Lecture 6: Equilibrium Search

Economics 712, Fall 2014

1 Firm Decision

Wage w, hours fixed at 1, either party can freely break contract.

J value of filled job, V value of vacant job.

Over short interval $\Delta > 0$:

$$V = -pc\Delta + e^{-r\Delta}[e^{-q(\theta)\Delta}V + (1 - e^{-q(\theta)\Delta})J]$$

Take limits as $\Delta \to 0$:

$$\begin{aligned} rV &= -pc + q(\theta)(J - V) \\ rJ &= p - w - sJ \end{aligned}$$

So we have:

$$J = \frac{p - w}{r + s}$$

Free entry of firms means $V = 0$. So we also have:

$$J = \frac{pc}{q(\theta)}$$

Equating gives the job creation condition:

$$p - w - (r + s)\frac{pc}{q(\theta)} = 0$$

If $c \to 0$ or if $q(\theta) \to \infty$ then $p = w$.

2 Workers' Decisions

Now no longer a distribution of offers, no reason to turn down wage as long as $W > U$.

Value of unemployed:

$$rU = z + \theta q(\theta)(W - U)$$

Value of employed:

$$rW = w + s(U - W(w))$$

We can solve:

$$W(w) = \frac{w}{r+s} + \frac{s}{r+s}U, \quad W'(w) = \frac{1}{r+s}$$

Use expression for W, solve for (W, U) explicitly:

$$\begin{aligned} U &= \frac{sz + \theta q(\theta)w + rz}{r^2 + r\theta q(\theta) + sr} \\ W &= \frac{sz + \theta q(\theta)w + rw}{r^2 + r\theta q(\theta) + sr} \end{aligned}$$

So $W > U \Leftrightarrow w > z$.

3 Wage Determination

Rents in equilibrium: surplus to both workers and firms once matched. Wage splits surplus, not pinned down.

Solve for wages by Nash bargaining solution:

$$w = \arg\max_{\hat{w}} \left(W(\hat{w}) - U\right)^{\beta} \left(J(\hat{w}) - V\right)^{1-\beta}$$

Optimality condition:

$$\beta \frac{W'(w)}{W - U} = -(1-\beta)\frac{J'(w)}{J - V}$$

Simplify:

$$W = U + \beta(W - U + J)$$

Use expression for J:

$$W - U = \frac{\beta}{1-\beta}\frac{pc}{q(\theta)}$$

Also can solve to show:

$$w = rU + \beta(p - rU)$$

Finally, can derive the wage equation:

$$w = (1-\beta)z + \beta(p + \theta pc)$$

4 Steady State Unemployment

Three key equations:

$$\begin{aligned} w &= (1-\beta)z + \beta(p + \theta pc) \\ p &- w - (r+s)\frac{pc}{q(\theta)} = 0 \\ u &= \frac{s}{s + \theta q(\theta)} \end{aligned}$$

Combine first two to determine steady state θ:

$$(1-\beta)(p - z) - \frac{r + s + \beta\theta q(\theta)}{q(\theta)}pc = 0$$

Then Beveridge curve determines steady state u given θ.