Lecture 3: Asset Pricing and the Equity Premium Puzzle

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1 Pricing Long-lived Assets

For multi-period claims, can chain together one-step claims:

$$q^j(s, s^j) = \int q(s, s')q^{j-1}(s', s^j)ds'$$

Ownership of tree is claim to entire $\{s_t\}$ so can define price as:

$$p(s) \quad = \quad \beta \int \frac{u'(s')}{u'(s)} s' f(s, s')ds' + \beta^2 \int \frac{u'(s'')}{u'(s)} s'' f^2(s, s'')ds'' + \ldots$$

Or in sequence notation:

$$p_t = E_t \left[\sum_{j=1}^{\infty} \beta^j \frac{u'(s_{t+j})}{u'(s_t)} s_{t+j} \right]$$

2 Equity Premium

2.1 Risk Neutrality

With linear utility $u'(c_t)$ constant, so risk free rate:

$$1 = E_t(\beta R) \quad \Rightarrow R = \frac{1}{\beta}$$

So then

$$p_t = E_t \left[\sum_{j=1}^{\infty} \beta^j s_{t+j} \right] = E_t \left[\sum_{j=1}^{\infty} \frac{s_{t+j}}{R^j} \right]$$

2.2 Risk Corrections

Risk free rate:

$$1 = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} R\right] \quad \Rightarrow R = \frac{1}{E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right]}$$

or $R = 1/E_t m_{t+1}$.

For general payoff x_{t+1},

$$\begin{aligned} p_t &= E_t(m_{t+1}x_{t+1}) \\ &= E_t m_{t+1} E_t x_{t+1} + cov_t(m_{t+1}, x_{t+1}) \\ &= \frac{E_t x_{t+1}}{R} + cov_t(m_{t+1}, x_{t+1}) \end{aligned}$$

Assets that payoff more when m_{t+1} is high, and hence when $u'(c_{t+1}$ is high or c_{t+1} is low, command a higher price. Such assets provide insurance and allow an agent to smooth consumption.

2.3 Power Utility and Risk-Free Rates

Now assume $u(c) = c^{1-\gamma}/(1-\gamma)$

Risk-free rate when c_{t+1} known:

$$R = \frac{1}{E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]} = \frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}$$

2.4 Equity Premium Characterization

Define $r^f = R - 1$, $\beta = \frac{1}{1+\delta}$, then (net) stock return r_{t+1} satisfies:

$$1 = E_t\left[\frac{1}{1+\delta}(1+\Delta c_{t+1})^{-\gamma}(1+r_{t+1})\right]$$

Take 2nd order Taylor approximation of right side, unconditional expectations:

$$E(r) = \delta + \gamma E(\Delta c_t) + \gamma cov(r_t, \Delta c_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(\Delta c_t)$$

For risk free rate $cov(r_t, \Delta c_t) = 0$, so:

$$r^f = \delta + \gamma E(\Delta c_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(\Delta c_t)$$

Then we can write excess returns on stocks over the risk free rate, or the equity premium as:

$$E(r_t) - r^f = \gamma cov(r_t, \Delta c_t)$$

Which in turn can be re-written to give the risk-return tradeoff:

$$\frac{E(r_t) - r^f}{\sigma(r)} = \gamma \sigma(\Delta c_t) corr(\Delta c_t, r_t)$$

Left side known as Sharpe ratio

Moments of aggregate consumption growth and level of risk aversion then determine equity premium. The equity premium puzzle is that to rationalize observed asset prices requires implausibly high levels of risk aversion. Volatility of consumption growth is low relative to the Sharpe ratio.

2.5 Attempted Resolutions

There has been a very large literature in macro asset pricing to try to explain the equity premium puzzle. These theories can be grouped into three broad classes:

- Change preferences: Get away from time additively separable power utility function. Examples: recursive preferences, robustness, habit persistence

- Change constraints: Get away from representative agent complete markets with no frictions. Examples:Limited participation, transaction costs, incomplete markets
- Change shocks: Get away from typical log-normal assumption for consumption growth. Examples: disaster models, long-run risk, learning