Job Search and Savings: Wealth Effects and Duration Dependence

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This article studies a risk-averse worker's optimal savings and job search behavior as she moves back and forth between employment and unemployment. We show that job search effort is negatively related to wealth under the assumption of additively separable utility. Consequently, job search exhibits positive unemployment duration dependence because wealth is drawn down to smooth consumption as the spell progresses. Finally, given optimal search, savings still provide imperfect insurance against income fluctuations; precautionary savings are built up during employment spells and run down during unemployment spells, but the consumption path will not be perfectly smooth over states.

I. Introduction

This article studies how a worker's job search and savings decisions are affected by wealth. The acts of both job search and saving are undertaken in order to increase future consumption possibilities, and a general theory would need to allow the savings and search behavior to influence each other. Yet job search and saving are rarely analyzed as interrelated choice problems. In the labor literature, the savings decision is typically moot because of risk neutrality of the worker or because wealth cannot be stored. As noted in Mortensen (1986), neither assumption is satisfactory in the context of a worker's job search decision. In

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the macro literature the savings problem is well understood, but here the search decision is ignored, since unemployed workers become reemployed at an exogenous rate.

Wealth can, of course, affect an individual's job search effort, just as the costs of search can affect her savings decisions: when she has a job she both plans for, and expects, any unemployment spell to be brief if search costs are low, and therefore fewer savings are needed for such an eventuality. And if she has little wealth and hence less ability to smooth consumption, she has greater incentive to search intensively after losing a job in order to regain an income. Thus, in order to understand the search and savings decisions better, they need to be analyzed as a joint choice problem, which is the aim of this article. Finally, it is imperative in assessing the welfare implications of, for instance, unemployment insurance and employment protection that not only search effort but also the worker's ability to self-insure via savings be accounted for.

However, the job search model with savings does not lend itself easily to characterization, which may account for the reluctance of the writers of search literature to include a savings choice in the model. This article provides an analytical characterization of a risk-averse worker's behavior in a sequential job search model with savings where the worker alternates back and forth between employment and unemployment and thus has incentives both to save and to search. In every period, the worker decides how much to consume and how much to save. When unemployed, the worker searches for jobs and faces convex search costs. Through her search choice, the worker affects the probability of her reemployment.

We prove that the chosen job search effort is monotonically decreasing with wealth under the assumption that the utility function is separable in consumption and search effort as well as a technical assumption of the existence of a simple wealth lottery. Furthermore, search effort exhibits positive unemployment duration dependence as a direct implication of the negative relationship between search effort and wealth: as an unemployment spell progresses, wealth is drawn down to smooth consumption and consequently search effort increases.

The marginal valuation of wealth is shown to be higher when the worker is unemployed compared to when she is employed. Consequently, an unemployed worker values the transition into employment less the more wealthy she is. Thus, if the costs of search are unaffected by wealth (as is the case when utility is separable in consumption and search effort), job search effort will decrease with wealth. If, however, for instance, marginal search costs are diminished as the worker becomes wealthier, then the net effect is ambiguous. Therefore, with specific assumptions concerning preferences and search costs one can generate a wide range of relationships between wealth and search effort, including nonmonotonic ones. Nevertheless, the case where search effort is negatively related to wealth is of particular interest, because it is a plausible way to account for the recent empirical finding that wealthier workers experience longer unemployment spells.[1]

Finally, allowing for income uncertainty in all states (also when the worker is employed) makes for a more interesting consumption choice framework. With the endogenous Markov income process and the borrowing constraint included in our model, it is shown that savings do provide some insurance against income fluctuations but that it is not perfect. In the light of the results in Deaton (1991), this is to be expected. Deaton characterizes the consumption choice under borrowing constraints and exogenously given income processes. Our results extend the consumption characterization to a case of endogenous income processes.

Under the assumption that employment is an absorbing state and that utility exhibits decreasing absolute risk aversion, a result related to ours is shown in Danforth (1979). Here it is shown that the reservation wage choice is positively related to wealth holdings.[2] The assumption that employment is an absorbing state, that is, that once employed the worker can never again lose her job, is crucial to the line of reasoning in Danforth (1979) but limits the scope of the analysis: unemployment is restricted to affecting only newly arrived workers. Once a worker is employed she has no precautionary motive for saving. Thus, the wealth that serves as insurance against unemployment in the model is never accumulated by the worker herself with the intention of self-insurance. Also, the study of many labor market policy related issues such as unemployment insurance, employment protection, and the like makes little sense if the worker is at no risk of losing her job. The characterization of how search behavior depends on wealth is considerably complicated by the assumption that jobs can be lost. We nevertheless insist that it is an important generalization, both because it facilitates the links between search and savings decisions to be studied and because it constitutes a comprehensive framework for studying labor insurance issues.

Our work is also related to the theoretical literature on duration dependence. The model we suggest implies that the unemployment hazard

[1] See, e.g., Algan et al. (2003), Lentz (2002), and Lentz and Tranæs (2004), in which unemployment spell duration is shown to be positively related to the worker's wealth holdings. Direct evidence of the impact of wealth holdings on the search decision is given in Bloemen and Stancanelli (2001) and Alexopoulos and Gladden (2002), where wealth holdings are shown in survey data to be negatively related to the reservation wage and positively related to the search intensity choice.

[2] We do not believe that it is crucial whether the problem is formulated as a choice of reservation wage given a fixed search intensity or (as here) as a choice of search intensity given a fixed wage.

rate increases with the duration of the unemployment spell.[3] The literature on duration dependence of the unemployment hazard rate includes arguments in favor of both positive and negative duration dependence. Berkovitch (1990) suggests that there is a stigma associated with long unemployment spells, so that the hazard rate would show negative duration dependence. Others have associated negative duration dependence with the loss of absolute or relative skills due to inactivity or separation from innovations. In Mortensen (1986) a simple liquidity constraint is built into a basic search model that generates a decreasing reservation wage as the unemployed worker moves closer and closer to the constraint, and thus the unemployment exit probability exhibits positive duration dependence. It is worthwhile noting that the duration dependence in our job search model with savings is an endogenous feature, whereas most other studies obtain duration dependence as a direct result of some sort of assumed duration dependence on exogenous parameters, such as offer arrival rates, unemployment benefits, and so on.

The article proceeds by presenting the model in Section II. The main analysis of the negative wealth effect case is presented in Section III. Section IV demonstrates the broad range of possible wealth effects on the search decision in the general model and gives a simple intuitive explanation for the causes of the different effects. Section V concludes.

II. A Model of Job Search and Savings

Consider a worker who moves back and forth between employment and unemployment according to a two-state Markov process set in discrete time with time indexed by t. The worker is risk averse in consumption and therefore wants to smooth consumption over the two states representing high and low income situations. The consumption smoothing is accomplished by use of capital markets where the worker's savings can be placed.

We assume that during unemployment the worker can affect the probability s_t of moving back into employment via the choice of search intensity. Assuming that there is a one-to-one relationship between search intensity and the transition probability, s_t, we will simply let the worker choose $s_t \in [0, 1]$ directly. Job separation is exogenous.

The worker derives utility from consumption and disutility from job

[3] We assume in our analysis that the interest rate is less than the subjective rate at which the worker discounts future utility. It is possible that the duration dependence can be reversed in special cases where the interest rate is sufficiently greater than the subjective discount rate and when the worker holds sufficiently high amounts of wealth. In this case, the worker may want to increase savings even when unemployed (but at a lower rate than when employed). Thus, in this special case the sign of the duration dependence will potentially be wealth dependent.

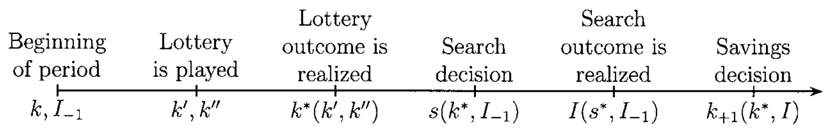

FIG. 1.—The timing of the worker's decisions within a period

search. It is assumed that utility is separable over time with the period-by-period utility given by $v(c_t, s_t)$, where c_t and s_t are consumption and search intensity, respectively, in period t. In periods of employment, the search choice is mute and $s = 0$. We assume throughout that $c_t \geq 0$ and that the worker discounts future utility by the rate $\rho > 0$. Savings are assumed to carry a rate of return, $r < \rho$.

When employed, the worker receives wage w, and during periods of unemployment the worker gets compensation b. The compensation will be referred to as unemployment benefits but could also include income from a secondary labor market, surplus from home production, and so on.

The worker's wealth at the beginning of period t is k_t. Let $I_t \in \{b, w\}$ be the income in period t. Thus, state variables at the beginning of period t are k_t and I_{t-1}, where I_{t-1} tracks the worker's employment status in the previous period. The timing of events in each period is as follows (see fig. 1 for the case of an unemployed worker): given (k_t, I_{t-1}), the worker first has a choice of putting her wealth into a lottery with immediate realization k_t^*. The lottery holds a zero risk premium and will therefore only be entered into if the worker's value function is convex at k_t and thus exhibits a risk-loving attitude to wealth. Then given k_t^*, a search and separation stage follows. If unemployed in the previous period, the worker decides on how much effort to put into job search so as to affect the probability of moving into employment in the current period. Immediately following the search effort choice the state of employment in period t is realized and, along with it, the period t income, I_t. If employed in the previous period, the worker does not search and the search effort is zero. Instead, the worker is (exogenously) either separated from her job or remains employed, which in this situation realizes I_t. Finally, based on k_t^* and I_t, the worker decides on how much to consume in the present period, or, equivalently, she decides how much to save for the future, which determines her wealth in the next period, k_{t+1}.

Unlike in the job search model without savings, one cannot in general rule out local convexities of the worker's value functions in the model with savings. Convexities of the value functions turn out to be highly problematic in terms of characterization of the worker's search and savings choices.[4] The lottery is introduced in order to mitigate this problem. It effectively smooths out any local convexities and ensures concavity of the value functions of the model with the lottery.[5]

The lottery is modeled as yielding return $k_t^l \leq k_t$ with probability α and return $k_t^h \geq k_t$ with probability $1 - \alpha$. The parameter α is set so that the expected return of the lottery equals k_t, that is, $\alpha = (k_t^h - k_t)/(k_t^h - k_t^l)$. The worker has access to a full menu of these lotteries in the sense of being able to choose k_t^l and k_t^h freely as long as the returns lie within the wealth bounds. Nonparticipation in the lottery is given by $k_t^l = k_t^h = k_t$.

The following analysis will exclusively focus on the worker's choice of savings and search intensity. We take no particular stand on the existence and determination of wages, the interest rate, or the lottery.

All in all, the worker is faced with the following problem:

$$\max_{\{s_t, c_t, k_t^l, k_t^h\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1 + \rho)^{-t} v(c_t, s_t),$$

$$\text{subject to } k_{t+1} = (1 + r)k_t^* + I_t - c_t,$$

where $k_t^* = k_t$, k_t^l, or k_t^h according to whether the worker chooses to participate in the period t lottery or not and what the outcome of it is. The income process is given by $I_t \equiv wn_t + (1 - n_t)b$, where $n_t \in \{0, 1\}$; $n_t = 0$ means that the worker is unemployed, and $n_t = 1$ means that the worker is employed. The random variable n_t follows a Markov process with transition function $P(n_{t-1}, n_t)$; $P(0, 1) = s_t$, $P(0, 0) = 1 - s_t$, $P(1, 0) = \eta$, and $P(1, 1) = 1 - \eta$.

To ensure that our problem is well behaved we will assume the following concerning wealth.

Assumption 1. The worker's wealth at the beginning of period t, k_t, is assumed to be bounded both above and below, that is, $k_t \in [\underline{k}, \overline{k}]$.

A lower bound per se can be justified as a borrowing limit imposed by the capital market. Aiyagari (1994) points out that a lower bound on

[4] Convexity of the value functions also implies rather perverse and counterintuitive behavior on the part of the worker. For instance, consumption will decrease with greater wealth in this case.

[5] As such, the assumption of the existence of lotteries can be viewed as a sufficient condition for our results. However, simulations of our model suggest that it is not a necessary condition since all value functions turn out to be perfectly concave for a very wide range of model parameters in the model without the lottery. The issue of possible problems with convexities of value functions or concavities of cost functions in these types of models is noted in papers such as Phelan and Townsend (1991), Hopenhayn and Nicolini (1997), and Gomes, Greenwood, and Rebelo (2001).

wealth can also be motivated by requiring asymptotic present value budget balance (i.e., $\lim_{t\to\infty} k_t/(1+r)^t \geq 0$) combined with nonnegative consumption. The upper bound is imposed in order to bound the problem and ensure the existence of a solution. The bound $\bar{k}$ will be set above the upper bound of the ergodic wealth distribution when possible. This can always be done as long as the upper bound on the ergodic wealth distribution is finite, which it is for any $r < \rho$, and in this case the assumption does not restrict the worker's savings problem from above. Finally, when we write "for all k_t" below, we mean for all $k_t \in [\underline{k}, \bar{k}]$.

Without any further structure the model can support various different relationships between wealth and search effort, as we discuss below. In the next section, attention is restricted to utility functions that are separable in consumption and search effort.

III. Separable Utility

In this section we will introduce some additional structure to the model of Section II that facilitates a precise characterization of utility and behavior in the two main states, employment and unemployment, as functions of wealth. The main difficulty is to show that the marginal valuation of wealth is higher when the worker is unemployed compared to when she is employed, proposition 1, which then almost immediately implies that search effort is inversely related to wealth. Specifically, we assume that utility is additively separable not only over time but also over consumption c_t and search s_t, that utility $u(\cdot)$ is strictly increasing and strictly concave in consumption c_t, and that search costs $e(\cdot)$ are strictly increasing and strictly convex in search effort s_t.

Assumption 2. In any period t, the worker's utility from consumption and search is $v(c_t, s_t) = u(c_t) - e(s_t)$. Furthermore, $u'(\cdot) > 0$, $u''(\cdot) < 0$, $e'(\cdot) > 0$, and $e''(\cdot) > 0$, with $e(0) = 0$.

For convenience it is assumed that both $u(\cdot)$ and $e(\cdot)$ are differentiable. In order to fully specify the details and orders of moves, the model is expressed in terms of Bellman equations. Let $V_g(k)$ and $U_g(k)$ be the value functions for the gambling stage of period t, given wealth k_t for $I_{t-1} = w$ and $I_{t-1} = b$, respectively. Thus,

$$V_g(k_t) = \max_{k_t^l \in [\underline{k}, k_t], k_t^h \in [k_t, \bar{k}]} \left[\frac{k_t^h - k_t}{k_t^h - k_t^l} V(k_t^l) + \frac{k_t - k_t^l}{k_t^h - k_t^l} V(k_t^h)\right],$$

$$U_g(k_t) = \max_{k_t^l \in [\underline{k}, k_t], k_t^h \in [k_t, \bar{k}]} \left[\frac{k_t^h - k_t}{k_t^h - k_t^l} U(k_t^l) + \frac{k_t - k_t^l}{k_t^h - k_t^l} U(k_t^h)\right],$$

where V and U are the value functions at the search and separation stage given the employment status of the previous period:

$$V(k_t) = (1-\eta)V_c(k_t) + \eta U_c(k_t),$$
$$U(k_t) = \max_{s\in[0,1]}[-e(s) + sV_c(k_t) + (1-s)U_c(k_t)],$$

and where, finally, V_c and U_c are the value functions at the consumption and savings stage, which depend on the employment status of the current period, period t:

$$V_c(k_t) = \max_{k_{t+1}\in[\underline{k},\min(k_t(1+r)+w,\bar{k})]}\left[u((1+r)k_t + w - k_{t+1}) + \frac{V_g(k_{t+1})}{1+\rho}\right],$$
$$U_c(k_t) = \max_{k_{t+1}\in[\underline{k},\min(k_t(1+r)+b,\bar{k})]}\left[u((1+r)k_t + b - k_{t+1}) + \frac{U_g(k_{t+1})}{1+\rho}\right],$$

where by definition $(1+r)k_t + I_t - k_{t+1} = c_t$. These six value functions represent the model and form the basis for the results we arrive at below.

Given the fact that the value functions are themselves time independent, we need only keep track of two periods at a time, and we suppress the index for the current period. For instance, k and k_{+1} will be the worker's wealth in two successive periods, the current and the next period.

To proceed, a little more notation is needed. Let $c(k; e)$ and $c(k; u)$ be the optimal choices of consumption given wealth k under employment and unemployment, respectively. Let $s(k)$ be the optimal choice of search given wealth k, and let the optimal choices of next period's wealth be defined by $k_{+1}(k; e) \equiv k + w - c(k; e)$ and $k_{+1}(k; u) \equiv k + b - c(k; u)$, again according to whether the worker is employed or unemployed in the current period. It is assumed that the utility function is such that the only constraint that may be binding is the lower bound on wealth. The first-order conditions associated with the optimal choices of consumption and search effort give us the following characterizations:

$$u'[c(k;\, e)] = \frac{V_g'(k_{+1}(k;e))}{1+\rho} + \lambda_V(k), \tag{1}$$

$$u'[c(k;\, u)] = \frac{U_g'(k_{+1}(k;\, u))}{1+\rho} + \lambda_U(k), \tag{2}$$

$$e'[s(k)] = V_c(k) - U_c(k), \tag{3}$$

where λ_V and λ_U are the nonnegative Lagrange multipliers associated with the lower bound on wealth. By the envelope theorem it follows that

$$V_g'(k) = \frac{V(k^h) - V(k^l)}{k^h - k^l} \text{ and } U_g'(k) = \frac{U(k^h) - U(k^l)}{k^h - k^l}, \ \forall\, k^h > k^l. \quad (4)$$

If the worker chooses not to participate in the lottery, that is, $k = k^l = k^h$, then (4) becomes

$$V_g'(k) = V'(k) \text{ and } U_g'(k) = U'(k), \ \forall\, k^h = k^l = k.$$

Furthermore,

$$V'(k) = (1-\eta)V_c'(k) + \eta U_c'(k),\ U'(k) = s(k)V_c'(k) + (1 - s(k))U_c'(k), \quad (5)$$

$$V_c'(k) = u'(c(k;\, e))(1+r), \text{ and } U_c'(k) = u'(c(k;\, u))(1+r). \quad (6)$$

In the next section, it is shown that assumptions 1 and 2 are sufficient to establish that $V_c'(k) < U_c'(k)$ for all k. To this end, a technical assumption is added to our description of the model above, namely, that the period length is sufficiently small so that the transition probabilities between states will be small relative to unity. Specifically, it is assumed that $s(k) + \eta \leq 1$.

A. The Key Proposition

In order to show the key proposition of the article, that a marginal wealth increase will reduce the value difference between the two states, employment and unemployment, we will need the following two lemmas. Lemma 1 establishes concavity of all value functions except $U(k)$, and lemma 2 associates first derivatives across value functions. Both lemmas are proven in the appendix.

Lemma 1. The value functions $V_g(k)$ and $U_g(k)$ are concave, and $V(k)$, $U_c(k)$, and $V_c(k)$ are strictly concave.

Lemma 2. $V_g'(k) = V'(k)$ and $U_g'(k) = U'(k^l(k)) = U'(k^h(k))$ for all k, where the optimal lottery for a worker who was unemployed in the previous period $(k^l(k), k^h(k))$ is an interior solution in $[\underline{k}, k] \times [k, \overline{k}]$ or if $k \in \{\underline{k}, \overline{k}\}$. Furthermore, in the case where $k > k^l(k) = \underline{k}$, it must be that $U_g'(k) \geq U'(k^l(k))$.

The key to the central results of the article and the main difficulty with providing characterizations of savings and search behavior when individuals alternate between employment and unemployment is to establish that $V_c'(k) - U_c'(k) < 0$ for all k, which proposition 1 does below. With this result in hand, it is then straightforward to establish interesting behavioral characteristics such as the sign of $s'(k)$, the marginal effect of more wealth on search effort.

Proposition 1. $V_c'(k) - U_c'(k) < 0$ for all k.

Proof. By concavity of $V(k)$ it follows that $V_g(k) = V(k)$. Thus it must be that

$$V_c(k) = \max_{k_{+1} \in \Gamma_w(k)} \left[u((1+r)k + w - k_{+1}) + \frac{V(k_{+1})}{1+\rho} \right]$$
$$= \max_{k_{+1} \in \Gamma_w(k)} \left[u((1+r)k + w - k_{+1}) + \frac{(1-\eta)V_c(k_{+1}) + \eta U_c(k_{+1})}{1+\rho} \right], \quad (7)$$

where $\Gamma_i(k) = \{\tilde{k} \in \mathbb{R} | \underline{k} \leq \tilde{k} \leq \min[(1+r)k + i, \overline{k}]\},\ i \in \{w, b\}$. Similarly, $U_c(k)$ can be written in terms of V_c and U_c. But in this case, one cannot disregard the lottery, so the expression is somewhat more complicated:

$$U_c(k) = \max_{k_{+1} \in \Gamma_b(k)} \left\{ \frac{k'' - k_{+1}}{k'' - k'} u((1+r)k + b - k_{+1}) \right.$$
$$+ \frac{1}{1+\rho} \max_{k_t^l \in [\underline{k}, k_t], k_t^h \in [k_t, \bar{k}]} \left[\frac{k^h - k_{+1}}{k^h - k^l} \max_{s \in [0,1]} [-e(s) + sV_c(k^l) + (1-s)U_c(k^l)] \right.$$
$$\left.\left. + \frac{k_{+1} - k^l}{k^h - k^l} \max_{s \in [0,1]} [-e(s) + sV_c(k^h) + (1-s)U_c(k^h)] \right] \right\}. \quad (8)$$

Define the state space, $X = [\underline{k}, \overline{k}] \times \{0, 1\}$ and $C(X)$ as the set of all bounded, continuous functions on X. Furthermore, define the mapping $T : C(X) \to C(X)$ such that $T(V_c, U_c)(k, 1)$ equals the right-hand side of (7) and $T(V_c, U_c)(k, 0)$ equals the right-hand side of (8). By the theorem of the maximum it is seen that T indeed maps $C(X)$ into itself since the right-hand sides of (7) and (8) are maximizations of bounded, continuous functions over compact sets.

The mapping T satisfies Blackwell's sufficient conditions for a contraction mapping, namely, monotonicity and discounting.[6] To see that monotonicity is satisfied, choose some $V_c^1(k) \geq V_c^2(k)$ and $U_c^1(k) \geq U_c^2(k)$ for all k. Then Blackwell's sufficient conditions state that it must be that $T(V_c^1, U_c^1)(x) \geq T(V_c^2, U_c^2)(x)$ for all $x \in X$. By examination of (7) and (8) this is seen to be trivially satisfied. The discounting condition states that it must be that for all $x \in X$ and for all $\lambda \geq 0$, $T(V_c + \lambda, U_c + \lambda)(x) \leq T(V_c, U_c)(x) + \lambda\beta$, for some $0 < \beta < 1$. It is seen from (7) and (8) that $T(V_c + \lambda, U_c + \lambda)(x) = T(V_c, U_c)(x) + \lambda/(1+\rho)$. Thus, discounting is satisfied for $\rho > 0$, which, by definition, is given. There-

[6] For a proof of Blackwell's sufficient conditions see, e.g., Stokey and Lucas (1989).

fore, it has been established by Blackwell's sufficient conditions that $T(V_c, U_c)$ is a contraction mapping.

By the contraction mapping theorem there exists a unique fixed point (V_c^*, U_c^*). Also, the contraction mapping property of T implies that for some closed set $S_1 \subseteq S$, if $T(S_1) \subseteq S_2 \subseteq S_1$, then $(V_c^*, U_c^*) \in S_2$.

In the following, it will be shown that T maps the closed set of functions S_1 defined by

$$S_1 = \{(V_c, U_c) \in C(X) | V_c'(k) - U_c'(k) \leq 0 \ \forall \, k\},$$

into the set S_2 defined by

$$S_2 = \{(V_c, U_c) \in C(X) | V_c'(k) - U_c'(k) < 0 \ \forall \, k\}.$$

Thus, by the argument above it must be that the fixed point of the mapping is characterized by $V_c' - U_c' < 0$.

The derivatives of the mapping are

$$\begin{aligned} T_k'(V_c, U_c)(k, 1) &= u'(c(k; e))(1 + r) \\ &= \frac{1 + r}{1 + \rho}[(1 - \eta)V_c'(k_{+1}(k; e)) + \eta U_c'(k_{+1}(k; e))] + (1 + r)\lambda_V(k), \end{aligned} \tag{9}$$

$$\begin{aligned} T_k'(V_c, U_c)(k, 0) &= u'(c(k; u))(1 + r) \\ &= \frac{1 + r}{1 + \rho} U_g'(k_{+1}(k; u)) + (1 + r)\lambda_U(k) \\ &\geq \frac{1 + r}{1 + \rho} U'(k_u^l) + (1 + r)\lambda_U(k) \qquad (10) \\ &= \frac{1 + r}{1 + \rho}[s(k_u^l)V_c'(k_u^l) + (1 - s(k_u^l))U_c'(k_u^l)] + (1 + r)\lambda_U(k), \end{aligned}$$

where $k_u^l \equiv k^l(k_{+1}(k; u))$. By lemma 2, (10) holds with equality if $k_u^l > \underline{k}$ but may hold with inequality if $k_u^l = \underline{k}$.

Assume that $V_c'(k) - U_c'(k) \leq 0$ for all k. It will then be shown that it must be that $T_k'(V_c, U_c)(k, 1) - T_k'(V_c, U_c)(k, 0) < 0$ for any $k \in [\underline{k}, \overline{k}]$.

Consider any given wealth level $k \in [\underline{k}, \overline{k}]$. Suppose that $\lambda_V(k) > \lambda_U(k)$, which is to say that, contrary to intuition, a marginal relaxation of the lower wealth bound is more valuable in the employed than in the unemployed state. By nonnegativity of the Lagrange multipliers it follows that $\lambda_V(k) > 0$, which implies that the lower wealth bound constraint must bind in the employed state, $k_{+1}(k; e) = \underline{k}$. The lower wealth bound may or may not bind in the unemployed state, $k_{+1}(k; u) \geq \underline{k}$. Thus, the assumption of $\lambda_V(k) > \lambda_U(k)$ necessarily implies that $k_{+1}(k; e) \leq k_{+1}(k; u)$, which by $w > b$ implies that $c(k; e) > c(k; u)$. By (9) and (10) and by strict concavity of $u(\cdot)$ it immediately follows that $T_k'(V_c, U_c)(k, 1) -$

$T_k'(V_c, U_c)(k, 0) < 0$. Generally, any case for which $k_{+1}(k; e) \leq k_{+1}(k; u)$ immediately yields this result.

Now, consider the alternative case of $k_{+1}(k; e) > k_{+1}(k; u)$, which by the above argument implies that $\lambda_V(k) \leq \lambda_U(k)$. Thus, it follows that $k_u^l \equiv k^l(k_{+1}(k; u)) \leq k_{+1}(k; u) < k_{+1}(k; e)$. Subtract (10) from (9):

$$\begin{aligned}
T_k'(V_c, U_c)(k, 1) - T_k'(V_c, U_c)(k, 0) \leq\ & T_k'(V_c,U_c)(k,1) \\
& - \frac{1+r}{1+\rho}[s(k_u')V_c'(k_u^l) + (1 - s(k_u'))U_c'(k_u^l)] \\
& - (1+r)\lambda_U(k) \\
<\ & T_V'(V_c,U_c)(k, 1) \\
& - \frac{1+r}{1+\rho}[s(k_u')V_c'(k_{+1}(k; e)) + (1 - s(k_u'))U_c'(k_{+1}(k; e))] \\
& - (1+r)\lambda_U(k) \\
=\ & \frac{1+r}{1+\rho}[1 - s(k_u') - \eta][V_c'(k_{+1}(k; e)) - U_c'(k_{+1}(k; e))] \\
& + (1+r)(\lambda_V(k) - \lambda_U(k)) \\
\leq\ & 0,
\end{aligned}$$

where the strict inequality follows from strict concavity of $V_c(\cdot)$ and $U_c(\cdot)$ and the fact that $k_u^l < k_{+1}(k; e)$. The last weak inequality follows from the assumption that the period length is sufficiently small that $1 - s(k_u') - \eta \geq 0$, from the assumption that $V_c'(k) - U_c'(k) \leq 0$ for all k, and finally from the assumption that $\lambda_V(k) \leq \lambda_U(k)$.

Thus, it has been shown that for a sufficiently small period length $T(S_1) \subseteq S_2$, and therefore that the fixed point of T must be characterized by $V_c'(k) - U_c'(k) < 0$. QED.

Notice that the presence of the lower wealth bound only strengthens the result of proposition 1. The arguments used in the proof do not rely on the presence of the lower bound except in showing that the problem is bounded.

One complication in establishing proposition 1 is that $U(k)$ need not be concave; compare lemma 1. By the envelope theorem,

$$U'(k) = s(k)V_c'(k) + (1 - s(k))U_c'(k).$$

This implies that

$$U''(k) = s(k)V_c''(k) + (1 - s(k))U_c''(k) + s'(k)[V_c'(k) - U_c'(k)].$$

By the first-order condition of the optimal search choice, $e'(s(k)) = V(k) - U(k)$, it follows that

$$U''(k) = s(k)V_c''(k) + (1 - s(k))U_c''(k) + \frac{[V_c'(k) - U_c'(k)]^2}{e''(s(k))}. \qquad (11)$$

The first two terms on the right-hand side of (11) are negative, but the last term is positive. Thus, concavity of $U(\cdot)$ does not follow directly. In simulations of numerous different specifications of the model, $U(\cdot)$ has consistently turned out to be concave, suggesting that the last term rarely dominates the two negative terms, in which case lotteries are never used. However, one has to allow for lotteries as it cannot generally be ruled out that $U(\cdot)$ is not concave.

B. Negative Wealth Effects on Job Search

One of the main interests of the article is to characterize the relationship between search intensity and wealth. To this end, equation (3) is differentiated with respect to k, which yields

$$s'(k) = \frac{\partial s(k)}{\partial k} = \frac{V_c'(k) - U_c'(k)}{e''(s(k))} \gtreqless 0 \text{ for } V_c'(k) \gtreqless -U_c'(k)0. \qquad (12)$$

From (6) and (12) it follows that $s'(k) \leq 0$ globally if and only if $c(k; e) \geq c(k; u)$ for all k, which is to say that search intensity increases when k falls. Furthermore, if k decreases over the unemployment spell, $k_{+1}(k; u) \leq k$, this will yield positive duration dependence of the worker's search intensity. In other words, the worker will search harder as a spell progresses. Notice that in this model search intensity of a risk-neutral worker is constant over the duration of an unemployment spell.

With the result of proposition 1 in hand, we can now characterize how search effort is affected by wealth by applying (12) and remembering that the effort function $e(s)$ is assumed to be strictly convex:

Proposition 2. Search effort increases as wealth decreases; $s'(k) < 0$ for all k.

Thus, under assumptions 1 and 2, an unemployed worker has less incentive to go back into employment the more wealth she holds. Since it is assumed that search costs are not affected by the level of wealth holdings, we obtain the result that wealthier workers will search less.

It is worthwhile noting that the special case of assumption 2 for which $u(c) = \ln(c)$ is consistent with a balanced growth steady state in the model if it is modified to allow for a positive growth trend in the income process and wealth bounds (see King, Plosser, and Rebelo [1988] for details).[7]

[7] While their model describes a consumption-labor supply choice, the basic methodology translates in a straightforward manner.

They also describe a nonseparable utility function class that is consistent with a balanced growth steady state in our search and savings model, $v(c, s) = [1/(1-\sigma)]c^{1-\sigma}e(1-s)$, where $e(\cdot)$ is increasing and concave for $\sigma < 1$ and decreasing and convex for $\sigma > 1$. We conjecture that this class of utility function will yield a monotonically decreasing relationship between wealth and search intensity in the case of $\sigma < 1$.[8]

C. Consumption Smoothing, Self-Insurance, and Duration Dependence

This section shifts the focus to the savings choice and the implied relationship between unemployment spell duration and the unemployment hazard rate. We also consider the role of savings as self-insurance against income fluctuations.

If wealth is steadily reduced as an unemployment spell progresses and there is a negative relationship between wealth and search, then search effort will increase during the spell. This is referred to as positive duration dependence. While it is a very intuitive result, it should be noted that it is a direct extension of the negative relationship between search intensity and wealth that was established in proposition 2. In fact, the duration dependence results can be reversed by having the utility and search cost structure chosen appropriately. This is discussed in more detail below.

Again we continue the analysis under assumptions 1 and 2. Beginning with the consumption decisions, it is seen by the following proposition that savings do indeed provide insurance against income fluctuations but that the insurance is imperfect.

PROPOSITION 3. For all k, consumption increases as wealth increases, $c'(k; e) > 0$ and $c'(k; u) > 0$, and consumption when employed is strictly greater than consumption when unemployed, $c(k; e) > c(k; u)$. Furthermore, for all $k > \underline{k}$ it must be that $b + rk < c(k; u) < c(k; e)$ and $k_{+1}(k; u) < k$.

Proof. Lemma 1 and the envelope conditions (6) immediately yield the fact that consumption must increase with wealth both when employed and unemployed. The conclusion that $c(k; u) < c(k; e)$ follows directly from lemma 1 and the envelope conditions (6).

The result that $b + rk < c(k; u)$ follows from the fact that for $r < \rho$ it must be that $k_{+1}(k; u) < k$ for all $k > \underline{k}$. This is seen from the following argument: in lemma 2 it was established that for an interior solution to the

[8] The cross-derivative $v''_{cs}(c, s)$ is such that marginal search costs are increasing in wealth for $\sigma < 1$. Combined with the result that gains from search decrease with greater wealth, one would expect an overall negative effect on search from an increase in wealth.

lottery, $U_g'(k_{+1}(k;\, u)) = U'(k^h(k_{+1}(k;\, u)))$. Now let $k_u^h \equiv k^h(k_{+1}(k;\, u))$. By the first-order and envelope conditions, it then follows that

$$U_c'(k) = \frac{1+r}{1+\rho}[s(k_u^h)V_c'(k_u^h) + (1 - s(k_u^h))U_c'(k_u^h)] + (1+r)\lambda_U(k). \quad (13)$$

Assuming that the period length is sufficiently small, it is given in lemma 1 that $V_c'(k) - U_c'(k) < 0$ for all k. Rewriting (13), it follows that

$$U_c'(k) - \frac{1+r}{1+\rho}U_c'(k_u^h) - (1+r)\lambda_U(k) = \frac{1+r}{1+\rho}s(k_u^h)[V_c'(k_u^h) - U_c'(k_u^h)]. \quad (14)$$

Thus, it must be that the right-hand side of (14) is strictly negative. Now suppose, contrary to the claim, that $k_{+1}(k;\, u) \geq k$. This first of all implies that $\lambda_U(k) = 0$ since the lower bound must not have been binding. Furthermore, it implies that $k_u^h \geq k$. But since $U_c'(\cdot)$ is strictly concave and $r < \rho$ this must mean that the left-hand side of (14) is positive, yielding a contradiction of the inequality. This establishes that $b + rk < c(k;\, u) < c(k;\, e)$ when $k > \underline{k}$. If $k = \underline{k}$, an unemployed worker is no longer able to insure against the low income state and will simply consume the income $b + r\underline{k}$. QED.

Hence, for $r < \rho$, unemployed workers always dissave as long as their wealth is above the minimum level, $\underline{k}$. The employed worker experiences two contradictory savings motives. Since $r < \rho$, there is a negative real return on savings and thus the speculative savings motive dictates that savings be reduced. However, the precautionary savings motive dictates an increase in savings during employment to insure against low income shocks. For low wealth levels, the precautionary motive dominates, and for sufficiently high wealth levels, the speculative savings motive eventually begins to dominate. In the special case where $r = \rho$, only the precautionary motive manifests itself, and by an argument analogous to the proof of proposition 3, one can show that $k_{+1}(k;\, e) \geq k$ for all k.

Proposition 3 shows that savings do allow for some insurance against income fluctuations but that the insurance is imperfect, since the consumption path is not perfectly smooth over states. This is because transitions between income states are never fully anticipated and the income process is a jump process.

As a direct extension of the arguments made in the proof of proposition 3, it can be shown that the search intensity of an unemployed worker will exhibit positive duration dependence throughout the unemployment spell. This result follows from the basic relationship between the choice of search intensity and the worker's wealth as established in proposition 2.

Proposition 4. The worker's search intensity exhibits positive duration dependence.

Proof. The result follows directly from two observations. (1) $k_{+1}(k;\, u) < k$, that is, an unemployed worker will monotonically decrease wealth. This was shown in the proof of proposition 3. (2) In proposition 2, it was established that the search intensity is decreasing in the worker's wealth. Thus, it must be that the search intensity increases as the unemployment spell progresses to the point where the worker's wealth reaches the lower bound. After this point, the search intensity remains constant. QED.

For an interest rate less than the worker's rate of time preferences, we have shown that an unemployed worker will monotonically decrease wealth, and, as such, the search intensity will strictly increase during spells of unemployment, up to the point where the lower bound on wealth has been reached.

IV. The General Case

In this section, we point out that a negative association between a risk-averse worker's search effort and wealth cannot be expected to function as a general theorem. We look at a more general model than the one studied above and present a counterexample. The unemployed worker chooses search intensity so as to equate marginal gains from search with marginal search cost. We cast the discussion in terms of how these two measures are affected by changes in wealth.

Before we look at the nonseparable case we touch upon a more technical modeling issue, namely, the timing of events. The particular choice of timing of events in the model provides a convenient framework for the analytical results. An alternative choice of timing would be to have the worker making the search and consumption choices simultaneously. There is, however, no reason to suspect that this alternative timing will change the savings and search choices in any qualitative way. This view is supported by the numerical simulations below, which yield the same characteristics of the search choices across the two different timing assumptions.

In our model, wealth allows the worker a high level of consumption during periods of unemployment. The purpose of search, on the other hand, is to reduce the duration of such periods. But the wealthier the worker is, the less utility difference there is between the two states. Therefore, at higher wealth levels the worker has less incentive to reduce the duration of an unemployment spell and, consequently, is less willing to suffer utility losses in order to increase the probability of transitioning back into employment.

Hence, in terms of characterizing how wealth influences search effort, the key relationship is how search costs are affected by wealth—directly as well as indirectly. While the gains from search decrease with greater wealth, if search costs also decrease with greater wealth, then the net wealth effect may be ambiguous. Generally, the cross-derivative $v''_{cs}(c, s)$

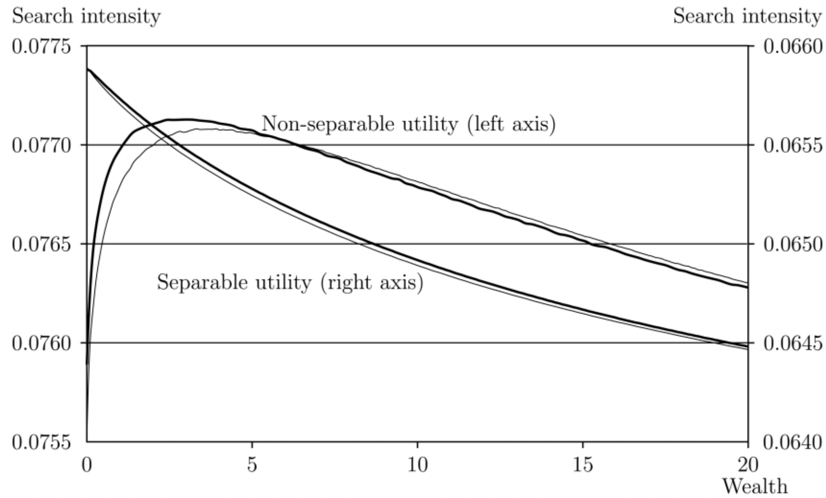

FIG. 2.—The unemployed worker's search intensity for separable and nonseparable utility. The thick line is the base model. The thin line is the alternative timing model, where the consumption and search decisions are made simultaneously.

will be of particular interest in this respect. A specific example for which search costs are affected by wealth is the case where search costs are monetary: the search costs will be evaluated in terms of a consumption loss via the budget constraint. For a concave utility function, the marginal utility loss from extra search will decrease the higher the consumption level. Higher wealth implies higher consumption, and consequently marginal search costs are decreasing in wealth in this case.

Figure 2 shows the choice of search intensity of an unemployed worker as a function of initial wealth for the model described in the previous section under two different utility functions, one where utility of consumption and disutility of search effort are separable and one where they are not.[9] The search choice is also shown for an alternative model differing

[9] The separable utility specification is given by

$$v(c, s) = c^{.5} - \frac{s}{1 - 10s},$$

and the nonseparable utility function is given by

$$v(c, s) = (c^{.5} + (1 - 10s)^{.5})^{.5}.$$

The simulations were performed under the following parameter specifications: the period length is weekly. The interest rate is set an annual rate of 4.5, and the subjective discount rate is set at an annual rate of 5.0. The benefit level is normalized at 1, and the wage is set at 2, yielding a replacement rate of 50. The lower bound on wealth is set at zero (implying no borrowing), and the upper bound only in that the savings and search choices are made simultaneously and not sequentially.

In the separable utility case, search costs are not affected by changes in wealth, and the search choice decreases monotonically with wealth due to the lower gains from search at higher wealth levels. In the nonseparable case, marginal search costs decrease with consumption. Since consumption is an increasing function of wealth, marginal search costs decrease with greater wealth. For low wealth levels (and, therefore, low consumption levels) the wealth effect on search costs is strong and more than offsets the wealth effect on gains from search. Consequently, at low wealth levels the search intensity increases with wealth. At higher wealth levels, changes in wealth affect search costs less, and the search intensity starts to decrease with greater wealth as the reduced gains to search eventually dominate the reduced costs. Thus, one finds a nonmonotonic search intensity with greater wealth for this particular case. Finally, it is seen that the choice of timing has very little effect on the choice of search intensity in this model.

In Flemming (1978) and section 5 of Acemoglu and Shimer (1999), it is assumed that search costs are monetary and that the basic utility function is characterized by constant absolute risk aversion.[10] As with the nonseparable example in figure 2, the cross-derivative is positive, that is $v''_{cs}(c, s) > 0$. This implies that, as consumption increases, the marginal search costs are reduced. Therefore, while greater wealth implies fewer incentives to move back into employment, it also increases consumption and consequently lowers the marginal search costs. It just so happens that in the constant absolute-risk-aversion case, the two effects exactly offset each other and eliminate wealth effects on the search decision altogether, given that there is no lower bound on wealth. The liquidity effects associated with a lower wealth bound will make the search decision depend negatively on wealth even in this case. Thus, in order to completely eliminate wealth effects on the search choice, it is necessary to allow negative consumption so as to eliminate the lower wealth bound.

The above examples should make it clear that one can establish a broad range of relationships between wealth and the search decision depending on the assumptions made in relation to $v''_{cs}(c, s)$. The results in Danforth (1979) even suggest that one can establish monotonically increasing search intensity with greater wealth by assuming monetary search costs, no lower

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is set so that it is above the upper bound of the ergodic wealth distribution. The job destruction rate is set so that the expected employment spell is 4.5 years.

[10] Acemoglu and Shimer (1999) assume a directed search technology. Jobs differ with respect to their wage, and the worker can choose which job to apply for. Higher-wage jobs will have longer queues. Thus, like the reservation wage decision, a choice of a higher probability of moving into employment (a choice of a lower-wage job) is associated with a future income loss.

bound on wealth, and a consumption utility function characterized by increasing absolute risk aversion.

V. Conclusion

In this article, we study a risk-averse worker who moves back and forth between employment and unemployment and thus faces a joint consumption-smoothing and job search (leisure-smoothing) problem. A major insight provided by the study is that both of the choices are affected in fundamental ways by allowing the analysis to treat them as interrelated problems. Some important aspects of search models are not affected much by the savings decision, but a general theory should allow savings and search behavior to influence each other.

Under the main assumption that utility is additively separable, we have derived a number of results regarding the worker's search and consumption choices. Our primary result is to identify conditions under which the unemployed worker's job search effort and thus her reemployment prospects are inversely related to wealth. This then implies that, as an unemployment spell stretches out and savings are reduced, the probability that the individual will find a job increases. The main difficulty in obtaining the result was to show that the marginal valuation of wealth is higher when the worker is unemployed compared to when she is employed. We conjecture that the results of the analysis can be carried over directly to conclusions about reservation wages instead of search intensities. The analysis in Gomes et al. (2001) suggests that this might very well be true.

Most of our results are quite intuitive and thus reassuring as to the usefulness of the modeling approach. For instance, consumption increases with wealth when the worker is both employed and unemployed. Also, it is found that precautionary savings are built up during employment spells and run down during unemployment spells. Furthermore, our finding that savings will not smooth consumption perfectly over states and over time extends similar results in Deaton (1991) to also hold in the case of endogenous income processes, here coming from endogenous search. The fact that insurance is less than perfect suggests that there may be room for welfare-improving labor market policies such as unemployment benefit programs and the like.

If one is interested in the savings decision from an insurance point of view, the case where the worker need not worry about insurance once employment is found (i.e., $\eta = 0$) is of course of limited interest. So even though many of the complications in the analysis above follow from the assumption that $\eta > 0$, it is crucial in this respect. Several recent papers deal with the insurance aspect, for instance, Acemoglu and Shimer (1999), Wang and Williamson (1999), and Lentz (2002), where optimal unemployment insurance schemes are derived in models where the workers are insured against unemployment via both their own savings and unemployment benefits. A similar problem is analyzed in Pissarides (2000), where instead of unemployment insurance, forms of employment protection such as severance pay and advance notice of job termination are the insurance mechanisms used to supplement workers' own savings. However, due to the analytical difficulties with the setup, these papers obtain analytical results only by assuming away the wealth effects on the search decisions or simply obtain results numerically instead. The present article should facilitate further analytical work on search models with job separation where wealth is allowed to affect search behavior.

Appendix

Proof of lemma 1. The value function $V_g(k)$ is the concavification of $V(k)$. Formally, let epi(V) be the epigraph of $V(k)$, that is, $\text{epi}(V) = \{(k, V) \in [\underline{k}, \overline{k}] \times \mathbb{R} \mid V \leq V(k)\}$, and let conv(epi($V$)) be its convex hull. The concavification of $V(k)$ is then simply the upper bound of the convex hull of V's epigraph, that is, the $\sup\{V \in \mathbb{R} \mid (k, V) \in \text{conv}(\text{epi}(V))\}$, which is exactly $V_g(k)$. Similarly, $U_g(k)$ is the concavification of $U(k)$. By concavity of V_g and U_g and by strict concavity of $u(c)$ it follows that $V_c(k)$ and $U_c(k)$ are strictly concave. To see this for $V_c(k)$, choose some $k_0, k_1 \in [\underline{k}, \overline{k}]$. Denote $\hat{k}_0 \equiv k_{+1}(k_0; e)$ and $\hat{k}_1 \equiv k_{+1}(k_1; e)$. Furthermore, define $k_\lambda \equiv \lambda k_0 + (1-\lambda)k_1$ and $\hat{k}_\lambda \equiv \lambda \hat{k}_0 + (1-\lambda)\hat{k}_1$ for some $\lambda \in [0, 1]$. Note that $\hat{k}_\lambda$ is not necessarily the optimal choice of next period's wealth given k_λ. Furthermore, it must be that $\hat{k}_\lambda$ is in the feasible set of choices of next period's wealth levels given k_λ. To see this, note that $\hat{k}_0 \leq \min[(1+r)k_0 + w, \overline{k}]$ and $\hat{k}_1 \leq \min[(1+r)k_1 + w, \overline{k}]$. Hence, it must be that $\hat{k}_\lambda \leq \min[(1+r)k_\lambda + w, \overline{k}]$. To show strict concavity, one must show that for all $k_0, k_1 \in [\underline{k}, \overline{k}]$, it must be that $V_c(k_\lambda) > \lambda V_c(k_0) + (1-\lambda)V_c(k_1)$. Thus, we get the following,

$$\begin{aligned}
\lambda V_c(k_0) + (1-\lambda)V_c(k_1) &= \lambda\left[u((1+r)k_0 + w - \hat{k}_0) + \frac{V_g(\hat{k}_0)}{1+\rho}\right] \\
&\quad + (1-\lambda)\left[u((1+r)k_1 + w - \hat{k}_1) + \frac{V_g(\hat{k}_1)}{1+\rho}\right] \\
&< u((1+r)k_\lambda + w - \hat{k}_\lambda) + \frac{V_g(\hat{k}_\lambda)}{1+\rho} \\
&\leq V_c(k_\lambda),
\end{aligned}$$

where the strict inequality follows from strict concavity of $u(\cdot)$ and concavity of $V_g(\cdot)$. The weak inequality follows from optimality. Thus, it must be that $V_c(\cdot)$ is strictly concave. A similar argument applies to $U_c(\cdot)$. $V(\cdot)$ is a convex combination of two strictly concave functions. Consequently, $V(k)$ must be strictly concave. QED.

Proof of lemma 2. First, we have that $V_g(k) = V(k)$ for all k, since $V(k)$ is concave by lemma 1, and thus $V_g'(k) = V'(k)$ for all k. Turning to $U_g'(k)$, lemma 2 claims that for interior choices of k^l and k^h it must be that the optimal solution is characterized by $U'(k^l) = U'(k^h) = U_g'(k)$. First of all, suppose that $k \in \{\underline{k}, \overline{k}\}$. In this case, the agent can effectively not participate in a lottery (with any real uncertainty) because the available lotteries are constrained to yield an expected value of k. Hence, it must be in this case that $k^l = k^h = k$ which immediately yields the result. Now, turn to the case where $k \in]\underline{k}, \overline{k}[$. The first-order condition for an interior choice of k^l having been unemployed in the previous period is given by

$$U'(k^l)\frac{k^h - k}{k^h - k^l} - \frac{1}{k^h - k^l}U(k^h) + \frac{k^h - k}{(k^h - k^l)^2}U(k^l) + \frac{k - k^l}{(k^h - k^l)^2}U(k^h) = 0$$

$$\Updownarrow$$

$$U'(k^l)(k^h - k) - \frac{k^h - k}{k^h - k^l}[U(k^h) - U(k^l)] = 0.$$

Via the envelope condition (4) this can be rewritten as

$$U'(k^l) = U_g'(k). \tag{A1}$$

The first-order condition for an interior choice of k^h is given by

$$\frac{k - k^l}{k^h - k^l}U'(k^h) + \frac{1}{k^h - k^l}U(k^l) - \frac{k^h - k}{(k^h - k^l)^2}U(k^l) - \frac{k - k^l}{(k^h - k^l)^2}U(k^h) = 0$$

$$\Updownarrow$$

$$U'(k^h)(k - k^l) - \frac{k - k^l}{k^h - k^l}[U(k^h) - U(k^l)] = 0.$$

Again using the envelope condition (4), this yields

$$U'(k^h) = U_g'(k). \tag{A2}$$

Hence, it follows directly from (A1) and (A2) that

$$U'(k^l) = U'(k^h) = U_g'(k).$$

The second part of the lemma is concerned with the case where $k > k^l = \underline{k}$. First of all, this implies that $k^h > k$ (or that $k^h = k$ and $\Pr(k^l) = 0$, which is the same as nonparticipation in the lottery and which is thus an uninteresting case). Now, suppose, contrary to the claim in lemma 2, that $U_g'(k) < U'(k^l) = U'(\underline{k})$. But in this case, it is seen by the arguments above that

$$\frac{\partial U_g(k)}{\partial k^l} = (k^h - k)[U'(\underline{k}) - U_g'(k)] > 0,$$

which contradicts that k^l is chosen optimally since the value of the lottery can be increased by increasing k^l, which is a feasible choice. This establishes the claim. QED.

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