1 Dynamics of Linear Difference Equations

1.1 Multivariate Models

\[ p_t = aE_t p_{t+1} + cm_t \]
\[ m_{t+1} = \rho m_t + \delta p_t + u_{t+1} \]

Write in matrix form:
\[ x_t = Ax_{t+1} + Bw_{t+1} \]

Diagonalize using eigenvector/eigenvalue decomposition:
\[ A = V\Lambda V^{-1} \]

Define \( z_t = V^{-1}x_t \), then:
\[ z_t = \Lambda z_{t+1} + V^{-1}Bw_{t+1} \]

If \( |\lambda_i| < 1 \), can solve forward:
\[ z_t^i = \lambda_i^{T-T} E_t z_{t+1}^i \]

So then \( z_t^i = 0 \).
If $|\lambda_i| > 1$, can solve backward:

$$z_i^t = \left(\frac{1}{\lambda_i}\right)^t z_0$$

For determinacy, need as many $|\lambda_i| > 1$ as there are predetermined variables.

1.2 Back to the example

For small $\delta > 0$ will have $0 < \lambda_1 < 1 < \lambda_2$ and therefore restriction:

$$v^{11}p_t + v^{21}m_t = 0$$

This gives determinate, saddlepath solution, determines $p_0$ given $m_0$.

But if $\delta$ large enough will have $\lambda_2 > \lambda_1 > 1$. Then need initial conditions for $m_0, p_0$.

Indeterminacy. Steady state is a sink.

2 Search Labor Model

2.1 McCall (1970) Model

Risk-neutral agent searches for job:

$$E_0 \sum_{t=0}^{\infty} \beta^t x_t$$

$x_t = w$ if employed, $x_t = z$ if unemployed

Job offers i.i.d. draw from $F(w)$.

Recursive formulation: state $s_t \in \{W, U\}$, control: accept, reject offer
Value of employed worker

\[ W(w) = E_0 \sum_{t=0}^{\infty} \beta^t x_t, \text{ s.t. } x_t = w \]
\[ = \frac{w}{1 - \beta} \]

Value of unemployed worker:

\[ U = z + \beta \int_0^{\infty} \max_{\text{acc, rej}} \left\{ U, \frac{w}{1 - \beta} \right\} dF(w) \]

Reservation wage \( w_R \):

\[ W(w_R) = U = \frac{w_R}{1 - \beta} \]

Characterize reservation wage:

\[ w_R - z = \frac{\beta}{1 - \beta} \int_{w_R}^{\infty} (w - w_R) dF(w) \]

Another characterization:

\[ w_R - z = \beta (E[w] - z) + \beta \int_0^{w_R} F(w) dw \]

Factors affecting reservation wage:

- value of unemployment \( z \)
- distribution of offers \( F \)