Lecture 2: Expectations and Dynamics

Economics 712, Fall 2014

1 Dynamics of Linear Difference Equations

1.1 Vector example: Dornbusch (1976) exchange rate model

Then re-write dynamics as deviations from steady state:

$$\begin{aligned} e_{t+1} - \bar{e} &= e_t - \bar{e} + \frac{1}{\lambda}(p_t - \bar{p}) \\ p_{t+1} - \bar{p} &= \frac{\delta\alpha}{1-\alpha\sigma}(e_t - \bar{e}) + \left[1 - \frac{\alpha}{1-\alpha\sigma}(\delta + \sigma/\lambda)(p_t - \bar{p})\right] \end{aligned}$$

Define vector $X_t = [e_t - \bar{e}, p_t - \bar{p}]'$, so can write this as a vector difference equation $X_{t+1} = AX_t$. What happens to X_t over time?

Qualitatively, can construct phase diagram.

$$\begin{aligned} \Delta e_t &= e_{t+1} - e_t = (p_t - \bar{p})\lambda \\ \Delta p_t &= \frac{\alpha}{1-\alpha\sigma}\left[\delta(e_t - \bar{e}) - (\delta + \sigma/\lambda)(p_t - \bar{p})\right] \end{aligned}$$

Analyze regions where $\Delta e_t > 0, \Delta e_t = 0, \Delta e_t < 0$, same for p_t.

Given p_t, there is a unique value of e_t such that the economy is stable. This mapping $e_t(p_t)$ defines the **saddle path**.

2 Forward Looking Model: Cagan Model with Rational Expectations

$$M_t^d = \exp\left(-\alpha \frac{E_t p_{t+1} - p_t}{p_t}\right)$$

Take logs, rewrite:

$$p_t = aE_t p_{t+1} + cm_t$$

What happens to prices over time? Solve forward if $|a| < 1$:

$$p_t = c\sum_{j=0}^{\infty} a^j E_t m_{t+j} + \lim_{T\to\infty} a^T E_t p_{t+T}$$

Example:

$$m_{t+1} = \rho m_t + w_{t+1}$$

Then:

$$p_t = \frac{c}{1 - a\rho} m_t$$

Lucas critique and cross-equation restrictions

3 Bubbles and Indeterminacy

3.1 Bubbles

Fundamentals solution:

$$p_t^f = c\sum_{j=0}^{\infty} a^j E_t m_{t+j}$$

Bubble component:

$$b_t = \lim_{T\to\infty} a^T E_t p_{t+T}$$

General solution:

$$p_t = p_t^f + b_t$$

Bubbles explode in expectation here:

$$E_t b_{t+j} = \left(\frac{1}{a}\right)^j b_t$$

3.2 Indeterminacy

What if $|a| > 1$? Can't solve forward

Say $m_t \equiv 1$

$$E_t p_{t+1} = \frac{1}{a} p_t - c$$

so

$$p_{t+1} = \frac{1}{a} p_t - c + e_{t+1}$$

Solve backward:

$$p_{t+1} = \frac{c}{1-a} + \left(\frac{1}{a}\right)^{t+1} p_0 + \sum_{j=0}^{t+1} \left(\frac{1}{a}\right)^j e_{t+1-j}$$

3.3 Multivariate Models

$$\begin{aligned} p_t &= aE_t p_{t+1} + cm_t \\ m_{t+1} &= \rho m_t + \delta p_t + u_{t+1} \end{aligned}$$

Write in matrix form:

$$x_t = Ax_{t+1} + Bw_{t+1}$$

Diagonalize using eigenvector/eigenvalue decomposition:

$$A = V\Lambda V^{-1}$$

Define $z_t = V^{-1}x_t$, then:

$$z_t = \Lambda z_{t+1} + V^{-1}Bw_{t+1}$$

If $|\lambda_i| < 1$, can solve forward:

$$z_t^i = \lambda_i^{t+T} E_t z_{t+T}^i$$

So then $z_t^i = 0$.

If $|\lambda_i| > 1$, can solve backward:

$$z_t^i = \left(\frac{1}{\lambda_i}\right)^t z_0$$

For determinacy, need as many $|\lambda_i| > 1$ as there are predetermined variables.

3.4 Back to the example

For small $\delta > 0$ will have $0 < \lambda_1 < 1 < \lambda_2$ and therefore restriction:

$$v^{11}p_t + v^{21}m_t = 0$$

This gives determinate, saddlepath solution, determines p_0 given m_0.

But if δ large enough will have $\lambda_2 > \lambda_1 > 1$. Then need initial conditions for m_0, p_0.

Indeterminacy. Steady state is a sink.