Lecture 2: More on Incomplete Markets Models

Economics 714, Spring 2015

1 Stationary Distributions

1.1 Background

We need to define the joint transition over assets a and labor l:

$$\begin{aligned} P((a,l),\hat{A}\times\hat{L}) &= Q(l,\hat{L}) \;\text{ if } a'(a,l)\in\hat{A}, \\ &= 0 \;\text{ if } a'(a,l)\notin\hat{A} \end{aligned}$$

To rule out multiple stationary distributions, need a mixing condition.

Assumption: $\exists c\in S$, $\varepsilon>0$, $N\geq 1$ such that $P^N(a,[c,b])\geq\varepsilon$ and $P^N(b,[a,c])\geq\varepsilon$.

This gives us:

Theorem: If $S=[a,b]\subset\mathbb{R}^n$, and P is monotone, has the Feller property, and satisfies the mixing condition, then there exists a unique invariant distribution μ^* and $(T^*)^n\mu_0\to\mu^*$ for all probability measures μ_0 on $(S,\mathcal{S})$.

1.2 Application

Huggett (1993) considers $l_t\in\{l_l,l_h\}$, shows that the key properties hold:

(i) $a'(a,l_l)<a \quad \forall a>\underline{a}$.

(ii) if $u(c)=c^{1-\gamma}/(1-\gamma)$ then $\exists\bar{a}$ such that $a'(\bar{a},l_h)=\bar{a}$.

Aiyagari (1994) considers l_t i.i.d., establishes existence of stationary distribution.

2 Incomplete Markets Model

2.1 Implications

1. Model generates lower risk free rate than complete markets, but effect not substantial.

2. Precautionary saving effect not very large

3. Model generates heterogeneity in wealth and income, but not (nearly) enough to match US data

4. Welfare costs of borrowing constraints and market incompleteness relatively small: self-insurance via saving able to smooth consumption relatively well.

3 Krusell-Smith (1998): Incomplete Markets with Aggregate Risk

Aggregate production function now has Markov productivity shock $z_t \sim Q_z$:

$$Y_t = z_t F(K_t, N_t)$$

Gives usual marginal productivity conditions: $w = zF_N(K, N)$, $r = zF_K(K, N) - \delta$

Idiosyncratic labor shocks are Markov conditional on z: $Q(l, dl'|z)$.

Joint distribution of (l, z) is Γ.

Attempt at recursive forumlation for individual agent problem:

$$V(a, l, z, K, N) = \max_{c, a'} \{u(c) + \beta E[V(a', l', z', K', N')|l, z, K, N]\}$$

subject to:

$$\begin{aligned} c + a' &= R(z, K, N)a + w(z, K, N)l \\ a &\geq \underline{\mathrm{a}} \\ (K', N') &= G(z, K, N) \end{aligned}$$

Problem: (z, K, N) not (in general) sufficient statistic for K', which depends on distribution of assets

Correct recursive formulation with state $\mu(a, l)$:

$$V(a, l, z, \mu) = \max_{c, a'} \{u(c) + \beta E[V(a', l', z', \mu')|l, z, \mu]\}$$

subject to:

$$\begin{aligned} c + a' &= R(z, K, N)a + w(z, K, N)l - c \\ a &\geq \underline{\mathrm{a}} \\ \mu' &= H(z, \mu) \end{aligned}$$

and then $K = \int a\mu(da, dl)$, $N = \int l\mu(da, dl)$.

Problem: infinite dimensional state μ. Also unknown H maps distributions into distributions

Krusell and Smith approximate μ by moments, assume H is log-linear function mapping moments to moments. Show that to forecast w,r, essentially enough to consider law of motion for K.