

Midterm Examination

Instructions: This is a 75 minute exam with two questions worth a total of 100 points. Points are indicated at the start of each question. **Allocate your time wisely.** In order to get full credit, you must give a clear, concise, and correct answer, including all necessary explanations and calculations. Notes, books, and calculators are not permitted.

1. **(50 points)** Consider the following version of a Lucas asset pricing model, in which an agent wants to minimize fluctuations in consumption around a mean. For simplicity we normalize the mean to zero. In other words, a representative agent has preferences:

$$-\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t c_t^2$$

over the single nonstorable consumption good c_t (“fruit”). The endowment process x_t is also mean zero and follows a Gaussian *AR* process:

$$x_{t+1} = \rho x_t + \varepsilon_{t+1}$$

where $0 < \rho < 1$ and ε_{t+1} is an i.i.d. $N(0, \sigma^2)$ random variable.

- (a) **(15 points)** Define a recursive competitive equilibrium with a market in claims to the endowment process (“trees”), with pricing function $p(x)$.
- (b) **(15 points)** Characterize the recursive competitive equilibrium and provide an expression for the pricing function $p(x)$.
- (c) **(10 points)** What is the risk-free one period interest rate in this economy? Interpret your answer.
- (d) **(10 points)** Show that the pricing function for the claims to the endowment can be written:

$$p(x) = \bar{p}x + \frac{\hat{p}}{x}.$$

Find expressions for the constants \bar{p} and \hat{p} .

2. **(50 points)** This problem considers a variation on the neoclassical growth model. Suppose that there is no technological change or population growth. However there are two types of households in the economy. Household 1 has the preferences:

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t})$$

over its consumption c_{1t} while household 2 has the preferences:

$$\sum_{t=0}^{\infty} \beta^t v(c_{2t}).$$

Note that β is the same across households, and suppose both u and v satisfy the usual smoothness and Inada conditions. As always, there is a representative firm who produces according to a production function $f(k)$ which satisfies $f' > 0, f'' < 0$, there is an initial capital stock k_0 , and capital depreciates at rate δ .

- (a) **(15 points)** Formulate the social planner's problem to determine efficient allocations, and derive the conditions for maximization.
- (b) **(15 points)** Show that to characterize the dynamics of the efficient allocation we can consider a system in capital k_t and aggregate consumption $c_t = c_{1t} + c_{2t}$. Plot a phase diagram in the (k, c) space.
- (c) **(10 points)** Does the system have a steady state? If so, is it unique? Find expressions for the steady state $(k_t, c_t, c_{1t}, c_{2t})$. How does the steady state (k_t, c_t) depend on u and v , the preferences of the households?
- (d) **(10 points)** Suppose that now the planner must finance a constant amount G of government spending every period. Suppose the planner has access to lump sum taxes, but can only tax household 2. What happens to the economy? Consider both long run effects and transitional dynamics.