1 Firm Decision

Wage \( w \), hours fixed at 1, either party can freely break contract.

\( J \) value of filled job, \( V \) value of vacant job.

Over short interval \( \Delta > 0 \):

\[
V = -pc\Delta + e^{-r\Delta}[e^{-q(\theta)\Delta}V + (1 - e^{-q(\theta)\Delta})J]
\]

Take limits as \( \Delta \to 0 \):

\[
rv = -pc + q(\theta)(J - V)
\]

\[
rJ = p - w - sJ
\]

So we have:

\[
J = \frac{p - w}{r + s}
\]

Free entry of firms means \( V = 0 \). So we also have:

\[
J = \frac{pc}{q(\theta)}
\]

Equating gives the job creation condition:

\[
p - w - (r + s)\frac{pc}{q(\theta)} = 0
\]

If \( c \to 0 \) or if \( q(\theta) \to \infty \) then \( p = w \).


2 Workers’ Decisions

Now no longer a distribution of offers, no reason to turn down wage as long as \( W > U \).

Value of unemployed:

\[
    rU = z + \theta q(\theta)(W - U)
\]

Value of employed:

\[
    rW = w + s(U - W(w))
\]

We can solve:

\[
    W(w) = \frac{w}{r + s} + \frac{s}{r + s}U, \quad W'(w) = \frac{1}{r + s}
\]

Use expression for \( W \), solve for \((W, U)\) explicitly:

\[
    U = \frac{sz + \theta q(\theta)w + rz}{r^2 + r\theta q(\theta) + sr}
    W = \frac{sz + \theta q(\theta)w + rw}{r^2 + r\theta q(\theta) + sr}
\]

So \( W > U \Leftrightarrow w > z \).

3 Wage Determination

Rents in equilibrium: surplus to both workers and firms once matched. Wage splits surplus, not pinned down.

Solve for wages by Nash bargaining solution:

\[
    w = \arg \max_{\hat{w}} (W(\hat{w}) - U)\beta (J(\hat{w}) - V)^{1-\beta}
\]

Optimality condition:

\[
    \beta \frac{W'(w)}{W - U} = -(1 - \beta) \frac{J'(w)}{J - V}
\]
Simplify:

\[ W = U + \beta(W - U + J) \]

Use expression for \( J \):

\[ W - U = \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)} \]

Also can solve to show:

\[ w = rU + \beta(p - rU) \]

Finally, can derive the wage equation:

\[ w = (1 - \beta)z + \beta(p + \theta pc) \]

### 4 Steady State Unemployment

Three key equations:

\[
\begin{align*}
    w &= (1 - \beta)z + \beta(p + \theta pc) \\
    p - w - (r + s) \frac{pc}{q(\theta)} &= 0 \\
    u &= \frac{s}{s + \theta q(\theta)} 
\end{align*}
\]

Combine first two to determine steady state \( \theta \):

\[
(1 - \beta)(p - z) - \frac{r + s + \beta \theta q(\theta)}{q(\theta)} pc = 0
\]

Then Beveridge curve determines steady state \( u \) given \( \theta \).