Lecture 6: Search Equilibrium

Economics 714, Spring 2018

1 Equilibrium Search Model

Pissarides (1985) model, later modified by Mortensen-Pissarides (1994)

Continuous time, constant interest rate r.

Continuum L of identical workers, risk neutral preferences:

$$\int_0^\infty e^{-rt} x_t dt$$

Endogenous number of firms, each with one job. Competitive producer of final good at price p.

Firms post vacancies, cost c per unit time, pc relative to output

uL unemployed, vL vacant jobs, fL jobs filled, related by matching function m:

$$fL = m(uL, vL)$$

Assume m increasing, concave, and has constant returns to scale, so $f = m(u, v)$. Define $\theta = v/u$ then

$$q(\theta) = m(u/v, 1) = m(1/\theta, 1)$$

(Poisson) Rate at which vacant jobs are filled. Mean duration of vacancy $= 1/q(\theta)$.

$\theta q(\theta)$ is rate at which unemployed workers find job, unemployment duration $1/(\theta q(\theta))$.

Properties:

$$\begin{aligned} q'(\theta) &\leq 0 \\ \frac{q'(\theta)\theta}{q(\theta)} &\in [-1, 0] \end{aligned}$$

Job creation when firm and worker meet, agree on wage. Jobs created:

$$fL = Lm(u, v) = Lvm(u/v, 1) = Lu\theta q(\theta)$$

Creation rate:

$$\frac{u\theta q(\theta)}{1 - u}$$

Jobs destroyed at exogenous Poisson rate s. Total job destruction: $s(1-u)L$.

Evolution of unemployment:

$$\dot{u} = s(1 - u) - u\theta q(\theta)$$

steady state:

$$u = \frac{s}{s + \theta q(\theta)}$$

2 Firm Decision

Wage w, hours fixed at 1, either party can freely break contract.

J value of filled job, V value of vacant job.

$$\begin{aligned} rV &= -pc + q(\theta)(J - V) \\ rJ &= p - w - sJ \end{aligned}$$

So we have:

$$J = \frac{p - w}{r + s}$$

Free entry of firms means $V = 0$. So we also have:

$$J = \frac{pc}{q(\theta)}$$

Equating gives the job creation condition:

$$p - w - (r + s)\frac{pc}{q(\theta)} = 0$$

2.1 Workers' Decisions

Now no longer a distribution of offers, no reason to turn down wage as long as $W > U$.

Value of unemployed:

$$rU = z + \theta q(\theta)(W - U)$$

Value of employed:

$$rW = w + s(U - W(w))$$

We can solve:

$$W(w) = \frac{w}{r + s} + \frac{s}{r + s}U, \quad W'(w) = \frac{1}{r + s}$$

Use expression for W, solve for (W, U) explicitly:

$$\begin{aligned} U &= \frac{sz + \theta q(\theta)w + rz}{r^2 + r\theta q(\theta) + sr} \\ W &= \frac{sz + \theta q(\theta)w + rw}{r^2 + r\theta q(\theta) + sr} \end{aligned}$$

So $W > U \Leftrightarrow w > z$.

2.2 Wage Determination

Solve for wages by Nash bargaining solution:

$$w = \arg\max_{\hat{w}} \left(W(\hat{w}) - U\right)^{\beta} \left(J(\hat{w}) - V\right)^{1-\beta}$$

Optimality condition:

$$\beta \frac{W'(w)}{W-U} = -(1-\beta)\frac{J'(w)}{J-V}$$

Simplify:

$$W = U + \beta(W - U + J)$$

Use expression for J:

$$W - U = \frac{\beta}{1-\beta}\frac{pc}{q(\theta)}$$

Also can solve to show:

$$w = rU + \beta(p - rU)$$

Finally, can derive the wage equation:

$$w = (1-\beta)z + \beta(p + \theta pc)$$

2.3 Steady State Unemployment

Three key equations:

$$\begin{aligned} w &= (1-\beta)z + \beta(p + \theta pc) \\ p &- w - (r+s)\frac{pc}{q(\theta)} = 0 \\ u &= \frac{s}{s + \theta q(\theta)} \end{aligned}$$

Combine first two to determine steady state θ:

$$(1-\beta)(p-z) - \frac{r+s+\beta\theta q(\theta)}{q(\theta)} pc = 0$$

Then Beveridge curve determines steady state u given θ.