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Economics 712
Macroeconomic Theory
Fall 2014

Problem Set 3

Due in Discussion on Nov. 14

1. Consider a version of the (deterministic) optimal growth model as in class, but to which we add a government. That is, there is an exogenous stream of government purchases $\{G_t\}$ that the planner takes as given. The household does not value government purchases, but they must be funded with real resources. So the planner chooses the allocation of consumption c_t and capital (next period) k_{t+1} to maximize the household utility (over consumption, with labor supplied inelastically) subject to the resource constraint:

$$k_{t+1} = (1-\delta)k_t + f(k_t) - c_t - G_t.$$

   (a) Suppose that government purchases are constant at $G_t = G$. How does the introduction of government spending affect the steady state levels of consumption and capital, relative to the case where $G = 0$?

   (b) Suppose that initially the economy is in a steady state with $G_t = G$, then there is a once-and-for-all unforeseen increase in purchases to a new higher level $G' > G$. What happens to consumption and capital immediately upon the impact of the change and in the long run?

   (c) Suppose that initially the economy is in a steady state with $G_t = G$, then at date T there is announcement that at the future date $T' > T$ purchases will increase to a new higher level $G' > G$ and remain there. What happens to consumption and capital at T, the date of the announcement? What happens between T and T'? What happens at T'?

2. Consider basic consumption-savings problem from class:

$$\max_{\{c_t, a_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t.} \quad c_t + a_{t+1} = Ra_t + y_t$$

   a_0, y_0 given, $c_t \geq 0, \quad a_t \geq \underline{a}$. Income $y \in Y$, where Y is compact, follows a Markov process with transition function Q on $(Y, \mathcal{Y})$. Suppose that u is bounded and C^2 with $u' > 0, u'' < 0$ and $\lim_{c \to 0} u'(c) = +\infty$.

   Prove that the value function $v(a, y)$ solving the Bellman equation is strictly increasing and concave in a, and that the optimal policy functions $a'(a, y)$ and $c(a, y)$ are continuous.

3. Consider an endowment economy with one good and two assets. Asset 1 pays a constant amount R in each period. Asset 2 pays a stochastic amount $x_t \in \{R/2, 2R\}$. Assume that the x is i.i.d. and so $x = R/2$ with probability θ and $x = 2R$ with probability $1 - \theta$. In equilibrium consumption of the nonstorable good is therefore $c = R + x$. The representative agent has preferences:

$$E \sum_{t=0}^{\infty} \beta^t \log c_t.$$

Find the equilibrium price/consumption ratios for the assets: $p_1(x)/c(x)$ and $p_2(x)/c(x)$. Which asset has the greater price/consumption ratio? Interpret your result.

4. This problem considers the computation of the optimal growth model. An infinitely-lived representative household owns a stock of capital which it rents to firms. The household's capital stock K depreciates at rate δ. Households do not value leisure and are endowed with one unit of time each period with which they can supply labor N to firms. They have standard time additive expected utility preferences with discount factor β and period utility $u(c)$. Firms produce output according to the production function $zF(K, N)$ where z is the level of technology .

   (a) First, write a computer program that solves the planners problem to determine the optimal allocation in the model. Set $\beta = 0.95$, $\delta = 0.1$, $z = 1$, $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$, and $F(K, N) = K^{0.35} N^{0.65}$. Plot the optimal policy function for K and the phase diagram with the $\Delta K = 0$ and $\Delta c = 0$ lines along with the saddle path (which is the decision rule $c(K)$).

   (b) Re-do your calculations with $\gamma = 1.01$. What happens to the steady state? What happens to the saddle path? Interpret your answer.

   (c) Now with $\gamma = 2$ assume that there is an unexpected permanent increase of 20% in total factor productivity, so now $z = 1.2$. What happens to the steady state levels of consumption and capital? Assuming the economy is initially in the steady state with $z = 1$, what happens to consumption and capital after the increase in z?