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Economics 714
Macroeconomic Theory
Spring 2018

Problem Set 2

Due in Class on 2/28

1. Consider a specification of the Pissarides model as in class, with a Cobb-Douglas matching function which in concentrated form leads to $q(\theta) = k\theta^{-\gamma}$ where the constants $k > 0$ and $0 < \gamma < 1$. Suppose also the vacancy cost is c, not pc as we specified in class.

   (a) Characterize the steady state equilibrium as sharply as you can, giving expressions for the equilibrium wage, unemployment rate, and vacancy rate.

   (b) Suppose that there is an increase in k, the effectiveness of matching. How does this affect the steady state equilibrium?

   (c) Suppose that the economy is initially in a steady state equilibrium, then there is a permanent reduction in productivity from p to $p' < p$. Suppose that instead of being reset via Nash bargaining, the wage remains constant. Provide conditions which ensure that this wage is feasible (gives both the worker and firm nonnegative surplus).

   (d) What happens to unemployment and vacancies following the negative productivity shock with a fixed wage?

2. Consider a two period endowment economy with a continuum of individuals of two types. The total population size is normalized to 2 and each type has measure 1. Endowments are i.i.d. over time, common across agent types, and take on the values 0 or 1 in each period with equal probability. The aggregate endowment is constant at 1 in each period, and the relevant state of nature $s_t \in \{0, 1\}, \quad t = 1, 2$ is an i.i.d. random variable indicating which type receives the high endowment in period t. All agents have common preferences, defined over their own consumption c_t^i $i = 1, 2$ at the two dates:
$$U(c_1^i, c_2^i) = -E\left[(c_1^i - 1)^2 + \beta(c_2^i - 1)^2\right].$$
   Assume that $0 \leq c_t^i \leq 1$, as it will be in equilibrium below, so that utility is increasing.

   (a) Suppose that agents can trade in a complete set of state-contingent securities (in zero net supply) at date 0.

      i. State explicitly what the relevant securities are, and solve fully for the equilibrium allocation of consumption for the two agent types.

      ii. Show how the equilibrium allocation is implemented – that is, what are the equilibrium holdings of the different securities by each type?

      iii. What is the equilibrium rate of return r on a risk-free bond?

(b) Now consider an incomplete markets economy, in which agents can only trade in a risk free bond in zero net supply. Let a_t^i be the holdings of this bond at the start of period t for each type i. Each agent is endowed with $a_1^i = 0$ initial holdings, so the holdings at date 2 are:

$$a_2^i = (1 + r)(e_1^i - c_1^i)$$

There is no reason to carry bonds after period 2 so:

$$c_2^i = a_2^i + e_2^i.$$

i. Taking the interest rate r as given, solve for each agent's optimal (state-contingent) consumption and bond holdings $\{c_t^i, a_t^i\}$.

ii. Discuss and interpret how the interest rates and the allocations compare to the complete markets case.

3. Consider an economy consisting of identical individuals with preferences given by

$$U(x_1, x_2, N) = \alpha_1 \log(x_1) + \alpha_2 \log(x_2) + (1 - \alpha_1 - \alpha_2) \log(1 - N).$$

Individuals supply labor, N, at a market wage of unity, and purchase goods 1 and 2, both of which have fixed producer prices equal to 1. The government must fund expenditure G, measured in the fixed producer prices.

(a) Assume that the government is restricted to using linear commodity taxes on goods 1 and2 to raise revenue. Set up the Ramsey optimal tax problem for this economy, and find the optimal tax rates on goods 1 and 2 as a function of G and the individual preference parameters.

(b) Obtain expressions for the marginal utility of income of the individual and the marginal cost of government revenue. How do changes in the government spending affect these costs and the difference between the two?