The New Keynesian Model

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Money, prices, and nominal rigidities

Flexible-price models

- Flexible price models share a common property – the inverse of the aggregate price level, $1/P_t$, behaves like a speculative asset price.
- Yet this seems at odds with the evidence.
- Many researchers accept that some degree of nominal rigidity in prices and/or wages is necessary if a dynamic general equilibrium model is going to have any chance of matching macro time series data and be useful for policy exercises.

Price Stickiness

- Tendency of prices to adjust slowly in economy.
- Sources: **Monopolistic competition** and **menu costs**.
- Under perfect competition, market forces prices to adjust rapidly. But in many markets, sellers produce differentiated goods with some market power: **monopolistic competition**. Sellers set prices.
- **Menu costs:** costs of changing prices may lead to price stickiness. Even small costs like these may prevent sellers from changing prices often.
- Since competition isn't perfect, having the wrong price temporarily won't affect the seller's profits much. The firm will change prices when demand or costs of production change enough to warrant the price change.
- We'll actually study the simpler time-dependent pricing rules, rather than menu cost models which lead to state-dependent pricing.

Empirical Evidence on Price Stickiness

- Carlton (1986): Industrial prices often fixed for several years, changed more often the more competitive the industry .
- Kashyap (1995): Catalog prices don't seem to change much from one issue to the next. Menu costs may not be cause of stickiness .
- Bils-Klenow (2004): Half of all goods prices last more that 5.5 months. Varies dramatically over types of goods, amount of competition in industry.
- Steinsson-Nakamura (2008): Excluding sales, frequency of price changes is 9-12 % per month. Median duration regular prices is 8-11 months.

Table 2

Monthly Frequency of Price Changes for Selected Categories

	% of Price Quotes with Price Changes	% of Price Quotes with Price Changes, excluding observations with item substitutions
All goods and services	26.1 (1.0)	23.6 (1.0)
Durable Goods	29.8 (2.5)	23.6 (2.5)
Nondurable Goods	29.9 (1.5)	27.5 (1.5)
Services	20.7 (1.5)	19.3 (1.6)
Food	25.3 (1.8)	24.1 (1.9)
Home Furnishings	26.4 (1.8)	24.2 (1.8)
Apparel	29.2 (3.0)	22.7 (3.1)
Transportation	39.4 (1.8)	35.8 (1.9)
Medical Care	9.4 (3.2)	8.3 (3.3)
Entertainment	11.3 (3.5)	8.5 (3.6)
Other	11.0 (3.3)	10.0 (3.3)
Raw Goods	54.3 (1.9)	53.7 (1.7)
Processed Goods	20.5 (0.8)	17.6 (0.7)

Notes: Frequencies are weighted means of category components. Standard errors are in parentheses. Durables, Nondurables and Services coincide with U.S. National Income and Product Account classifications. Housing (reduced to home furnishings in our data), apparel, transportation, medical care, entertainment, and other are BLS Major Groups for the CPI. Raw goods include gasoline, motor oil and coolants, fuel oil and other fuels, electricity, natural gas, meats, fish, eggs, fresh fruits, fresh vegetables, and fresh milk and

FIGURE 3: PRICE OF TRISCUIT 9.5 oz IN

DOMINICK'S FINER FOODS SUPERMARKET IN CHICAGO

Figure 1

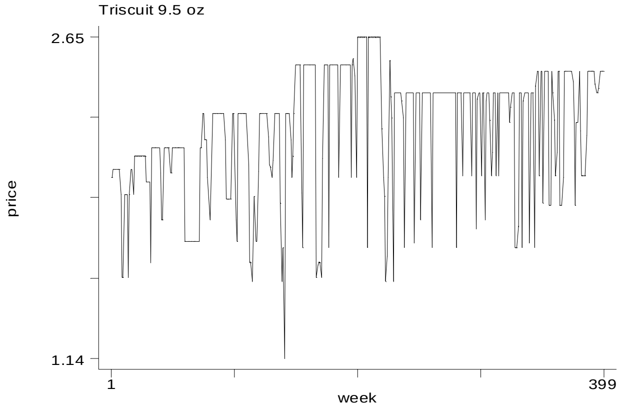

TABLE I
FREQUENCY OF PRICE CHANGE IN THE CPI

	Median frequency		Median implied duration		Mean frequency		Mean implied duration	
	1988–1997 (%)	1998–2005 (%)	1988–1997 (months)	1998–2005 (months)	1988–1997 (%)	1998–2005 (%)	1988–1997 (months)	1998–2005 (months)
A. Including sales								
Excluding substitutions	20.3	19.4	4.4	4.6	23.9	26.5	8.3	9.0
Including substitutions	21.7	20.5	4.1	4.4	25.2	27.7	7.5	7.7
B. Excluding sales and substitutions								
Contiguous observations	11.1	8.7	8.5	11.0	18.7	21.1	11.6	13.0
Carry regular price forward during sales and stockouts	11.2	9.0	8.4	10.6	18.6	20.9	11.0	12.3
Estimate frequency of price change during sales	11.5	9.6	8.2	9.9	19.0	21.3	11.2	12.5
Estimate frequency of price change during sales and stockouts	11.9	9.9	7.9	9.6	18.9	21.5	10.8	11.7
C. Excluding sales, including substitutions								
Contiguous observations	12.7	10.9	7.4	8.7	20.4	22.8	9.3	9.8
Carry regular price forward during sales and stockouts	12.3	10.6	7.6	8.9	19.7	22.0	9.6	10.4
Estimate frequency of price change during sales	12.8	11.3	7.3	8.3	20.8	22.8	9.2	9.8
Estimate frequency of price change during sales and stockouts	13.0	11.8	7.2	8.0	20.7	23.1	9.0	9.3

Notes. All frequencies are reported in percent per month. Implied durations are reported in months. "Median frequency" denotes the weighted median frequency of price change. It is calculated by first calculating the mean frequency of price change for each ELI and then taking a weighted median across ELIs within the major group using CPI expenditure weights. The "Median implied duration" is equal to $-1/\ln(1-f)$, where f is the median frequency of price change. "Mean frequency" denotes the weighted mean frequency of price change. "Mean implied duration" denotes the weighted implied duration of price change. It is calculated by first calculating the implied duration for each ELI as $-1/\ln(1-f)$, where f is the frequency of price change for a particular ELI, and then taking a weighted mean across ELI's using CPI expenditure weights.

Adding nominal rigidities

Objectives

1. To examine how the introduction of nominal rigidity affects analysis of macro issues.
2. To see how models employed in policy analysis can be derived when some degree of nominal rigidity is introduced into the dynamic general equilibrium models examined so far.

Basic new Keynesian model

Three basic components

1. An expectational "IS" curve (Euler equation)
2. An inflation adjustment equation (Phillips curve/price setting)
3. A specification of policy behavior

Overview of the Model

- The model consists of households who supply labor, purchase goods for consumption, and hold money and bonds, and firms who hire labor and produce and sell differentiated products in monopolistically competitive goods markets.
- The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977).
- Each firm set the price of the good it produces, but not all firms reset their price each period.
- Households and firms behave optimally: households maximize the expected present value of utility and firms maximize profits.

Households

- The preferences of a representative household defined over a composite consumption good C_t, real money balances M_t/P_t, and leisure $1 - N_t$, where N_t is the time devoted to market employment.
- Households maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \quad (1)$$

- The composite consumption good consists of differentiate products produced by monopolistically competitive final goods producers (firms). There are a continuum of such firms of measure 1, and firm j produces good c_j.

Households

- The composite consumption good that enters the household's utility function is defined as

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1. \tag{2}$$

  The parameter θ governs the price elasticity of demand for the individual goods.

Households

- The household's decision problem can be dealt with in two stages.
  1. Regardless of the level of C_t, it will always be optimal to purchase the combination of the individual goods that minimize the cost of achieving this level of the composite good.
  2. Given the cost of achieving any given level of C_t, the household chooses C_t, N_t, and M_t optimally.

Households

- Dealing first with the problem of minimizing the cost of buying C_t, the household's decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t, \tag{3}$$

where p_{jt} is the price of good j. Letting ψ_t be the Lagrange multiplier on the constraint, the first order condition for good j is

$$p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0.$$

Households

- Rearranging, $c_{jt} = (p_{jt}/\psi_t)^{-\theta} C_t$. From the definition of the composite level of consumption (2), this implies

$$C_t = \left[\int_0^1 \left[\left(\frac{p_{jt}}{\psi_t}\right)^{-\theta} C_t\right]^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} = \left(\frac{1}{\psi_t}\right)^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta} dj\right]^{\frac{\theta}{\theta-1}} C_t.$$

Solving for ψ_t,

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \equiv P_t. \tag{4}$$

Households

- The Lagrange multiplier is the appropriately aggregated price index for consumption.
- The demand for good j can then be written as

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t. \tag{5}$$

The price elasticity of demand for good j is equal to θ. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and, as a consequence, individual firms will have less market power.

Households

- Given the definition of the aggregate price index in (4), the budget constraint of the household is, in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + R_{t-1}\left(\frac{B_{t-1}}{P_t}\right) + \Pi_t, \quad (6)$$

  where M_t (B_t) is the household's nominal holdings of money (one period bonds). Bonds pay a gross nominal rate of interest R_t. Real profits received from firms are equal to Π_t.
- In the second stage of the household's decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (1) subject to (6).

Households

The following conditions must also hold in equilibrium

1. the Euler condition for the optimal intertemporal allocation of consumption
$$C_t^{-\sigma} = \beta R_t \mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}; \tag{7}$$
2. the condition for optimal money holdings:
$$\frac{\gamma \left(\frac{M_t}{P_t} \right)^{-b}}{C_t^{-\sigma}} = \frac{R_t - 1}{R_t}; \tag{8}$$
3. the condition for optimal labor supply:
$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{9}$$

Firms

- Firms maximize profits, subject to three constraints:

  1. The first is the production function summarizing the technology available for production. For simplicity, we have ignored capital, so output is a function solely of labor input N_{jt} and an aggregate productivity disturbance Z_t:

  $$c_{jt} = Z_t N_{jt}, \qquad \mathrm{E}(Z_t) = 1.$$

  2. The second constraint on the firm is the demand curve each faces. This is given by equation (5).
  3. The third constraint is that each period some firms are not able to adjust their price. The specific model of price stickiness we will use is due to Calvo (1983).

Price adjustment

- Each period, the firms that adjust their price are randomly selected: a fraction $1 - \omega$ of all firms adjust while the remaining ω fraction do not adjust.
  - The parameter ω is a measure of the degree of nominal rigidity; a larger ω implies fewer firms adjust each period and the expected time between price changes is longer.
- For those firms who do adjust their price at time t, they do so to maximize the expected discounted value of current and future profits.
  - Profits at some future date $t + s$ are affected by the choice of price at time t only if the firm has not received another opportunity to adjust between t and $t + s$. The probability of this is ω^s.

Price adjustment

The firm's decision problem

- First consider the firm's cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written as

$$\min_{N_{jt}} W_t N_{jt} + \varphi_t^n \left(c_{jt} - Z_t N_{jt} \right).$$

where φ_t^n is equal to the firm's nominal marginal cost. The first order condition implies

$$W_t = \varphi_t^n Z_t,$$

or $\varphi_t^n = W_t / Z_t$. Dividing by P_t yields real marginal cost as $\varphi_t = W_t / \left(P_t Z_t \right)$.

Price adjustment

The firm's decision problem

- The firm's pricing decision problem then involves picking p_{jt} to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \Pi \left(\frac{p_{jt}}{P_{t+i}}, \varphi_{t+i}, c_{t+i} \right) =$$

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i},$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$ and profits are

$$\Pi(p_{jt}) = \left[\left(\frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right]$$

Price adjustment

- All firms adjusting in period t face the same problem, so all adjusting firms will set the same price.
- Let p_t^* be the optimal price chosen by all firms adjusting at time t. The first order condition for the optimal choice of p_t^* is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta) \left(\frac{1}{p_{jt}} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{1-\theta} + \theta \varphi_{t+i} \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} \right]$$

- Using the definition of $\Delta_{i,t+i}$,

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta}}{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\theta - 1}}. \tag{10}$$

The case of flexible prices

- If all firms are able to adjust their prices every period ($\omega = 0$):

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta - 1}\right)\varphi_t = \mu\varphi_t. \tag{11}$$

- Each firm sets its price p_t^* equal to a markup $\mu > 1$ over nominal marginal cost $P_t\varphi_t$.
- When prices are flexible, all firms charge the same price, and $\varphi_t = \mu^{-1}$.

The case of flexible prices

- Using the definition of real marginal cost, this means

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu}.$$

- However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consist with household optimization:

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{Z_t}{\mu}. \tag{12}$$

The case of flexible prices

Flexible-price output

- Let a $\hat{x}_t$ denote the percent deviation of a variable X_t around its steady-state. Then, the steady-state yields

$$\eta \hat{n}_t + \sigma \hat{c}_t = \hat{z}_t.$$

- Now using the fact that $\hat{y}_t = \hat{n}_t + \hat{z}_t$ and $\hat{y}_t = \hat{c}_t$, flexible-price equilibrium output $\hat{y}_t^f$ can be expressed as

$$\hat{y}_t^f = \left[\frac{1+\eta}{\eta+\sigma} \right] \hat{z}_t. \tag{13}$$

The case of sticky prices

- When prices are sticky ($\omega > 0$), the firm must take into account expected future marginal cost as well as current marginal cost when setting p_t^*.
- The aggregate price index is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period t and the average of the remaining fraction ω of all firms who set prices in earlier periods.
- Because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that was prevailing in period $t - 1$.
- Thus, the average price in period t satisfies

$$P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \tag{14}$$

Inflation adjustment

- Using the first order condition for p_t^* and approximating around a zero average inflation, flexible-price equilibrium,

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\hat{\varphi}_t \tag{15}$$

where

$$\tilde{\kappa} = \frac{(1-\omega)\left[1-\beta\omega\right]}{\omega}$$

- Equation (15) is often referred to as the New Keynesian Phillips curve.

Forward-looking inflation adjustment

- The New Keynesian Phillips curve is forward-looking; when a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods.
- Solving forward,

$$\pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \hat{\varphi}_{t+i},$$

- Inflation is a function of the present discounted value of current and future real marginal cost.
- Inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.

Real marginal cost and the output gap

- The firm's real marginal cost is equal to the real wage it faces divided by the marginal product of labor: $\varphi_t = W_t / P_t Z_t$.
- Because nominal wages have been assumed to be completely flexible, the real wage must equal the marginal rate of substitution between leisure and consumption.
- In a flexible price equilibrium, all firms set the same price, so (11) implies that $\varphi = \mu^{-1}$. From equation (9), $\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \sigma \hat{y}_t$ Recalling that $\hat{c}_t = \hat{y}_t$, $\hat{y}_t = \hat{n}_t + \hat{z}_t$, the percentage deviation of real marginal cost around the flexible price equilibrium is

$$\hat{\varphi}_t = [\eta \hat{n}_t + \sigma \hat{y}_t] - \hat{z}_t = (\eta + \sigma) \left[\hat{y}_t - \left(\frac{1+\eta}{\eta+\sigma} \right) \hat{z}_t \right].$$

Real marginal cost and the output gap

- But from (13), this can be written as

$$\hat{\varphi}_t = (\eta + \sigma)\left(\hat{y}_t - \hat{y}_t^f\right). \tag{16}$$

- Using these results, the inflation adjustment equation (15) becomes

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t \tag{17}$$

  where $\kappa = (\eta + \sigma)\tilde{\kappa} = (\eta + \sigma)(1 - \omega)[1 - \beta\omega]/\omega$ and $x_t \equiv \hat{y}_t - \hat{y}_t^f$ is the gap between actual output and the flexible-price equilibrium output.
- This inflation adjustment or forward-looking Phillips curve relates output, in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity, to inflation.

The demand side of the model

- Start with Euler condition for optimal consumption choice

$$C_t^{-\sigma} = \beta R_t \mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}$$

- Linearize around steady-state:

$$-\sigma \hat{c}_t = (\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t) - \sigma \mathrm{E}_t \hat{c}_{t+1}$$

or

$$\hat{c}_t = \mathrm{E}_t \hat{c}_{t+1} - \left(\frac{1}{\sigma} \right) (\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t) .$$

- Goods market equilibrium (no capital)

$$Y_t = C_t .$$

The demand side of the model

Linearization

- Euler condition becomes

$$\hat{y}_t = \mathrm{E}_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right) \left(\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t\right).$$

- This is often called an "expectational IS curve", to make the comparisons with old-style Keynesian models clear.

Demand and the output gap

- Express in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$:

$$\hat{y}_t - \hat{y}_t^f = \mathrm{E}_t \left(\hat{y}_{t+1} - \hat{y}_{t+1}^f \right) - \left(\frac{1}{\sigma} \right) \left(\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t \right) + \left(\mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f \right)$$

  or

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma} \right) \left(r_t - r_t^n \right),$$

  where $r_t = \hat{\imath}_t - E_t p_{t+1} + p_t$ and

$$r_t^n \equiv \sigma \left(\mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f \right).$$

- Notice that the nominal interest rate affects output through the interest rate gap $r_t - r_t^n$.

The general equilibrium model

- Two equation system

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t$$

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right)$$

The general equilibrium model

- Consistent with
  - optimizing behavior by households and firms
  - budget constraints
  - market equilibrium
- Two equations but three unknowns: x_t, π_t, and i_t – need to specify monetary policy

Instrument Rules

- A common approach to policy is in terms of simple rules.
- The most famous of such instrument rules is the Taylor Rule (Taylor 1993):

$$i_t = \pi_t + 0.5x_t + 0.5\left(\pi_t - \pi^T\right) + r^*,$$

  where π^T was the target level of average inflation (Taylor assumed it to be 2%) and r^* was the equilibrium real rate of interest (Taylor assumed this too was equal to 2%).
- The Taylor Rule for general coefficients is

$$i_t = r^* + \pi^T + \delta_x x_t + \delta_\pi \left(\pi_t - \pi^T\right). \tag{18}$$

Solving the model for the rational expectations equilibrium

- Suppose $i_t = r_t^n + \delta\pi_t$.
- Write system as

$$\begin{bmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} \delta\pi_t$$

- or

$$\begin{bmatrix} \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \frac{\beta\delta-1}{\sigma\beta} & 1 + \frac{\kappa}{\sigma\beta} \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}$$

- Two eigenvalues outside the unit circle if and only if

$$\delta > 1$$

The Taylor Principle

- Policy must respond sufficiently strongly to inflation.

Definition

The condition that the nominal interest rate respond more than one-for-one to inflation is called the Taylor Principle.