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Economics 712
Macroeconomic Theory
Fall 2014

Problem Set 4

Due in class on December 10

1. When real balances enter the utility function, an agent seeks to maximize:

$$E_t \sum_{i=0}^{\infty} \beta^i U\left(C_{t+i}, m_{t+i}, L_{t+i}\right)$$

where L_t is leisure and $m_t = M_t/P_t$ is real balances, subject to the budget constraint in real terms:

$$w_t(1-L_t)+(r_t+1-\delta)\,K_{t-1}+(1+i_{t-1})\frac{P_{t-1}}{P_t}b_{t-1}+\frac{P_{t-1}}{P_t}m_{t-1}+\tau_t = C_t+K_t+b_t+m_t.$$

Here r_t is the real interest rate, i_t is nominal risk-free interest rate (on nominal bonds), b_t is real holdings of nominal bonds, and τ_t is lump sum taxes or transfers. Note that m_t which enters the utility function is real money balances at the end of the period, but some have argued that a preferable model specification has utility beginning of period real balances, as in the cash in advance model. For this latter, define a_t as real money balances after the purchase of bonds but prior to the receipt of income or purchase of consumption goods as:

$$a_t = (1+i_{t-1})\frac{P_{t-1}}{P_t}b_{t-1}+\frac{P_{t-1}}{P_t}m_{t-1}+\tau_t - b_t.$$

(a) Find the optimality conditions for the choice of leisure, consumption, and the holdings of capital, bonds, and money under the two specifications, when m_t and a_t enter the utility function.

(b) From the optimality conditions, find three expressions (Euler equations) relating: (i) the intertemporal marginal rate of substitution and the real interest rate, (ii) the intertemporal marginal rate of substitution, the nominal interest rate, and inflation, and (iii) the marginal rate of substitution between consumption and real balances and the nominal interest rate. How do these differ across the specifications?

2. Consider a cash-in-advance model in which there are two types of goods: c_1 requires money M_t to purchase, while c_2 can be purchased on credit. The two goods are technologically equivalent, as the endowment e_t can be converted one-for-one into either of them, so $e_t = c_{1t} + c_{2t}$. Suppose that e_t follows a Markov process with transition density $Q(e'|e)$. A representative agent in this economy thus solves:

$$\max_{\{c_{1t},c_{2t},M_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t})$$

subject to the technological constraint, the budget constraint:

$$P_t c_{1t} + P_t c_{2t} = P_t e_t + M_t - M_{t+1},$$

and the cash-in-advance constraint:

$$P_t c_{1t} \leq M_t.$$

(a) Write down the Bellman equation for the representative household and find the optimality conditions.

(b) Consider a steady state equilibrium in which the endowment is constant $e_t = e$, the money supply grows at a constant rate: $M_{t+1} = \mu M_t$, and real balances M_t/P_t are constant. What is the minimal level of μ that will support a steady state monetary equilibrium? Is such an equilibrium Pareto efficient?

3. Consider a simple search model of the housing market. Time is continuous, and there is a population of agents of mass 1 who are risk neutral and discount at rate r. A fraction H of the population owns (indivisible) houses, which are identical and yield flow utility u to their owners each instant. The fraction $(1 - H)$ of people without houses get no flow utility, but search for a house. Agents in the economy meet each other with Poisson arrival rate α. When a potential buyer meets a potential seller, they trade the house at price p (which they take as given) with probability $\pi = \pi_0 \pi_1$, where π_0 is the buying probability and π_1 is the selling probability.

   (a) Write down the (Hamilton-Jacobi) Bellman equations determining V_1, the value of the owner of a house, and V_0, the value of an agent searching to buy a house. What is the welfare gain of owning a house $V_1 - V_0$?

   (b) Characterize the optimal buying and selling strategies which determine π_0 and π_1. Assume that agents trade when they are indifferent.

   (c) Find the equilibrium price p. How does it depend on the housing supply H?

4. Reconsider the New Keynesian model from class, but suppose the period utility function is given by:

$$U(C_t, M_t/P_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \left(\frac{M_t}{P_t} \right)^{1-b} - \frac{N_t^{1+\eta}}{1+\eta},$$

also suppose the aggregate production function is:

$$Y_t = Z_t N_t^a.$$

   (a) Derive the household's optimality conditions and their log-linearizations.

   (b) Derive an expression for the flexible-price equilibrium output and the output gap.

   (c) Does money affect the flexible-price equilibrium? Does the nominal interest rate? Explain.