# Lecture 3: Incomplete Markets Models

# Limited Commitment

### Economics 714, Spring 2015

## 1 Krusell-Smith (1998): Incomplete Markets with Aggregate Risk

Correct recursive formulation with state $\mu(a, l)$:

$$V(a, l, z, \mu) = \max_{c, a'} \{u(c) + \beta E[V(a', l', z', \mu')|l, z, \mu]\}$$

subject to:

$$\begin{aligned} a' &= R(z, K, N)a + w(z, K, N)l - c \\ a &\geq \underline{\mathrm{a}} \\ \mu' &= H(z, \mu) \end{aligned}$$

and then $K = \int a\mu(da, dl)$, $N = \int l\mu(da, dl)$.

Problem: infinite dimensional state $\mu$. Also unknown $H$ maps distributions into distributions

Krusell and Smith approximate $\mu$ by moments, assume $H$ is log-linear function mapping moments to moments. Show that to forecast $w$,$r$, essentially enough to consider law of motion for $K$.

# 2 Limited Commitment Models

Previously had exogenously incomplete markets, now endogenize via a default option.

## 2.1 Kehoe and Levine (2001)

Two representative individuals, $i = 1, 2$ continuum of each type.

Individual $i$ consumes $c_t^i$. Standard preferences:

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i)$$

with $u' > 0, u'' < 0$, Indada

Endowments are time varying but (for now) deterministic, $e_t^i \in \{e^g, e^b\}$, and $e_t^1 = e^g$ in periods 0,2,4. $e^g + e^b = 2$.

No durable goods (for now) but Arrow securities with price $q_{t,t+1}$, holdings $\theta_{t,t+1}^i$.

Flow budget constraint:

$$c_t^i + q_{t,t+1}\theta_{t,t+1}^i = e_t^i + \theta_{t-1,t}^i$$

Lifetime budget constraint:

$$\sum_{s=0}^{\infty} q_{t,t+s} c_{t+s}^i = \sum_{s=0}^{\infty} q_{t,t+s} e_{t+s}^i + \theta_{t-1,t}^i$$

New addition: **Solvency constraint**

$$\sum_{s=0}^{\infty} \beta^s u(c_{t+s}^i) \geq \sum_{s=0}^{\infty} \beta^s u(e_{t+s}^i)$$

## 2.2 Equilibrium

Equilibrium definition is standard. Note $c_t^1 + c_t^2 = e^g + e^b = 2$, $\theta_{t,t+1}^1 + \theta_{t,t+1}^2 = 0$.

Look for symmetric steady state (SSS) equilibrium: $c_t^i = c^g \quad (c^b)$ when $e_t^i = e^g \quad (e^b)$.

Characterize SSS as functions of $c^g$:

$$\begin{aligned} U^g(c^g) &= u(c^g) + \beta U^b(c^g) \\ U^b(c^g) &= u(2 - c^g) + \beta U^g(c^g) \end{aligned}$$

Solvency/participation constraints: $U^g(c^g) \geq U^g(e^g)$, $U^b(c^g) \geq U^b(e^g)$

Combining, define:

$$f(c^g) = u(c^g) - u(e^g) + \beta \left[ u(2 - c^g) - u(2 - e^g) \right]$$

Full insurance: $c^g = 1$, partial insurance: $f(c^g) = 0$, $1 < c^g < e^g$.

## 2.3 Characterize SSS

When $c^g > 1 > c^b$, low endowment consumer is constrained. Calculate interest rate from unconstrained consumer:

$$q = \beta \frac{u'(c^b)}{u'(c_g)} > \beta$$

So $R = 1/q < 1/\beta$.

If $f(1) < 0$ then there is no equilibrium with more consumption smoothing. Autarky is always an equilibrium, but apart from that there is no other equilibrium with less consumption smoothing.

## 2.4 Adding Physical Capital

Suppose there is one unit of capital, giving (net) return $r$ each period. Then resource constraint becomes:

$$c_t^1 + c_t^2 = e^g + e^b + r = 2 + r$$

When default, lose access to capital market.

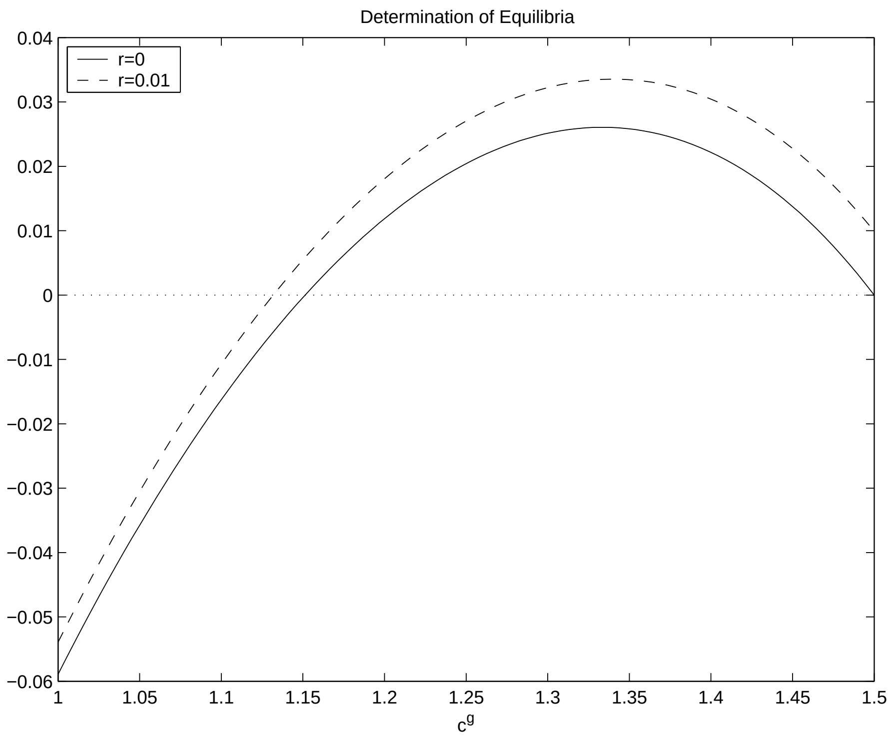
Analysis goes through, but now equilibrium determined by:

$$f(c^g) = u(c^g) - u(e^g) + \beta \left[ u(2 + r - c^g) - u(2 - e^g) \right]$$

When $r > 0$ there is a unique equilibrium, not autarky.

## 2.5 An Example

- $f(c^g) = u(c^g) - u(e^g) + \beta[u(2 + r - c^g) - u(2 - e^g)]$
- Example: $u(c) = \log(c)$, $\beta = 0.5$, $e^g = 1.5$.
- When $r = 0$, equilibria at $c^g = e^g = 1.5$ **AND** at $c^g = 1.15 < e^g$ First-best: $c^g = c^b = 1$.
- When $r = .01$, unique equilibrium at $c^g = 1.13 < e^g$. First-best: $c^g = c^b = 1.005$.


Determination of Equilibria
r=0
r=0.01
0.04
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
c^g