1 Krusell-Smith (1998): Incomplete Markets with Aggregate Risk

Correct recursive formulation with state $\mu(a, l)$:

$$V(a, l, z, \mu) = \max_{c, a'} \{ u(c) + \beta E[V(a', l', z', \mu') | l, z, \mu] \}$$

subject to:

$$a' = R(z, K, N)a + w(z, K, N)l - c$$

$$a \geq \bar{a}$$

$$\mu' = H(z, \mu)$$

and then $K = \int a\mu(da, dl), N = \int l\mu(da, dl)$.

Problem: infinite dimensional state $\mu$. Also unknown $H$ maps distributions into distributions.

Krusell and Smith approximate $\mu$ by moments, assume $H$ is log-linear function mapping moments to moments. Show that to forecast $w, r$, essentially enough to consider law of motion for $K$. 

1
2 Limited Commitment Models

Previously had exogenously incomplete markets, now endogenize via a default option.

2.1 Kehoe and Levine (2001)

Two representative individuals, $i = 1, 2$ continuum of each type.

Individual $i$ consumes $c_i^t$. Standard preferences:

$$\sum_{t=0}^{\infty} \beta^t u(c_i^t)$$

with $u' > 0, u'' < 0$, Indada.

Endowments are time varying but (for now) deterministic, $e_i^t \in \{e^g, e^b\}$, and $e_i^1 = e^g$ in periods 0, 2, 4. $e^g + e^b = 2$.

No durable goods (for now) but Arrow securities with price $q_{t,t+1}$, holdings $\theta_{t,t+1}$.

Flow budget constraint:

$$c_i^t + q_{t,t+1}\theta_{t,t+1}^i = e_i^t + \theta_{t-1,t}^i$$

Lifetime budget constraint:

$$\sum_{s=0}^{\infty} q_{t,t+s}c_{t+s}^i = \sum_{s=0}^{\infty} q_{t,t+s}c_{t+s}^i + \theta_{t-1,t}^i$$

New addition: Solvency constraint

$$\sum_{s=0}^{\infty} \beta^s u(c_{t+s}^i) \geq \sum_{s=0}^{\infty} \beta^s u(e_{t+s}^i)$$
2.2 Equilibrium

Equilibrium definition is standard. Note \( c_t^1 + c_t^2 = e^g + e^b = 2, \theta_{t,t+1}^1 + \theta_{t,t+1}^2 = 0 \).

Look for symmetric steady state (SSS) equilibrium: \( c_t^i = c^g \ (e^b) \) when \( e_t^i = e^g \ (e^b) \).

Characterize SSS as functions of \( c^g \):

\[
U^g(c^g) = u(c^g) + \beta U^b(c^g)
\]

\[
U^b(c^g) = u(2 - c^g) + \beta U^g(c^g)
\]

Solvency/participation constraints: \( U^g(e^g) \geq U^g(c^g), \ U^b(e^g) \geq U^b(c^g) \)

Combining, define:

\[
f(c^g) = u(c^g) - u(e^g) + \beta [u(2 - c^g) - u(2 - e^g)]
\]

Full insurance: \( c^g = 1 \), partial insurance: \( f(e^g) = 0, \ 1 < c^g < e^g \).

2.3 Characterize SSS

When \( c^g > 1 > e^b \), low endowment consumer is constrained. Calculate interest rate from unconstrained consumer:

\[
q = \beta \frac{u'(e^b)}{u'(c^g)} > \beta
\]

So \( R = 1/q < 1/\beta \).

If \( f(1) < 0 \) then there is no equilibrium with more consumption smoothing. Autarky is always an equilibrium, but apart from that there is no other equilibrium with less consumption smoothing.
2.4 Adding Physical Capital

Suppose there is one unit of capital, giving (net) return $r$ each period. Then resource constraint becomes:

$$c_t^1 + c_t^2 = e^g + e^b + r = 2 + r$$

When default, lose access to capital market.

Analysis goes through, but now equilibrium determined by:

$$f(c^g) = u(c^g) - u(e^g) + \beta [u(2 + r - c^g) - u(2 - e^g)]$$

When $r > 0$ there is a unique equilibrium, not autarky.

2.5 An Example

- $f(c^g) = u(c^g) - u(e^g) + \beta [u(2 + r - c^g) - u(2 - e^g)]$

- Example: $u(c) = \log(c)$, $\beta = 0.5$, $e^g = 1.5$.

- When $r = 0$, equilibria at $c^g = e^g = 1.5$ AND at $c^g = 1.15 < e^g$ First-best: $c^g = c^b = 1$.

- When $r = .01$, unique equilibrium at $c^g = 1.13 < e^g$. First-best: $c^g = c^b = 1.005$. 
Determination of Equilibria

\begin{align*}
\text{c} &= \text{g} \\
r &= 0 \\
r &= 0.01
\end{align*}