Lecture 17
Real Business Cycles: Quantitative Results

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Economics 712
Simulations from a Quantitative Version

- We have seen the qualitative behavior of the model, showing that the real business cycle model is consistent with the data.
- Apart from the special case we studied, to fully solve the model we need to use numerical methods.
- Calibrate the model: choose parameters to match some key economic data. Example: set $\beta$ so that steady state real interest rate matches US data.
- Program up on computer and simulate: use random number generator to draw technology shocks, feed them through the model.
- Compute correlations and volatilities and compare to US data.
Figure 1

Output and HP Trend

HP Band Pass (6,32) Cyclical Components

HP Growth Component and Linear Trend Residual

Figure 2

Note: Sample period is 1947:1 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.
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Figure 4

Note: Sample period is 1947:1 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.
We will show how to choose some parameters of a RBC model to match the data, a process known as calibration. Suppose preferences are given by:

\[
\sum_{t=0}^{\infty} \beta^t (1 + n)^t [(1 - a) \log C_t + a \log (1 - N_t)]
\]

here \( n > 0 \) is the population growth rate and \( C_t \) and \( N_t \) are per capita consumption and hours. Suppose labor-augmenting technology grows at rate \( g \) so \( A_t = (1 + g)^t \). Thus the aggregate resource constraint is:

\[
C_t + I_t = (1 + g)^{(1-\alpha)t} K_t^\alpha N_t^{1-\alpha},
\]

where \( I_t \) is per capita investment an \( K_t \) is the per capita capital stock. Finally the law of motion for the capital in per capita terms is:

\[
(1 + g)(1 + n)K_{t+1} = (1 - \delta)K_t + I_t
\]
We will use that for Cobb-Douglas production $F_K = \alpha Y / K$, $F_N = (1 - \alpha) Y / N$. The optimality conditions are:

$$\frac{a C_t}{(1 - a)(1 - N_t)} = (1 - \alpha) z_t K_t^\alpha N_t^{-\alpha}$$

$$\frac{1}{C_t} = \beta E_t \left[ \frac{\alpha z_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}}{C_{t+1}} \right]$$

$$K_{t+1} = z_t K_t^\alpha N_t^{1-\alpha} + (1 - \delta) K_t - C_t$$

Now we consolidate these equations:

$$\frac{a}{1 - N_t} = (1 - \alpha) \frac{1 - a}{C_t} \frac{Y_t}{N_t}$$

$$\left(1 + g\right) \frac{C_{t+1}}{C_t} = \beta \left[ 1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \right]$$
This model has a balanced growth path (BGP) in which hours worked $N_t$ is constant and all other per capita variables grow at the constant rate $g$, i.e. $K_{t+1} = (1 + g)K_t$ and so on. Using the equilibrium conditions, we can find three equations relating the hours $N$, the capital/output ratio $K/Y$, the consumption/output ratio $C/Y$, and the investment/capital ratio $I/K$ to each other and the parameters of the model.

First, we divide the law of motion for capital by $K_t$, and use $K_{t+1}/K_t = 1 + g$ to obtain

$$\begin{align*}
(1 + g)^2(1 + n) &= 1 - \delta + \frac{I}{K} 
\end{align*}$$

(3)

Then using (1),

$$\begin{align*}
\frac{a}{1 - N} &= (1 - \alpha) \frac{1 - a}{N} \frac{Y}{C} 
\end{align*}$$

(4)

Finally from (2),

$$\begin{align*}
(1 + g)^2 &= \beta \left[ 1 - \delta + \alpha \frac{Y}{K} \right] 
\end{align*}$$

(5)
Suppose $\alpha = 0.4$, $n = 0.012$ and $g = 0.0156$, which are estimated from US data.

1. Given a value of $I/K = 0.076$ in the data, we find a value of $\delta$ consistent with this in the BGP. Using (3), we obtain $\delta = 0.0321$.

2. Given a value of $K/Y = 3.32$ and the value of $\delta$, we find a value of $\beta$ from the BGP relations. Now using (5) and the previously obtained value of $\delta$, we can calculate $\beta = 0.9478$.

3. Given a value of $N = 0.31$ and $Y/C = 1.33$ find a value of $a$ from the BGP relations. Finally from (4), $a = 0.640$. 
A basic real business cycle (RBC) model

Simulations using a linear approximation to the equilibrium conditions

- Specify functional forms

\[
U = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{(1-L_t)^{1+\eta}}{1+\eta}
\]

\[
F = Z_t N_t^\alpha K_{t-1}^{1-\alpha}
\]

- First order conditions become (with some simplification)

\[
\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \alpha Z_t N_t^{\alpha-1} K_{t-1}^{1-\alpha} = \alpha \left( \frac{Y_t}{N_t} \right)
\]

\[
C_t^{-\sigma} = \beta E_t \left[ (1-\alpha) \left( \frac{Y_t}{K_{t-1}} \right) + 1-\delta \right] C_{t+1}
\]

\[
Y_t = Z_t N_t^\alpha K_{t-1}^{1-\alpha}
\]

\[
Y_t = C_t + K_t - (1-\delta) K_{t-1}
\]
Steady state for $Y$, $C$, $K$, $N$, $R$:

$$\frac{\chi N^n}{C^{-\sigma}} = \alpha \left( \frac{Y}{N} \right)$$

$$1 = \beta R$$

$$R \equiv (1 - \alpha) \left( \frac{K}{N} \right)^{-\alpha} + 1 - \delta$$

$$\left( \frac{Y}{N} \right) = \left( \frac{K}{N} \right)^{1-\alpha}$$

$$Y = C + \delta K$$
A basic real business cycle (RBC) model

Linearizing around a $Z = 1$ steady state (lower case denotes % deviation)

$$\eta n_t + \sigma c_t = z_t + y_t - n_t$$

$$c_t = E_t c_{t+1} - \sigma^{-1}(1 - \alpha) \left( \frac{Y}{K} \right) \beta E_t (y_{t+1} - k_t)$$

$$y_t = z_t + \alpha n_t + (1 - \alpha) k_{t-1}$$

$$y_t = \left( \frac{C}{Y} \right) c_t + \left( \frac{K}{Y} \right) [k_t - (1 - \delta) k_{t-1}]$$

Assume

$$z_t = \rho z_{t-1} + e_t$$
A basic real business cycle (RBC) model

- **Calibration**
  - Parameters: $\eta, \sigma, \sigma, \alpha, \delta, \rho, \sigma_e^2$
  - Do not use business cycle evidence to calibrate parameters

- **Simulations**
Figure 10.03  Small shocks and large cycles

Abel/Bernanke, Macroeconomics, © 2001 Addison Wesley Longman, Inc. All rights reserved
Capital

Williams Economics 712
Impulse Responses

\[ y_{kl} \]

Williams
Economics 712
Figure 10.01  Actual versus simulated volatilities of key macroeconomic variables
Figure 10.02  Actual versus simulated correlations of key macroeconomic variables with GNP
Assessment of the Basic Real Business Model

- It accounts for a substantial amount of the observed fluctuations. Accounts for the covariances among a number of variables. Has some problems accounting for hours worked.
- Are fluctuations in TFP really productivity fluctuations?
- Factor utilization rates vary over the business cycle. During recessions, firms reduce the number of shifts. Similarly, firms are reluctant to fire trained workers.
- Neither is well-measured. They show up in the Solow residual.
- There is no direct evidence of technology fluctuations.
- Is intertemporal labor supply really so elastic?
- All employment variation in the model is voluntary, driven by intertemporal substitution.
- Deliberate monetary policy changes appear to have real effects.
Figure 7

Note: Sample period is 1947:2 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.
Figure 8

Output with small A shocks

Labor Input with small A shocks

Output with smaller labor elasticity

Labor Input with smaller labor elasticity

Note: Sample period is 1947:2 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.
Great Depression is a unique event in US history.
Timing 1929-1933.
Major changes in the US Economic policy: New Deal.
Can we use the theory to think about it?
## Data on the Great Depression

<table>
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<tr>
<th>Year</th>
<th>$u$</th>
<th>$Y$</th>
<th>$C$</th>
<th>$I$</th>
<th>$G$</th>
<th>$i$</th>
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<td>139.6</td>
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<td>1.7</td>
<td>−2.2</td>
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<td>118.1</td>
<td>9.4</td>
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<td>1937</td>
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<td>35.2</td>
<td>0.6</td>
<td>−1.6</td>
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</table>
Output, Inputs and TFP During the Great Depression

Theory

\[
\frac{\dot{z}}{z} = \frac{\dot{Y}}{Y} - \alpha \frac{\dot{K}}{K} - (1 - \alpha) \frac{\dot{N}}{N}
\]

Data (1929=100)

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<tr>
<th>Year</th>
<th>Y</th>
<th>N</th>
<th>K</th>
<th>z</th>
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<td>83.4</td>
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<td>65.3</td>
<td>73.5</td>
<td>98.4</td>
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Why did TFP fall so much?
Figure 1: Real Output, Consumption and Private Hours
(Per Adult, Index 1929 =100)
Predicted and Actual Output in 1929–39

Detrended Levels, With Initial Capital Stock in the Model Equal to the Actual Capital Stock in 1929
Potential Reasons

- Changes in Capacity Utilization.
- Changes in Quality of Factor Inputs.
- Changes in Composition of Production.
- Labor Hoarding.
- Increasing Returns to Scale.
Other Reasons for Great Depression

- Based on Cole and Ohanian (1999)
- Monetary Shocks: Monetary contraction, change in reserve requirements too late
- Banking Shocks: Banks that failed too small
- Fiscal Shocks: Government spending did rise (moderately)
- Sticky Nominal Wages: Probably more important for recovery
Cole and Ohanian (2001). Data (1929=100); data are detrended

<table>
<thead>
<tr>
<th>Year</th>
<th>Y</th>
<th>z</th>
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<tbody>
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</tr>
<tr>
<td>1939</td>
<td>73.2</td>
<td>103.1</td>
</tr>
</tbody>
</table>

Fast Recovery of $z$, slow recovery of output. Why?
Predicted and Actual Recovery of Output in 1934–39

Detrended Levels, With Initial Capital Stock in the Model Equal to the Actual Capital Stock in 1934
Figure 2: Comparing Output in the Models to the Data