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Economics 712
Macroeconomic Theory
Fall 2014

Problem Set 1

Due in Discussion on Sept. 26

1. Suppose that the process for income y_t can be written as:

$$y_t = \epsilon_t + \beta \sum_{i=1}^{\infty} \epsilon_{t-i},$$

where ϵ_t is an i.i.d. mean zero random variable. As of date $t-1$, an agent observes his whole past sequence of income realizations $\{y_{t-1}, y_{t-2}, ...\}$.

   (a) Find the rational expectations forecast $y_t^e = E_{t-1} y_t$.

   (b) Show that the forecast y_t^e can be written as a recursion in terms of y_{t-1}^e and y_{t-1}. Interpret your result.

2. Reconsider a version of the Cagan-type model that we discussed in class:

$$p_t = \alpha + \beta E_t p_{t+1} + \gamma m_t$$

where variables are in logs, $\alpha > 0$ and $0 < \beta < 1$ are constants, and expectations are rational. The money supply process follows:

$$m_{t+1} = \bar{m} + \lambda m_t + \delta p_t + u_{t+1}$$

where m_0 is given, $\bar{m} > 0$, $0 < \lambda < 1$, and δ are constants and u_t is an i.i.d. random variable with zero mean.

   (a) Find the steady state of the model. Then express the dynamics in terms of deviations from the steady state, putting the system into the form of an equation:

$$x_t = A x_{t+1} + B w_{t+1}$$

   (b) Diagonalize the system by finding the eigenvector-eigenvalue decomposition of $A = V \Lambda V^{-1}$, and find the dynamics of the transformed system:

$$z_t = \hat{A} E_t z_{t+1}.$$

   (c) Under what conditions (on δ) is the equilibrium determinate? What is the relationship between p_t and m_t which holds in a determinate equilibrium? What happens if this condition fails?

3. In the lecture notes we gave (without proof) the following characterization of the reservation wage in an environment with perfect job finding ($p = 1$) and no separations ($s = 0$).

$$w_R - z = \beta(E[w] - z) + \beta \int_0^{w_R} F(w)dw$$

   (a) Derive the analogue of this condition in an environment with imperfect job finding ($p < 1$) and separations ($s > 0$).

   (b) Prove that if an offer distribution G is a mean preserving spread of F then the reservation wage is greater for G than F. Note that we can suppose that both F and G have finite support $[0, \bar{w}]$, and they share the same mean so $\int_0^{\bar{w}} wdF(w) = \int_0^{\bar{w}} wdG(w)$. But G is a mean-preserving spread of F, which we can characterize as $\int_0^b [G(w) - F(w)]dw \geq 0$ for $0 \leq b \leq \bar{w}$.

   (c) Prove that if the job offer rate p falls then the steady state unemployment rate increases, even though the reservation wage falls.

4. Consider the following variations on the basic McCall search model. In each case, suppose that workers are risk-neutral, unemployed workers get constant payments z, jobs last forever (unless specified), and search yields at least one job offer.

   (a) Suppose that an unemployed worker had the option to recall a previous wage offer. That is, if the worker received an offer of w_0 at some date, he could turn it down and search again, but would have the option at future dates to work at that previously made offer w_0. Find the unemployed worker's Bellman equation and characterize his optimal decision rule. How does it compare to the case without recall?

   (b) Suppose that employed workers receive unsolicited offers, that is a worker currently employed at wage w receives an offer w' from a distribution $G(w'|w)$. If the employed worker takes the job at wage w' he must spend one period in transit, earning no income, before beginning the job. Find the Bellman equations of employed and unemployed workers and characterize their decision rules. How does the the possibility of offers on the job change the reservation wage of an unemployed worker?

   (c) Suppose that we allowed workers to quit, but they would not otherwise separate from firms. That is, at any date an employed worker would have the option to quit his job, become unemployed, and search for a new job. Find the Bellman equations for employed and unemployed workers and characterize their optimal decision rules. Would employed workers ever quit?

   (d) Continue to allow quits, but now suppose that wages are not known with certainty when a worker accepts a job. That is, unemployed workers receive offers of an expected wage w drawn from $F(w)$. But once employed, the jobs pay $w + \Delta$ with probability 1/2 and $w - \Delta$ with probability 1/2 for $\Delta > 0$. The uncertainty is all resolved once the job has begun. That is, wages are constant over time but uncertain at the time of acceptance. Find the Bellman equations for employed and unemployed workers and characterize their optimal decision rules. Would employed workers ever quit?