Lecture 5: Equilibrium Search

Economics 712, Fall 2014

1 Setup


Continuous time, constant interest rate $r$.

Continuum $L$ of identical workers, risk neutral preferences:

$$\int_0^\infty e^{-rt}x_i dt$$

Endogenous number of firms, each with one job. Competitive producer of final good at price $p$.

Firms post vacancies, cost $c$ per unit time, $pc$ relative to output

$uL$ unemployed, $vL$ vacant jobs, $fL$ jobs filled, related by matching function $m$:

$$fL = m(uL, vL)$$

Assume $m$ increasing, concave, and has constant returns to scale, so $f = m(u, v)$.

Define $\theta = v/u$ then

$$q(\theta) = m(u/v, 1) = m(1/\theta, 1)$$

(Poisson) Rate at which vacant jobs are filled. Mean duration of vacancy $= 1/q(\theta)$.

$\theta q(\theta)$ is rate at which unemployed workers find job, unemployment duration $1/(\theta q(\theta))$. 
Properties:

\[ q'(\theta) \leq 0 \]

\[ \frac{q'(\theta)\theta}{q(\theta)} \in [-1,0] \]

Job creation when firm and worker meet, agree on wage. Jobs created:

\[ fL = Lm(u,v) = Lvm(u/v,1) = Lu\theta q(\theta) \]

Creation rate:

\[ \frac{u\theta q(\theta)}{1-u} \]

Jobs destroyed at exogenous Poisson rate \( s \). Total job destruction: \( s(1-u)L \).

Evolution of unemployment:

\[ \dot{u} = s(1-u) - u\theta q(\theta) \]

steady state:

\[ u = \frac{s}{s+\theta q(\theta)} \]

\section{Firm Decision}

Wage \( w \), hours fixed at 1, either party can freely break contract. 

\( J \) value of filled job, \( V \) value of vacant job.

\[ rV = -pc + q(\theta)(J-V) \]

\[ rJ = p-w-sJ \]
So we have:

\[ J = \frac{p - w}{r + s} \]

Free entry of firms means \( V = 0 \). So we also have:

\[ J = \frac{pc}{q(\theta)} \]

Equating gives the job creation condition:

\[ p - w - (r + s)\frac{pc}{q(\theta)} = 0 \]

3 Workers’ Decisions

Now no longer a distribution of offers, no reason to turn down wage as long as \( W > U \).

Value of unemployed:

\[ rU = z + \theta q(\theta)(W - U) \]

Value of employed:

\[ rW = w + s(U - W(w)) \]

We can solve:

\[ W(w) = \frac{w}{r + s} + \frac{s}{r + s}U, \quad W'(w) = \frac{1}{r + s} \]

Use expression for \( W \), solve for \((W, U)\) explicitly:

\[ U = \frac{sz + \theta q(\theta)w + rz}{r^2 + r\theta q(\theta) + sr} \]
\[ W = \frac{sz + \theta q(\theta)w + rw}{r^2 + r\theta q(\theta) + sr} \]

So \( W > U \iff w > z \).
4 Wage Determination

Solve for wages by Nash bargaining solution:

$$w = \arg \max_w (W(\hat{w}) - U)^\beta (J(\hat{w}) - V)^{1-\beta}$$

Optimality condition:

$$\frac{\beta W''(w)}{W - U} = -(1 - \beta) \frac{J'(w)}{J - V}$$

Simplify:

$$W = U + \beta(W - U + J)$$

Use expression for $J$:

$$W - U = \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)}$$

Also can solve to show:

$$w = rU + \beta(p - rU)$$

Finally, can derive the wage equation:

$$w = (1 - \beta)z + \beta(p + \theta pc)$$

5 Steady State Unemployment

Three key equations:

$$w = (1 - \beta)z + \beta(p + \theta pc)$$

$$p = w - (r + s) \frac{pc}{q(\theta)} = 0$$

$$u = \frac{s}{s + \theta q(\theta)}$$
Combine first two to determine steady state $\theta$:

$$(1 - \beta)(p - z) - \frac{r + s + \beta q(\theta)}{q(\theta)} p_c = 0$$

Then Beveridge curve determines steady state $u$ given $\theta$. 