Lecture 7: More on Search

Economics 714, Spring 2018

1 Equilibrium Search Model

1.1 From last time

Beveridge curve:

$$u = \frac{s}{s + \theta q(\theta)}$$

Job creation condition:

$$p - w - (r + s)\frac{pc}{q(\theta)} = 0$$

1.2 Workers' Decisions

There is a single wage w rather than a distribution of offers, so no reason to turn down wage as long as wage is greater than the unemployment benefit.

Value of unemployed:

$$rU = z + \theta q(\theta)(W - U)$$

Value of employed:

$$rW = w + s(U - W(w))$$

We can solve:

$$W(w) = \frac{w}{r + s} + \frac{s}{r + s}U, \quad W'(w) = \frac{1}{r + s}$$

Use expression for W, solve for (W, U) explicitly:

$$\begin{aligned} U &= \frac{sz + \theta q(\theta) w + rz}{r^2 + r\theta q(\theta) + sr} \\ W &= \frac{sz + \theta q(\theta) w + rw}{r^2 + r\theta q(\theta) + sr} \end{aligned}$$

So $W > U \Leftrightarrow w > z$. So clearly as long as the wage is above the benefit, it's better to be employed.

1.3 Wage Determination

Because of the search frictions, both workers and firms have some power in their match. To determine the wage, we need to use some bargaining protocol to determine how to split up the joint surplus in the match.

The most common approach is to solve for wages by Nash bargaining, where each party has a clear reservation value and we assume workers have (relative) bargaining power β:

$$w = \arg\max_{\hat{w}} \left(W(\hat{w}) - U\right)^{\beta} \left(J(\hat{w}) - V\right)^{1-\beta}$$

Optimality condition:

$$\beta \frac{W'(w)}{W - U} = -(1-\beta) \frac{J'(w)}{J - V}$$

Simplify:

$$W = U + \beta(W - U + J)$$

Use expression for J:

$$W - U = \frac{\beta}{1-\beta} \frac{pc}{q(\theta)}$$

Also can solve to show:

$$w = rU + \beta(p - rU)$$

Finally, can derive the wage equation:

$$w = (1 - \beta)z + \beta(p + \theta pc)$$

1.4 Steady State Unemployment

Three key equations:

$$\begin{aligned} w &= (1 - \beta)z + \beta(p + \theta pc) \\ p &- w - (r + s)\frac{pc}{q(\theta)} = 0 \\ u &= \frac{s}{s + \theta q(\theta)} \end{aligned}$$

Combine first two to determine steady state θ:

$$(1 - \beta)(p - z) - \frac{r + s + \beta\theta q(\theta)}{q(\theta)} pc = 0$$

Then Beveridge curve determines steady state u given θ.

We can then trace out changes in steady state equilibrium employment if there is a change in the underlying parameters like productivity p or the unemployment benefit z.

2 Mortensen-Pissarides (1994)

2.1 Setup

We now endogenize job destruction.

Each job has productivity px where x idiosyncratic.

New x arrives at rate λ, drawn from distribution G on $[0,1]$. Initial draw is $x = 1$.

Value of a job now is $J(x)$. If $J(x) \geq 0$ job kept, if $J(x) < 0$ destroyed.

Reservation productivity R such that $J(R) = 0$.

Job destruction rate: $\lambda G(R)(1-u)$.

Unemployment flow:

$$\dot{u} = \lambda G(R)(1-u)) - u\theta q(\theta)$$

Steady state:

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$$

2.2 Value Functions

Value functions for the firm:

$$\begin{aligned} rV &= -pc + q(\theta)(J(1) - V) \\ rJ(x) &= px - w(x) + \lambda \left[\int_R^1 J(s) dG(s) - J(x) \right] \end{aligned}$$

Value functions for the worker:

$$\begin{aligned} rU &= z + \theta q(\theta)(W(1) - U) \\ rW(x) &= w(x) + \lambda \left[\int_R^1 W(s) dG(s) + G(R)U - W(x) \right] \end{aligned}$$

Nash bargaining:

$$W(x) - U = \beta[W(x) - U + J(x)]$$

2.3 Equilibrium Relations

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$$

$$J(R) = 0$$

Free entry: $V = 0$ so:

$$J(1) = \frac{pc}{q(\theta)}$$

$$W(x) - U = \beta[W(x) - U + J(x)]$$

Need to solve to characterize R in addition to θ.