Money in an RBC framework

Noah Williams

University of Wisconsin

A basic real business cycle (RBC) model

Representative household

- Takes prices as given and maximizes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i U\left(C_{t+1}, L_{t+i}\right)$$

subject to

$$w_t(1 - L_{t+i}) + r_t K_{t-1} + (1-\delta)K_{t-1} = C_t + K_t.$$

- First order conditions:

$$U_C - \lambda_t = 0$$

$$U_L - w_t \lambda_t = 0 \Rightarrow \frac{U_L}{U_C} = w_t$$

$$-\lambda_t + \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta\right) \lambda_{t+1} = 0$$

- Combining the first and third conditions yields the Euler condition:

$$U_C(C_t, L_t) = \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta\right) U_C(C_{t+1}, L_{t+1})$$

A basic real business cycle (RBC) model

Firms

- Firms maximize profits in competitive goods and factor markets, subject to a production technology.

$$Y_t = F(N_t, K_{t-1}, Z_t)$$

$$\max \left[F(N_t, K_{t-1}, Z_t) - w_t N_t - r_t K_{t-1} \right]$$

- First order conditons:

$$F_N = w_t$$

$$F_K = r_t$$

A basic real business cycle (RBC) model

Market clearing:

- Goods market:

$$Y_t = C_t + K_t - (1 - \delta) K_{t-1}$$

- Labor market:

$$N_t = 1 - L_t$$

A basic real business cycle (RBC) model

Equilibrium

- Given an exogenous process for the productivity shock Z, equilibrium consists of time paths for output, consumption, capital, employment, wages and rental rates that satisfy

$$\frac{U_L(C_t, 1 - N_t)}{U_C(C_t, 1 - N_t)} = w_t$$

$$U_C(C_t, 1 - N_t) = \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta\right) U_C(C_{t+1}, 1 - N_{t+1})$$

$$Y_t = F(N_t, K_{t-1}, Z_t)$$

$$F_N(N_t, K_{t-1}, Z_t) = w_t$$

$$F_K(N_t, K_{t-1}, Z_t) = r_t$$

$$Y_t = C_t + K_t - (1 - \delta) K_{t-1}$$

A basic real business cycle (RBC) model

- Specify functional forms

$$U = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{(1-L_t)^{1+\eta}}{1+\eta}$$

$$F = Z_t N_t^{\alpha} K_{t-1}^{1-\alpha}$$

- First order conditions become (with some simplification)

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \alpha Z_t N_t^{\alpha-1} K_{t-1}^{1-\alpha} = \alpha \left(\frac{Y_t}{N_t} \right)$$

$$C_t^{-\sigma} = \beta \mathrm{E}_t \left[(1-\alpha) \left(\frac{Y_t}{K_{t-1}} \right) + 1 - \delta \right] C_{t+1}^{-\sigma}$$

$$Y_t = Z_t N_t^{\alpha} K_{t-1}^{1-\alpha}$$

$$Y_t = C_t + K_t - (1-\delta) K_{t-1}$$

A basic real business cycle (RBC) model

Steady state for Y, C, K, N, R:

$$\frac{\chi N^{\eta}}{C^{-\sigma}} = \alpha \left(\frac{Y}{N}\right)$$

$$1 = \beta R$$

$$R \equiv (1-\alpha)\left(\frac{K}{N}\right)^{-\alpha} + 1 - \delta$$

$$\left(\frac{Y}{N}\right) = \left(\frac{K}{N}\right)^{1-\alpha}$$

$$Y = C + \delta K$$

Incorporating money

- To employ the neoclassical framework to analyze monetary issues, a role for money must be specified so that the agents will wish to hold positive quantities of money.
- Three general approaches to incorporating money into general equilibrium models have been followed:
  1. Assume that money yields direct utility by incorporating money balances directly into the utility functions of the agents of the model (Sidrauski 1967).
  2. Impose transactions costs
     1. Make asset exchanges costly (Baumol 1952, Tobin 1956)
     2. Require that money be used for certain types of transactions (Clower 1967)
     3. Assume time and money can be combined to produce transaction services that are necessary for obtaining consumption goods
     4. Assume direct barter of commodities is costly (Kiyotaki and Wright 1989).
  3. Treat money like any other asset used to transfer resources intertemporally (Samuelson 1958).

A basic MIU model

First approach: MIU Sidrauski (1967)

- Real money balances enter directly in the utility function.
- Given suitable restrictions on the utility function, such an approach can guarantee that, in equilibrium, agents choose to hold positive amounts of money so that money will be positively valued.
- Representative household takes prices as given and maximizes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i U\left(C_{t+i}, m_{t+i}, L_{t+i}\right)$$

subject to a budget constraint.

A basic MIU model

- Budget constraint, in nominal terms:

$$\begin{array}{c} W_t(1-L_t) + (r_t + 1 - \delta) P_t K_{t-1} \\ +(1+i_{t-1}) B_{t-1} + M_{t-1} + P_t \tau_t \end{array} = P_t C_t + P_t K_t + B_t + M_t$$

where τ are real, lump-sum transfers to the household (can be negative).

- In real terms:

$$\begin{array}{c} w_t(1-L_t) + (r_t + 1 - \delta) K_{t-1} \\ +(1+i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \tau_t \end{array} = C_t + K_t + \frac{B_t}{P_t} + \frac{M_t}{P_t}$$

or

$$\begin{array}{c} w_t(1-L_t) + (r_t + 1 - \delta) K_{t-1} \\ +(1+i_{t-1}) \frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} + \tau_t \end{array} = C_t + K_t + b_t + m_t$$

A basic MIU model

- First order conditions:

$$U_C - \lambda_t = 0$$

$$U_L - w_t \lambda_t = 0 \Rightarrow \frac{U_L}{U_C} = w_t$$

$$-\lambda_t + \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta \right) \lambda_{t+1} = 0$$

$$-\lambda_t + \beta (1 + i_t) \mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) \lambda_{t+1} = 0$$

$$U_m - \lambda_t + \beta \mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) \lambda_{t+1} = 0$$

- First three look just as before (except U_C and U_L may depend on m).

A basic MIU model

- The FOC for money can be written

$$\frac{\lambda_t}{P_t} = \frac{U_m}{P_t} + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{P_{t+1}} \right)$$

- Solve forward to yield

$$\frac{1}{P_t} = \left(\frac{1}{\lambda_t} \right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{U_m \left(C_{t+i}, m_{t+i}, L_{t+i} \right)}{P_{t+i}} \right]$$

- Value of money equals expected present value of discounted utility return.

A basic MIU model

- Combining the first and third conditions yields the Euler condition:

$$U_C(C_t, m_t, L_t) = \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta\right) U_C(C_{t+1}, m_{t+1}, L_{t+1})$$

- Similarly,

$$U_C(C_t, m_t, L_t) = \beta(1 + i_t)\mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) U_C(C_{t+1}, m_{t+1}, L_{t+1})$$

- From fourth and fifth,

$$\frac{U_m}{U_C} = \beta i_t \mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) \frac{U_C(C_{t+1}, m_{t+1}, L_{t+1})}{U_C(C_t, m_t, L_t)}$$

- But

$$1 = \beta(1 + i_t)\mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) \frac{U_C(C_{t+1}, m_{t+1}, L_{t+1})}{U_C(C_t, m_t, L_t)}$$

so

$$\frac{U_m}{U_C} = \beta i_t \mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) \frac{U_C(C_{t+1}, m_{t+1}, L_{t+1})}{U_C(C_t, m_t, L_t)} = \frac{i_t}{1 + i_t}.$$

A basic MIU model

- Firm side of model is same as in basic RBC model.
- Collecting equilibrium conditions
  - Given an exogenous process for the productivity shock and the nominal stock of money, equilibrium consists of time paths for output, real money balances, consumption, capital, employment, wages, the nominal rate of interest, the price level, and rental rates that satisfy

A basic MIU model

$$\frac{U_L(C_t, m_t, 1 - N_t)}{U_C(C_t, m_t, 1 - N_t)} = w_t$$

$$\frac{U_m}{U_C} = \frac{i_t}{1 + i_t}.$$

$$U_C(C_t, m_t, 1 - N_t) = \beta \mathrm{E}_t \left(r_{t+1} + 1 - \delta\right) U_C(C_{t+1}, m_{t+1}, 1 - N_{t+1})$$

$$U_C(C_t, m_t, L_t) = \beta(1 + i_t)\mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) U_C(C_{t+1}, m_{t+1}, L_{t+1})$$

$$Y_t = F(N_t, K_{t-1}, Z_t)$$

$$F_N(N_t, K_{t-1}, Z_t) = w_t$$

$$F_K(N_t, K_{t-1}, Z_t) = r_t$$

$$Y_t = C_t + K_t - (1 - \delta)K_{t-1}$$

$$P_t m_t = M_t$$

A basic MIU model

Steady-state

- Standard RBC relationships plus

$$1 = \beta \left(\frac{1+i}{1+\pi} \right)$$

$$\frac{U_m(C, m, 1-N)}{U_C(C, m, 1-N)} = \frac{i}{1+i} = 1 - \frac{\beta}{1+\pi}$$

$$P_t = \frac{M_0 (1+\pi)^t}{m}$$

- These imply the Fisher equation $1 + i = (r + 1 - \delta)(1 + \pi)$ or

$$i \approx r + \pi$$

Superneutrality

- If U_L and U_C independent of m, real side identical to basic RBC model. (Steady state and dynamics)
  - Neutrality: One time changes in the nominal quantity of money affect only the price level.
  - Superneutrality: Changes in growth rate of money do not affect real variables.
- Changes in the rate of growth of money affect only the inflation rate, the nominal interest rate, and real money balances.
  - If F displays constant returns to scale and U_L and U_C depend on m, still get steady-state capital-labor ratio independence of money growth.

Welfare cost of inflation

- Because money holdings yield direct utility and higher inflation reduces real money balances, inflation generates a welfare loss.
- Questions:
  - How large is the welfare cost of inflation?
  - Is there an optimal rate of inflation that maximizes the steady-state welfare of the representative household?

Welfare cost of inflation

- The optimal rate of inflation addressed by Bailey (1956) and M. Friedman (1969).
  - Basic intuition: the private opportunity cost of holding money depends on the nominal rate of interest. The social marginal cost of producing money is essentially zero.
  - The wedge that arises between the private marginal cost and the social marginal cost when the nominal rate of interest is positive generates an inefficiency.
  - This inefficiency would be eliminated if the nominal rate of interest were zero.
  - So the optimal rate of inflation is a rate of *deflation* approximately equal to the real return on capital.
- In the steady state, real money balances are directly related to the inflation rate, so the optimal rate of inflation is also frequently discussed under the heading of the *optimal quantity of money*.

Welfare cost of inflation

- The major criticism of this result is that of Phelps (1973), who pointed out that money growth generates revenue for the government–the inflation tax.
- The implicit assumption so far has been that variations in money growth are engineered via lump-sum transfers.
- Any effects on government revenue can be offset by a suitable adjustment in these lump-sum transfers (taxes).
- But if governments only have distortionary taxes available for financing expenditures, then reducing inflation tax revenues to achieve the Friedman rule for optimal inflation requires that the lost revenue be replaced through increases in other distortionary taxes.
- To minimize the total distortions associated with raising a given amount of revenue, it may be optimal to rely to some degree on the inflation tax.
- More recent work has reexamined these results (see Chari, Christiano, and Kehoe 1991, 1996, Correia and Teles 1996, 1999, Mulligan and Sala-i-Martin 1997).

Cash-in-advance models

- Clower (1967), goods buy money and money buys goods, but goods don't buy goods.
- And because goods don't buy goods, a medium of exchange that serves to aid the process of transacting will have value.
- The demand for money is then determined by the nature of the economy's transactions technology.
- Rather than allowing substitutability between time and money in carrying out transactions, *cash-in-advance* (CIA) models simply require that money balances be held to finance certain types of purchases; without money, these purchases cannot be made. CIA models.

Cash-in-advance models

- Timing assumptions also are important in CIA models.
- In Lucas (1982), agents are able to allocate their portfolio between cash and other assets at the start of each period, after observing any current shocks but prior to purchasing goods.
  - The asset market opens first and then the goods market opens.
  - If there is a positive opportunity cost of holding money and the asset market opens first, agents will only hold an amount of money that is just sufficient to finance their desired level of consumption.
- In Svensson (1985), the goods market opens first.
  - This implies that agents have available for spending only the cash carried over from the previous period, and so cash balances must be chosen before agents know how much spending they will wish to undertake.
  - If uncertainty is resolved *after* money balances are chosen, an agent may find that she is holding cash balances that are too low to finance her desired spending level. Or she may be left with more cash than she needs, thereby forgoing interest income.

Cash-in-advance models

The representative agent

- Maximizes present discounted value of expected utility subject to a sequence of new constraints.
- If the goods markets open first and the agent enters the period with money holdings M_{t-1} and receives a lump-sum transfer T_t (in nominal terms), the CIA constraint takes the form

$$P_t c_t \leq M_{t-1} + T_t,$$

  where c is real consumption, P is the aggregate price level, and T is the nominal lump-sum transfer.
- In real terms,

$$c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} = \frac{m_{t-1}}{\Pi_t} + \tau_t,$$

  where $m_{t-1} = M_{t-1}/P_{t-1}$, $\Pi_t = P_t/P_{t-1}$ is 1 plus the inflation rate, and $\tau_t = T_t/P_t$.
- Income from production during period t will not be available for consumption purchases during period t.

Cash-in-advance models

- Budget constraint in real terms, $\omega_t \geq c_t + m_t + b_t + k_t$ where

$$\omega_t \equiv f(k_{t-1}) + (1-\delta)k_{t-1} + \tau_t + \frac{m_{t-1} + (1+i_{t-1})b_{t-1}}{\Pi_t},$$

  and m and b are real cash and bond holdings.
- Let $a_t = b_t + m_t$. Then budget constraint implies

$$\omega_{t+1} = f(k_t) + (1-\delta)k_t + \tau_{t+1} + R_t a_t - \left(\frac{1+i_t}{\Pi_{t+1}}\right) m_t.$$

- This form highlights that there is a cost to holding money when the nominal interest rate is positive. This cost is $(1+i_t)/\Pi_{t+1}$.
- This is the same expression for the opportunity cost of money in an MIU model.

Cash-in-advance models

The decision problem

- Define the value function

$$V(\omega_t, m_{t-1}) = \max\{u(c_t) + \beta V(\omega_{t+1}, m_t)\},$$

where the maximization is subject to the budget constraint

$$\omega_t \geq c_t + m_t + b_t + k_t,$$

the CIA constraint, and the definition of ω_{t+1}.

Cash-in-advance models

The decision problem

- Letting λ_t (μ_t) denote the Lagrangian multiplier associated with the budget constraint (the CIA constraint), the first order necessary conditions for the agent's choice of consumption, capital, bond, and money holdings take the form

$$u_c(c_t) - \lambda_t - \mu_t = 0$$

$$\beta \left[f_k(k_t) + 1 - \delta \right] V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0$$

$$\beta R_t V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0$$

$$\beta \left[R_t - \frac{i_t}{\Pi_{t+1}} \right] V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) - \lambda_t = 0.$$

Cash-in-advance models

From the envelope theorem,

$$V_\omega(\omega_t, m_{t-1}) = \lambda_t$$

$$V_m(\omega_t, m_{t-1}) = \left(\frac{1}{\Pi_t}\right)\mu_t.$$

- λ_t is equal to the marginal utility of wealth.
- The marginal utility of consumption exceeds the marginal utility of wealth by the value of liquidity services, μ_t.
- The individual must hold money in order to purchase consumption, so the "cost," to which the marginal utility of consumption is set equal, is the marginal utility of wealth plus the cost of the liquidity services needed to finance the transaction.

Cash-in-advance models

The first order conditions imply

$$\lambda_t = \beta \left(\frac{\lambda_{t+1} + \mu_{t+1}}{\Pi_{t+1}} \right) \Rightarrow \frac{1}{P_t} = \left(\frac{1}{\lambda_t} \right) \beta \left(\frac{\lambda_{t+1} + \mu_{t+1}}{P_{t+1}} \right).$$

- This equation can also be interpreted as an asset pricing equation for money.
- The price of a unit of money in terms of goods is just $1/P_t$ at time t; its value in utility terms is λ_t / P_t.
- Solving this equation forward implies that

$$\frac{1}{P_t} = \left(\frac{1}{\lambda_t} \right) \sum_{i=1}^{\infty} \beta^i \left(\frac{\mu_{t+i}}{P_{t+i}} \right).$$

- Value of money related to present discounted value of Lagrangian multipliers on CIA constraint.

Cash-in-advance models

The nominal rate of interest

- Since $\lambda_t = \beta R_t \lambda_{t+1} = \beta \left(\lambda_{t+1} + \mu_{t+1}\right) / \Pi_{t+1}$, or $R_t \Pi_{t+1} \lambda_{t+1} = \left(\lambda_{t+1} + \mu_{t+1}\right)$ and $1 + i_t = R_t \Pi_{t+1}$, the nominal interest rate is given by

$$i_t = \left(\frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_{t+1}}\right) - 1 = \frac{\mu_{t+1}}{\lambda_{t+1}}.$$

- The nominal rate of interest is positive if and only if money yields liquidity services ($\mu_{t+1} > 0$).
- In particular, if the nominal interest rate is positive, the CIA constraint is binding ($\mu > 0$).

Cash-in-advance models

Marginal utility of consumption

- We can use the relationship between the nominal rate of interest and the Lagrangian multipliers to rewrite the expression for the marginal utility of consumption as $u_c = \lambda(1 + \mu/\lambda) = \lambda(1 + i) \geq \lambda$.
- Since λ represents the marginal value of income, the marginal utility of consumption exceeds that of income whenever the nominal interest rate is positive.
- Even though the economy's technology allows output to be directly transformed into consumption, the "price" of consumption is not equal to 1; it is $1 + i$ since the household must hold money to finance consumption.
- Thus, in this CIA model, a positive nominal interest rate acts as a tax on consumption; it raises the price of consumption above its production cost.

Cash-in-advance models

Modifications

- Cash and Credit Goods
  - Lucas and Stokey (1983, 1987) introduced the idea that the CIA constraint may only apply to a subset of consumption goods. The marginal utility of cash goods will be equated to $\lambda + \mu \geq \lambda$, while the marginal utility of credit goods will be equated to λ.
  - Hence, the CIA requirement for cash goods drives a wedge between the marginal utilities of the two types of goods.
  - It is exactly as if the consumer faces a tax of $\mu/\lambda = i$ on purchases of the cash good. Higher inflation, by reducing holdings of real cash balances, serves to raise the tax on cash goods and generates a substitution away from the cash good and toward the credit good.
- CIA and Investment Goods
  - The inflation tax applies to both consumption and investment goods. Higher rates of inflation will tend to discourage capital accumulation lower the steady-state capital-labor ratio.

Welfare costs of inflation

- In CIA models, inflation acts as a tax on goods or activities whose purchase requires cash.
- This tax then introduces a distortion by creating a wedge between the marginal rates of transformation implied by the economy's technology and the marginal rates of substitution faced by consumers.
- Since the CIA model, like the MIU model, offers no reason for such a distortion to be introduced (there is no inefficiency that calls for Pigovian taxes or subsidies on particular activities, and the government's revenue needs can be met through lump-sum taxation), optimality calls for setting the inflation tax equal to zero.
- The inflation tax is directly related to the nominal rate of interest; a zero inflation tax is achieved when the nominal rate of interest is equal to zero.