Lecture 3: More on Dynamics

Labor Market Search

Economics 712, Fall 2014

1 Dynamics of Linear Difference Equations

1.1 Multivariate Models

$$\begin{aligned} p_t &= aE_t p_{t+1} + cm_t \\ m_{t+1} &= \rho m_t + \delta p_t + u_{t+1} \end{aligned}$$

Write in matrix form:

$$x_t = Ax_{t+1} + Bw_{t+1}$$

Diagonalize using eigenvector/eigenvalue decomposition:

$$A = V\Lambda V^{-1}$$

Define $z_t = V^{-1}x_t$, then:

$$z_t = \Lambda z_{t+1} + V^{-1}Bw_{t+1}$$

If $|\lambda_i| < 1$, can solve forward:

$$z_t^i = \lambda_i^{t+T} E_t z_{t+T}^i$$

So then $z_t^i = 0$.

If $|\lambda_i| > 1$, can solve backward:

$$z_t^i = \left(\frac{1}{\lambda_i}\right)^t z_0$$

For determinacy, need as many $|\lambda_i| > 1$ as there are predetermined variables.

1.2 Back to the example

For small $\delta > 0$ will have $0 < \lambda_1 < 1 < \lambda_2$ and therefore restriction:

$$v^{11} p_t + v^{21} m_t = 0$$

This gives determinate, saddlepath solution, determines p_0 given m_0.

But if δ large enough will have $\lambda_2 > \lambda_1 > 1$. Then need initial conditions for m_0, p_0. Indeterminacy. Steady state is a sink.

2 Search Labor Model

2.1 McCall (1970) Model

Risk-neutral agent searches for job:

$$E_0 \sum_{t=0}^{\infty} \beta^t x_t$$

$x_t = w$ if employed, $x_t = z$ if unemployed

Job offers i.i.d. draw from $F(w)$.

Recursive formulation: state $s_t \in \{W, U\}$, control: accept, reject offer

Value of employed worker

$$
\begin{aligned}
W(w) &= E_0 \sum_{t=0}^{\infty} \beta^t x_t, \text{ s.t. } x_t = w \\
&= \frac{w}{1-\beta}
\end{aligned}
$$

Value of unemployed worker:

$$U = z + \beta \int_0^{\infty} \max_{acc,rej} \left\{ U, \frac{w}{1-\beta} \right\} dF(w)$$

Reservation wage w_R:

$$W(w_R) = U = \frac{w_R}{1-\beta}$$

Characterize reservation wage:

$$w_R - z = \frac{\beta}{1-\beta} \int_{w_R}^{\infty} (w - w_R) dF(w)$$

Another characterization:

$$w_R - z = \beta(E[w] - z) + \beta \int_0^{w_R} F(w) dw$$

Factors affecting reservation wage:

- value of unemployment z
- distribution of offers F