hazard rate over unemployment duration.

The policy analysis emphasized the importance of including transitional dynamics in the analysis. Failure to do so, will introduce a significant downward bias in the results because the wealth distribution in the economy depends on the benefit scheme. A lower level of benefits will result in greater wealth holdings in steady state. By not including transitional dynamics in the analysis one will be ignoring the consumption cost associated with building these greater wealth holdings and consequently lower benefit levels look more attractive. The analysis showed that the downward bias can be quite large even in the case where the interest rate is zero and the steady state wealth distribution is less sensitive to the benefit scheme.

A Appendix

A.1 Numerical Model Solution

The model is solved via value function iteration where a cubic spline projection is used to approximate the value functions.[17] The value function iteration is accelerated by performing a number of value function iterations for fixed policy functions between policy function updates.[18] For more on this see for example Judd (1998).

Denote by $\tilde{V}_e(k)$ and $\tilde{V}_u(k)$ the cubic spline projections that approximate the fix point of the mappings in (1) and (2). Based on the spline projections, one can then evaluate the two relevant policy function $s(k)$ and $k_u(k)$ for any k by simply solving the maximization problem in (2) using $\tilde{V}_e(k)$ and $\tilde{V}_u(k)$.

In order to evaluate the likelihood function (5) for a given θ, one must be able to evaluate the search intensity choice over 4 dimensions: The wealth level at the outset of the spell, the wage, the offer arrival rate and the duration of the spell to date. To this end, the model is first solved for each point in the grid $\{w_i, \lambda_j\}_{i,j=1}^{M}$. This amounts to M^2 times that one must solve the model. For each grid point (w, λ), the policy functions are then evaluated on a wealth grid $\{k_l\}_{l=1}^{M}$. The procedure leads to a full evaluation of the search and savings policy functions on the 3-dimensional grid $\{k_i, w_j, \lambda_l\}_{i,j,l=1}^{M}$. Interpolation is subsequently performed between the grid points which yields the continuous approximations to the policy functions denoted by $\tilde{s}(k, w, \lambda)$ and $\tilde{k}_u(k, w, \lambda)$.

The duration dependence of the search choice is determined via the evolution of the worker's wealth over the unemployment spell. The wealth level at a particular time during the unemployment spell is inferred via the observed wealth level at the outset of the spell and then by simple iteration on $\tilde{k}_u(\cdot)$. Denote the wealth of a worker with initial wealth k, wage w and offer arrival rate λ at

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[17] Discretization methods are in this case impractical given the potentially very wide support of the state space. The upper bound on the ergodic wealth distribution can be very high when the interest rate is close to the subjective discount rate. Experimentation suggests that to properly capture the savings decisions via discretization methods, one will need such a large number of points in the state space that computation times become unacceptable. On the other hand, projection methods based on cubic splines turn out to be extremely effective.

[18] The expensive part of the value function iteration step is to solve the maximization problem on the right hand side of (1) and (2). Even though one may need more iteration steps, one can still greatly accelerate the solution of the model by not solving for new policy functions in every iteration.

spell duration t by $\hat{k}\left(k,w,\lambda,t\right)$. This is naturally defined by:

$$\begin{aligned}\hat{k}\left(k,w,\lambda,1\right) &= k \\ \hat{k}\left(k,w,\lambda,t\right) &= \tilde{k}_u\left(\hat{k}\left(k,w,\lambda,t-1\right),w,\lambda\right),\ t=2,3,\ldots.\end{aligned}$$

Finally, denote the search intensity for a worker with initial wealth k, wage w and offer arrival rate λ at duration t by $\hat{s}\left(k,w,\lambda,t\right)$. This is defined by:

$$\hat{s}\left(k,w,\lambda,t\right)=\tilde{s}\left(\hat{k}\left(k,w,\lambda,t\right),w,\lambda\right),\ t=1,2,\ldots.$$

The method of approximating the policy functions over not only wealth and wages but also over the offer arrival rate allows for easy estimation of many β and unobserved heterogeneity parameters because one does not have to simulate the model for any new choice of β and G. The only time one must re-simulate the model is when a new θ is tried. Simulating the model is quite computation intensive and it imposes a natural limit on how many structural parameters one can include in θ.

A.2 Optimal Group Wide UI Schemes

This appendix presents an alternative optimal unemployment insurance policy analysis to complement the results in section 4.2. The analysis in this section differs from that in section 4.2 only in that the lower wealth bound has been set at $\underline{k}=-41.3$ to match the bound in the empirical analysis. This establishes consistency between the policy analysis and the model estimate.

As can be seen the optimal UI results are generally not sensitive to the two different choices of lower wealth bound. This is not surprising since the analysis is initialized by the steady state for the given model parameters. Of course, the implied consumption level distribution across agents in different states shifts with the choice of lower wealth bound and since the utility function is concave one would expect a quantitative impact. It happens to be second order in this case.

The lower wealth bound restricts the set of possible policy choices since the natural lower wealth bound $\hat{k}$ exceeds $\underline{k}=-41.3$ for sufficiently low benefit levels. This is not consistent with the no-default setup in this paper. For the given specification, the lower wealth bound of $\underline{k}=-41.3$ and a 5% annual interest rate imply a lower bound on feasible replacement rate policies and initial state replacement rates of about 20-25%. The bound varies slightly with the particular implementation in Figure 6 because the balanced budget requirement impacts the net benefit level. In the analysis I have set a fixed lower bound of 25% in all cases. The bound is restrictive only in the quadratic cost function combined with a 5% interest rate case.

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Table 1: Summary Statistics

Variable	Minimum	Maximum	Mean	Standard Deviation
Spell duration	3.00	52.00	16.16	13.94
Spells per worker	1.00	32.00	7.05	5.02
Years of education	9.00	18.00	11.99	2.64
Female	0.00	1.00	0.47	0.50
Spouse's yearly income	0.00	1,046,000.00	159,760.00	137,138.00
Number of children	0.00	12.00	1.66	1.88
Age	18.00	66.00	42.51	8.86
Upper management	0.00	1.00	0.15	0.35
Lower management	0.00	1.00	0.21	0.40
Salaried worker	0.00	1.00	0.29	0.45
Skilled worker	0.00	1.00	0.12	0.32
Hourly wage	85.36	443.99	144.32	49.29
Owner of real estate	0.00	1.00	0.53	0.50
Net wealth	-1,046,000.00	1,046,000.00	115,313.00	307,580.00

N=12,865

All income and wealth amounts are in 1994 DKK.

Table 2: Results of Structural Estimation

	Estimate	Std err
α	2.2081*	0.0053
γ	13.3610*	0.0113
β-estimates:		
Years of Education	−0.0052	0.0078
1=Female	0.0277	0.0466
Spouse's Income	0.0486*	0.0187
(1=Female)*(Spouse's Income)	−0.0763*	0.0230
Number of Children	0.0052	0.0110
(1=Female)*(Number of Children)	−0.0138	0.0159
Age	−0.0846*	0.0196
1=Owner of Real Estate	−0.0132	0.0291
1=Upper Management	−0.0074	0.0629
1=Lower Management	0.0605	0.0528
1=Salaried Worker	0.0174	0.0446
1=Skilled Worker	0.0091	0.0597
Constant	−1.8173*	0.1496
G-estimates:		
σ_1	−0.6560*	0.0547
σ_2	0.0000	–
σ_3	0.8257*	0.0717
σ_4	1.6126*	0.1372
$\Pr(\sigma_1)$	0.2780*	0.0709
$\Pr(\sigma_2)$	0.4362*	0.0615
$\Pr(\sigma_3)$	0.2383*	0.0298
$\Pr(\sigma_4)$	0.0474*	0.0173

*Significantly different from zero at the 1% level.

Table 3: Optimal Individual Benefit Schemes ($\lambda = .0983$).

	Employed					
	Replacement Rate			Income Tax Rate		
	$k = 0$	$k = 10$	$k = 100$	$k = 0$	$k = 10$	$k = 100$
$w = .20$	68.0	47.1	28.7	4.4	3.0	1.9
$w = .40$	68.2	56.1	32.9	4.6	3.8	2.3

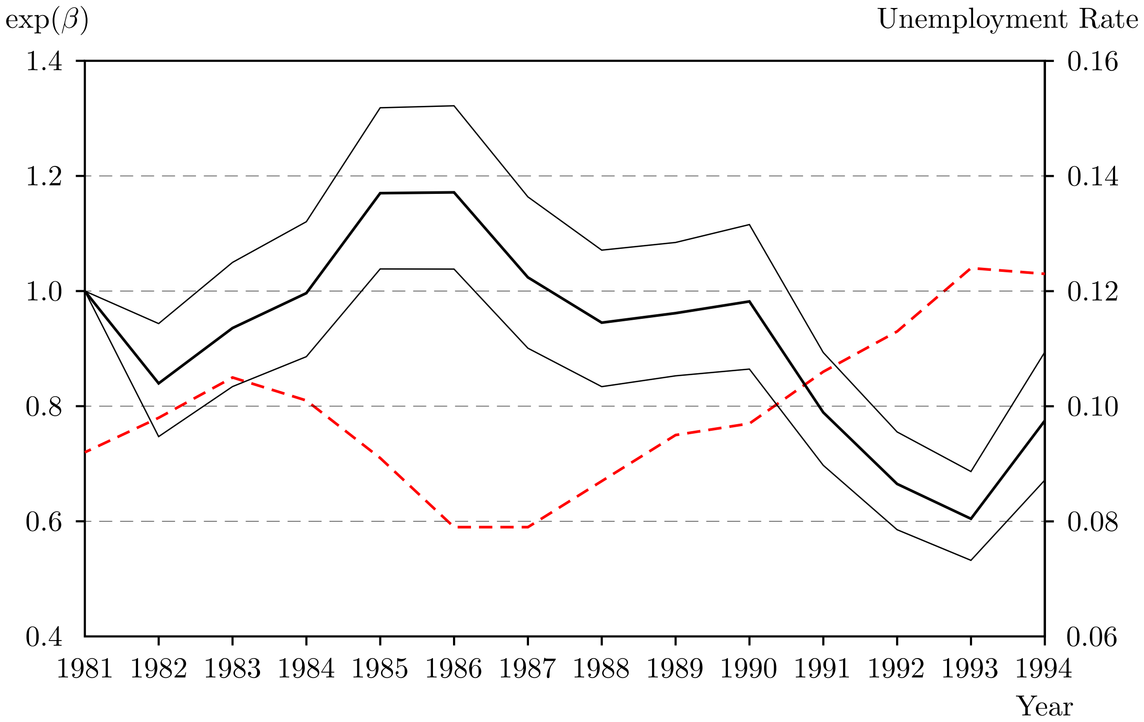
	Unemployed					
	Replacement Rate			Income Tax Rate		
	$k = 0$	$k = 10$	$k = 100$	$k = 0$	$k = 10$	$k = 100$
$w = .20$	75.5	48.1	27.3	6.0	3.8	2.3
$w = .40$	75.4	58.0	32.0	6.4	4.8	2.6

Table 4: Sensitivity Study ($w = 0.2$).

	Employed					
	Replacement Rate			Income Tax Rate		
	$k = 0$	$k = 10$	$k = 100$	$k = 0$	$k = 10$	$k = 100$
$\begin{pmatrix} \alpha = 1.5 \\ \gamma = 2.0 \\ \lambda = 0.83 \end{pmatrix}$	30.95	11.61	3.94	1.28	0.46	0.19
$\begin{pmatrix} \alpha = 2.2 \\ \gamma = 2.0 \\ \lambda = 0.45 \end{pmatrix}$	36.71	14.74	6.23	1.64	0.63	0.37
$\begin{pmatrix} \alpha = 2.5 \\ \gamma = 2.0 \\ \lambda = 0.33 \end{pmatrix}$	39.19	14.93	7.42	1.80	0.65	0.48

	Unemployed					
	Replacement Rate			Income Tax Rate		
	$k = 0$	$k = 10$	$k = 100$	$k = 0$	$k = 10$	$k = 100$
$\begin{pmatrix} \alpha = 1.5 \\ \gamma = 2.0 \\ \lambda = 0.83 \end{pmatrix}$	45.89	12.60	3.40	2.54	0.62	0.21
$\begin{pmatrix} \alpha = 2.2 \\ \gamma = 2.0 \\ \lambda = 0.45 \end{pmatrix}$	51.84	16.20	5.50	3.11	0.85	0.40
$\begin{pmatrix} \alpha = 2.5 \\ \gamma = 2.0 \\ \lambda = 0.33 \end{pmatrix}$	54.28	17.55	6.70	3.56	0.95	0.53

Figure 1: Yearly Dummy Estimates and the Unemployment Rate.



Note: Yearly dummy estimates and 95% confidence bounds drawn on left axis with solid lines. The unemployment rate is drawn on the right axis with a dashed line.

Figure 2: Estimated Unemployment Hazard Rate.

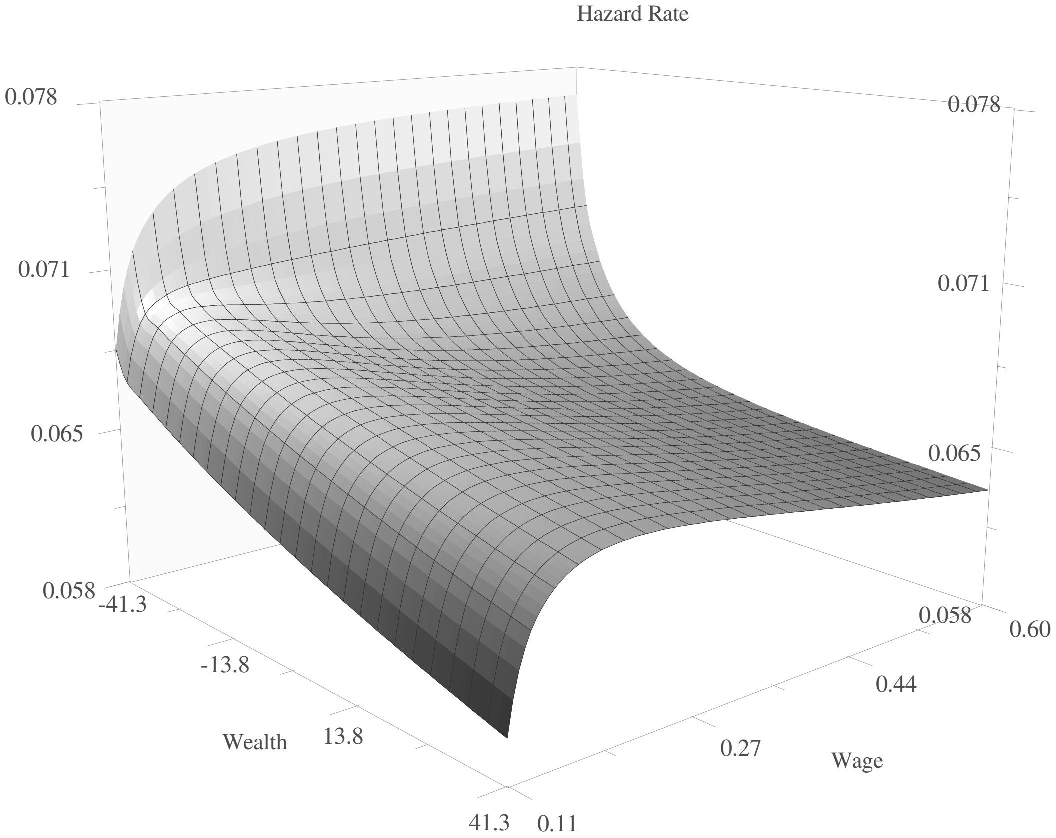
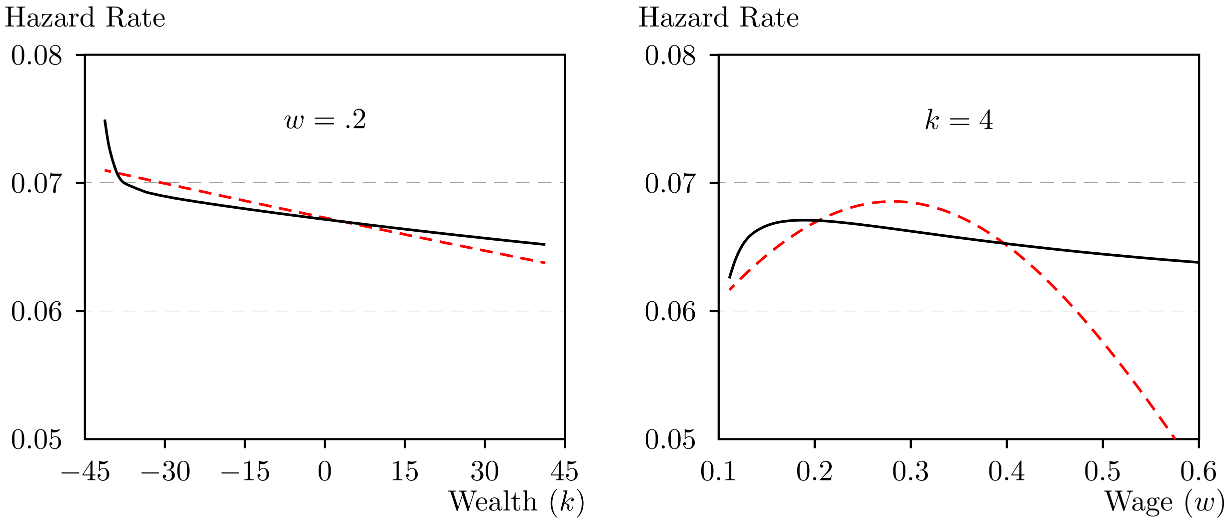


Figure 3: Estimated Unemployment Hazard Rate (other 2 dimensions fixed)



Note: Model estimated hazard rate drawn in solid pen. Reduced form estimates based on a standard proportional hazard model are drawn in dashed line.

Figure 4: Estimated Hazard Rate at Different Positions in the Wealth Distribution.

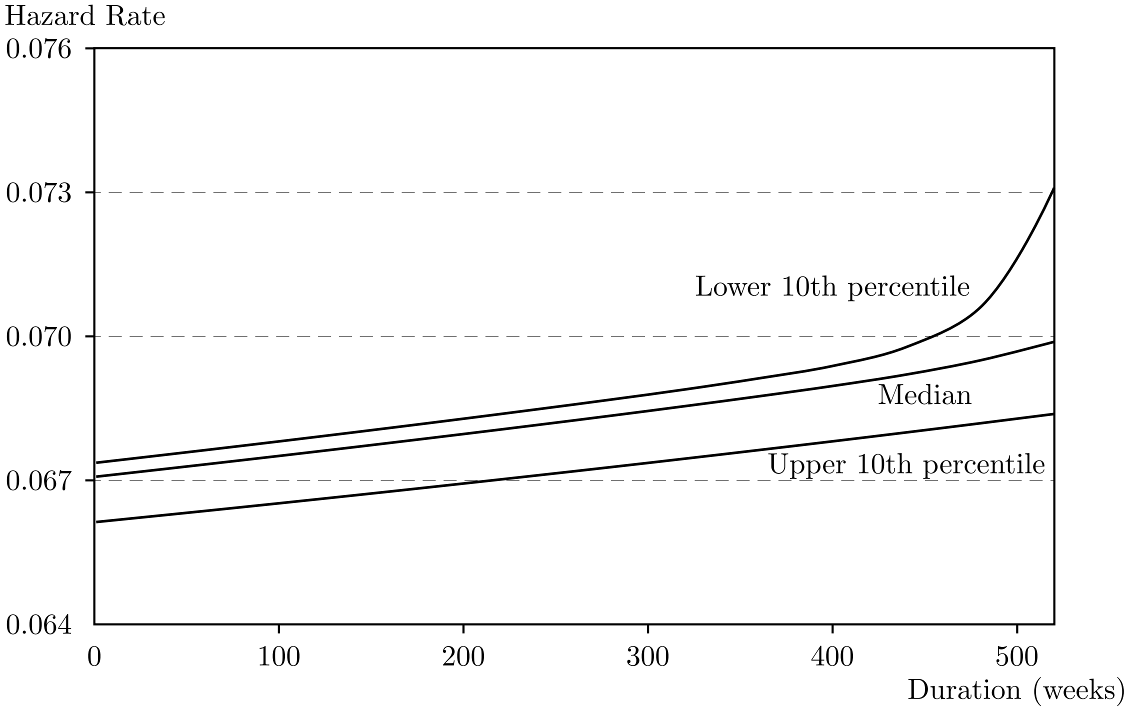
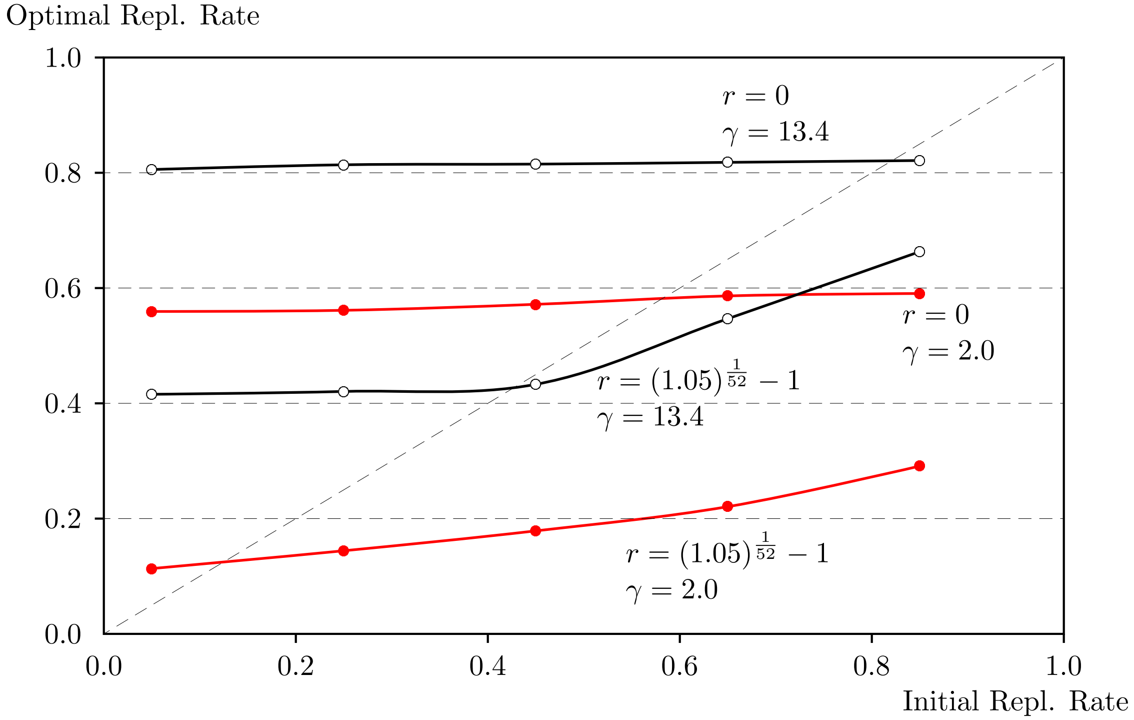
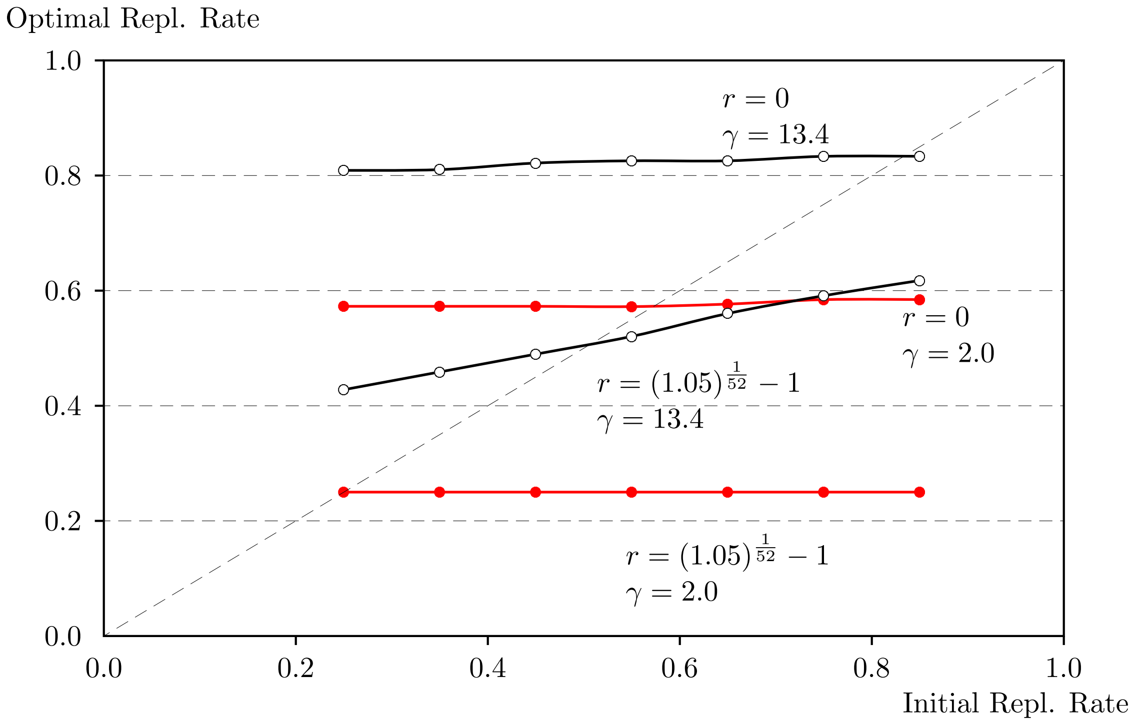


Figure 5: Optimal Group Wide Replacement Rates.



Note: The curves show the solution to the optimal utilitarian social planner problem set out in (9) subject to the inter-temporal budget constraint (10). The initial state φ^0 is the balanced budget steady state distribution associated with the initial replacement rate. All curves are determined for the group of $w = 0.2$ type workers and a lower wealth bound of $\underline{k} = 0$.

Figure 6: Optimal Group Wide Replacement Rates.



Note: The curves show the solution to the optimal utilitarian social planner problem set out in (9) subject to the inter-temporal budget constraint (10). The initial state φ^0 is the balanced budget steady state distribution associated with the initial replacement rate. All curves are determined for the group of $w = 0.2$ type workers and a lower wealth bound of $\underline{k} = -41.3$.