Lecture 17
Real Business Cycles: Quantitative Results

Noah Williams

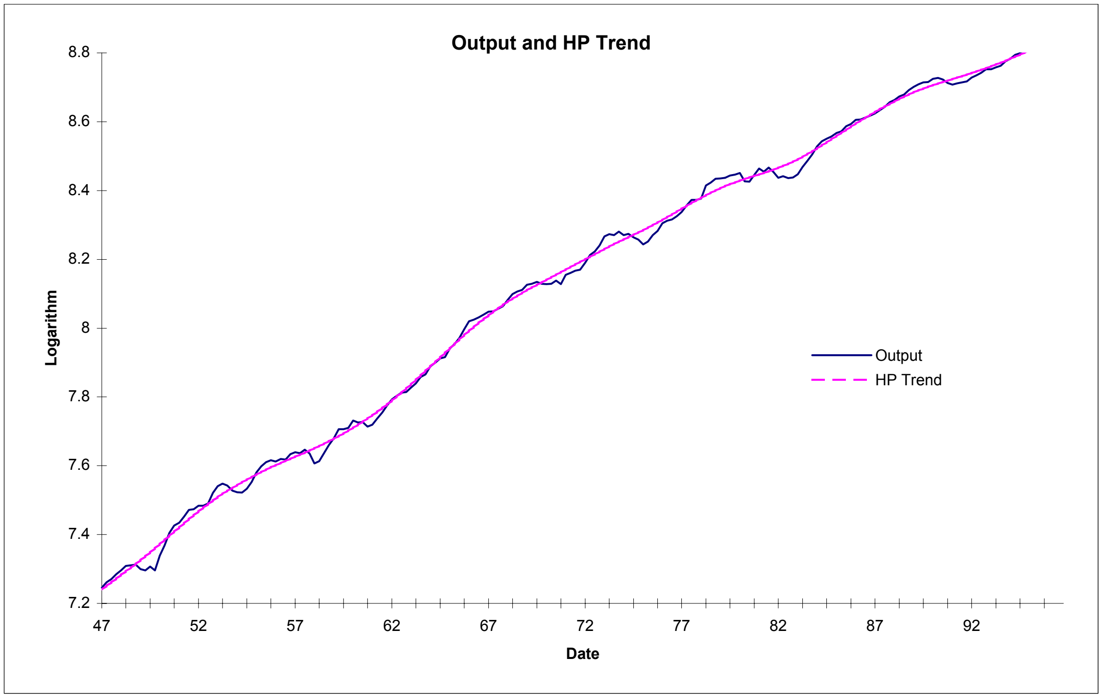
University of Wisconsin - Madison

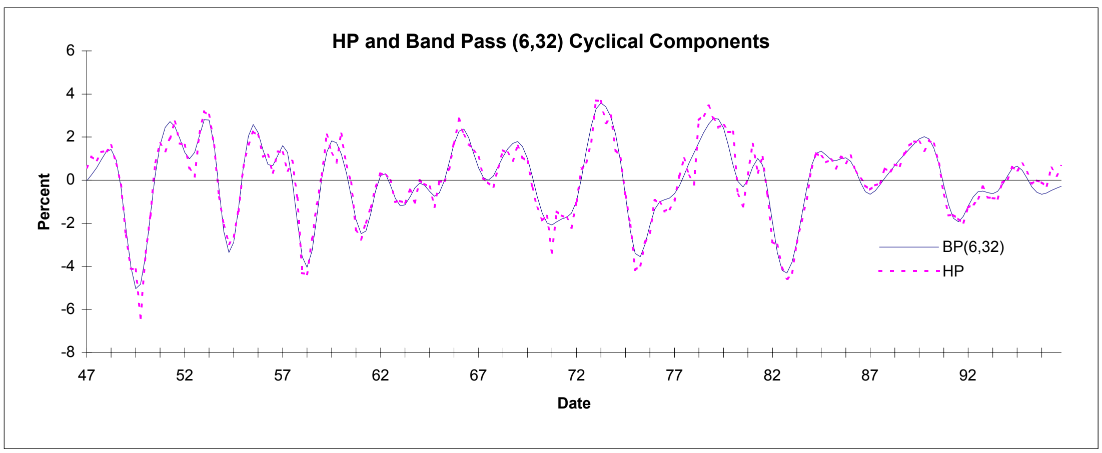
Economics 712

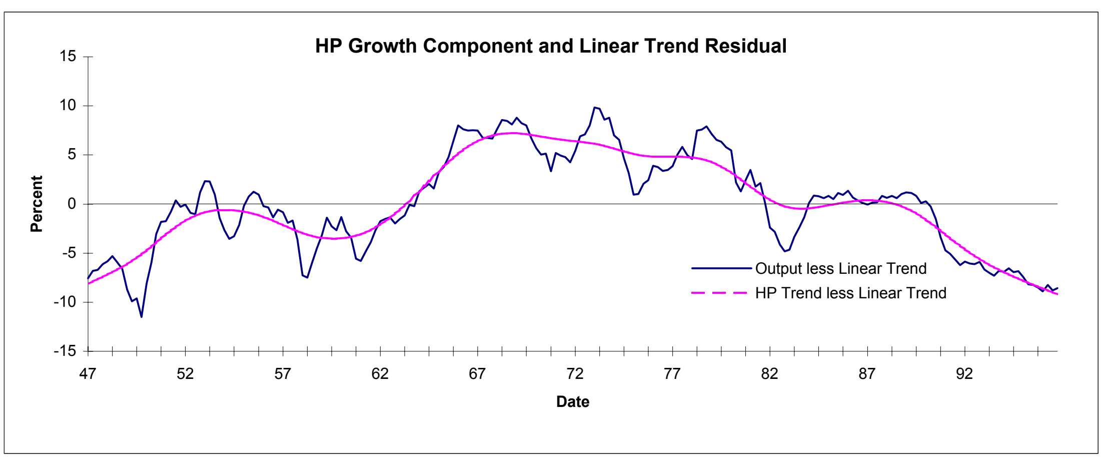
Simulations from a Quantitative Version

- We have seen the qualitative behavior of the model, showing that the real business cycle model is consistent with the data.
- Apart from the special case we studied, to fully solve the model we need to use numerical methods.
- Calibrate the model: choose parameters to match some key economic data.
  Example: set β so that steady state real interest rate matches US data.
- Program up on computer and simulate: use random number generator to draw technology shocks, feed them through the model.
- Compute correlations and volatilities and compare to US data.

Figure 1





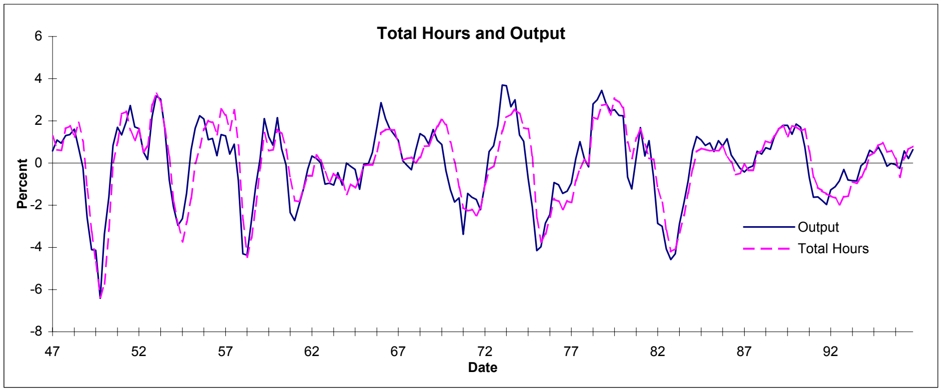


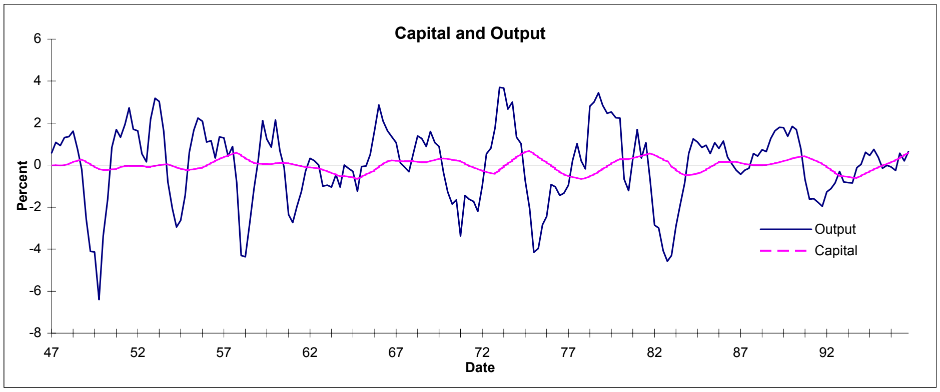
Note: Sample period is 1947:1 - 1996:4.

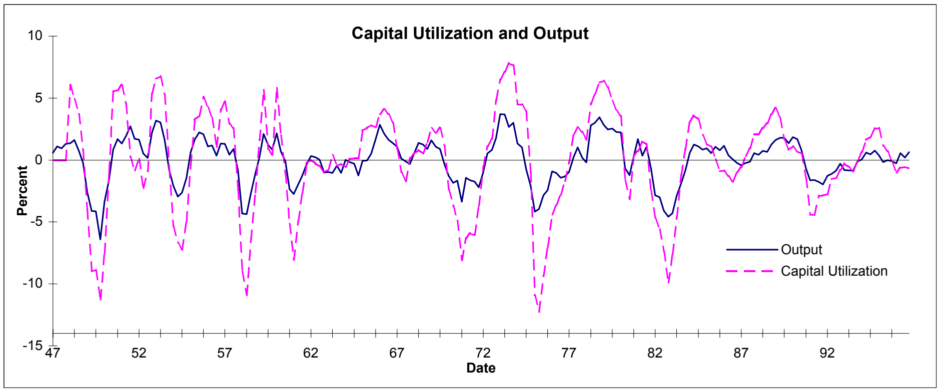
Figure 2

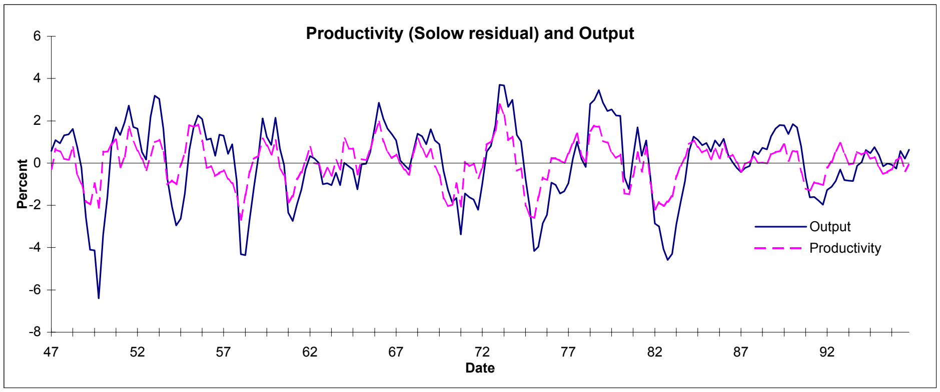
Note: Sample period is 1947:1 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.

Figure 3



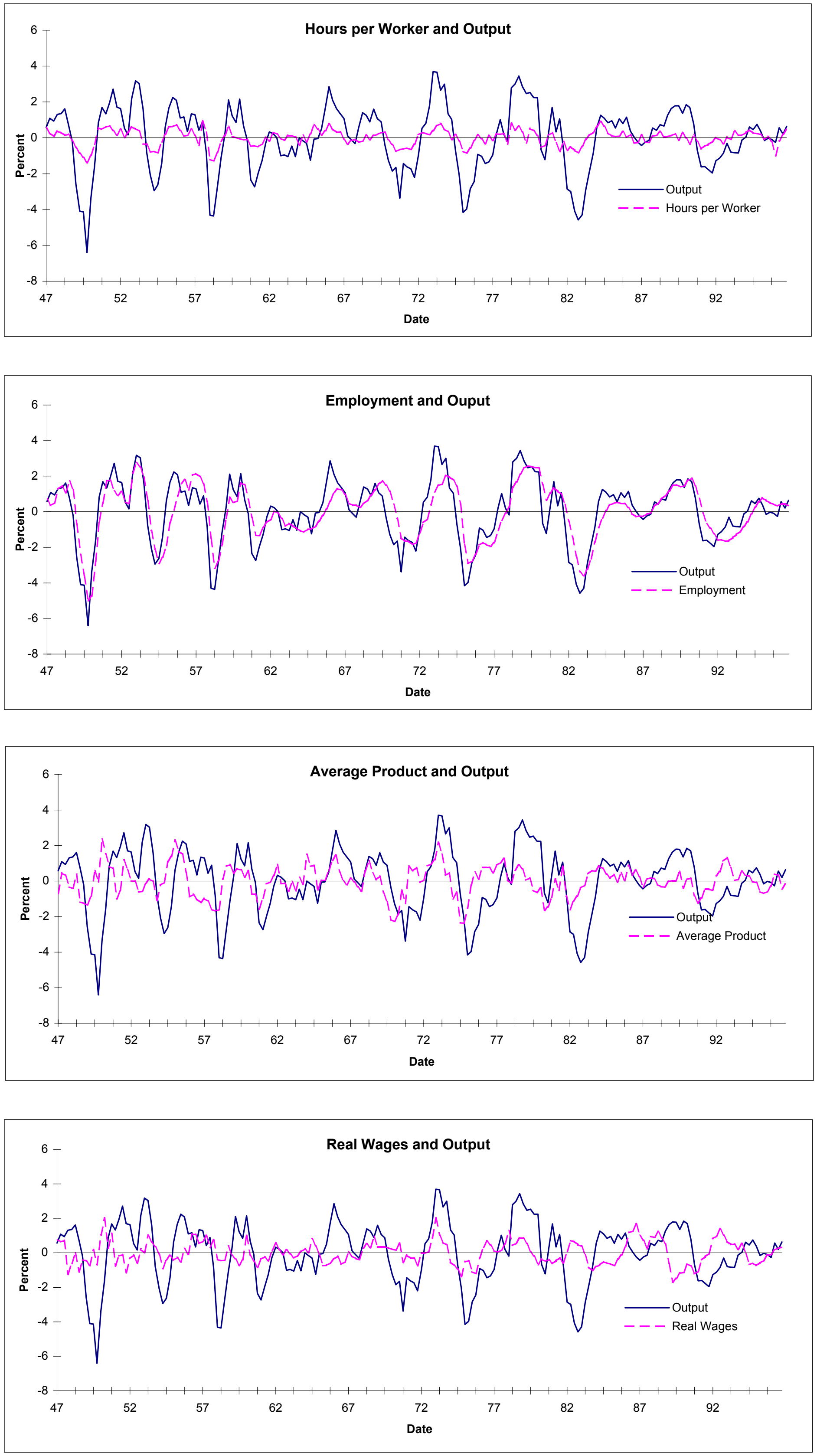






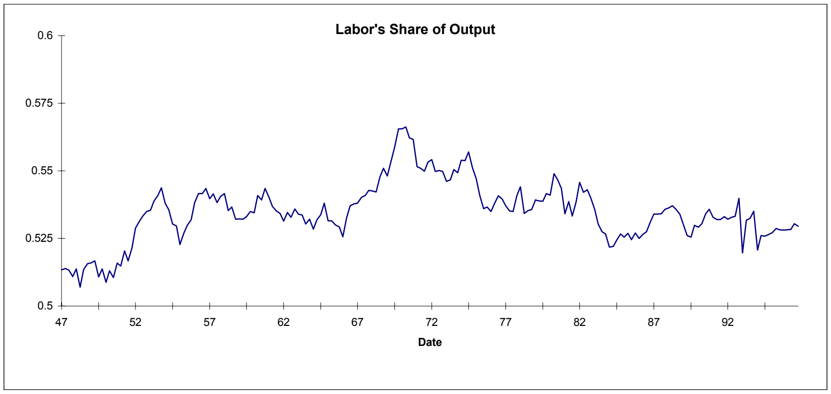
Note: Sample period is 1947:1 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.

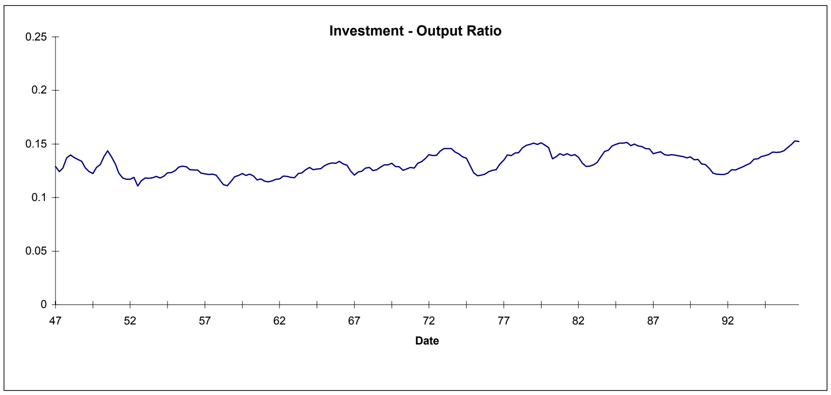
Figure 4

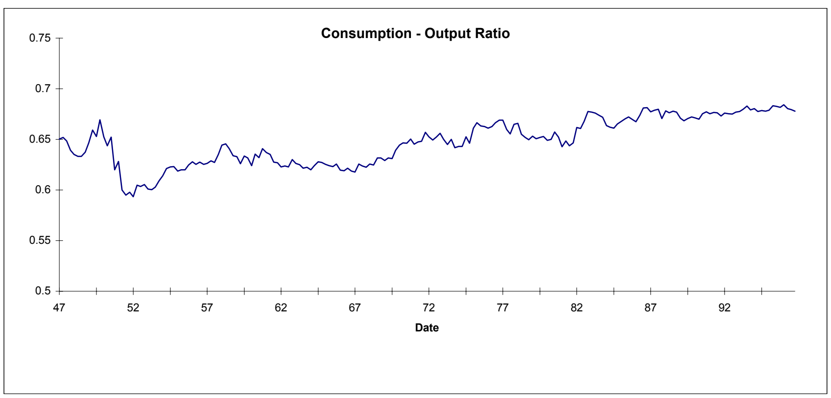


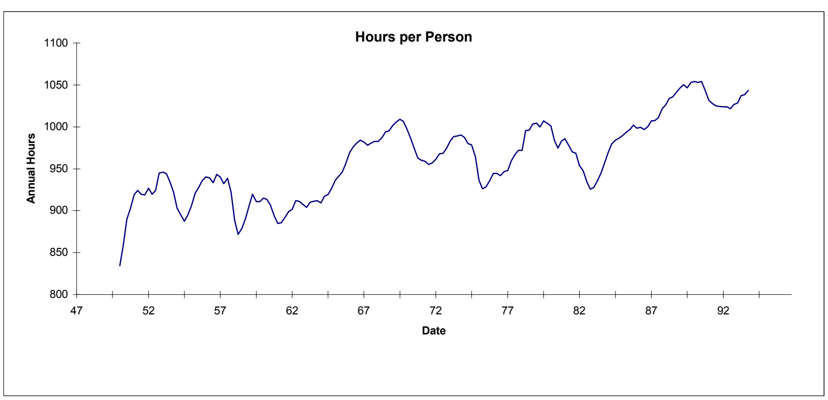
Note: Sample period is 1947:1 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.

Figure 5


Labor's Share of Output
0.6
0.575
0.55
0.525
0.5
47
52
57
62
67
72
77
82
87
92
Date


Investment - Output Ratio
0.25
0.2
0.15
0.1
0.05
0
47
52
57
62
67
72
77
82
87
92
Date


Consumption - Output Ratio
0.75
0.7
0.65
0.6
0.55
0.5
47
52
57
62
67
72
77
82
87
92
Date


Hours per Person
Annual Hours
1100
1050
1000
950
900
850
800
47
52
57
62
67
72
77
82
87
92
Date

Calibrating an RBC Model

We will show how to choose some parameters of a RBC model to match the data, a process known as calibration. Suppose preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t (1+n)^t [(1-a)\log C_t + a\log(1-N_t)]$$

here $n > 0$ is the population growth rate and C_t and N_t are per capita consumption and hours. Suppose labor-augmenting technology grows at rate g so $A_t = (1+g)^t$. Thus the aggregate resource constraint is:

$$C_t + I_t = (1+g)^{(1-\alpha)t} K_t^{\alpha} N_t^{1-\alpha},$$

where I_t is per capita investment an K_t is the per capita capital stock. Finally the law of motion for the capital in per capita terms is:

$$(1+g)(1+n)K_{t+1} = (1-\delta)K_t + I_t$$

We will use that for Cobb-Douglas production $F_K = \alpha Y/K$, $F_N = (1-\alpha)Y/N$. The optimality conditions are:

$$\begin{aligned}
\frac{aC_t}{(1-a)1-N_t} &= (1-\alpha)\, z_t K_t^{\alpha} N_t^{-\alpha} \\
\frac{1}{C_t} &= \beta E_t \left[\frac{\alpha z_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}}{C_{t+1}} \right] \\
K_{t+1} &= z_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta) K_t - C_t
\end{aligned}$$

Now we consolidate these equations:

$$\frac{a}{1-N_t} = (1-\alpha)\frac{1-a}{C_t}\frac{Y_t}{N_t} \tag{1}$$

$$(1+g)\frac{C_{t+1}}{C_t} = \beta\left[1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right] \tag{2}$$

This model has a balanced growth path (BGP) in which hours worked N_t is constant and all other per capita variables grow at the constant rate g, i.e. $K_{t+1} = (1+g)K_t$ and so on. Using the equilibrium conditions, we can find three equations relating the hours N, the capital/output ratio K/Y, the consumption/output ratio C/Y, and the investment/capital ratio I/K to each other and the parameters of the model.
First, we divide the law of motion for capital by K_t, and use $K_{t+1}/K_t = 1+g$ to obtain

$$(1+g)^2(1+n) = 1 - \delta + \frac{I}{K} \tag{3}$$

Then using (1),

$$\frac{a}{1-N} = (1-\alpha)\frac{1-a}{N}\frac{Y}{C} \tag{4}$$

Finally from (2),

$$(1+g)^2 = \beta\left[1 - \delta + \alpha\frac{Y}{K}\right] \tag{5}$$

Suppose $\alpha = 0.4$, $n = 0.012$ and $g = 0.0156$, which are estimated from US data.

1. Given a value of $I/K = 0.076$ in the data, we find a value of δ consistent with this in the BGP.

   Using (3), we obtain $\delta = 0.0321$.

2. Given a value of $K/Y = 3.32$ and the value of δ, we find a value of β from the BGP relations.

   Now using (5) and the previously obtained value of δ, we can calculate $\beta = 0.9478$.

3. Given a value of $N = 0.31$ and $Y/C = 1.33$ find a value of a from the BGP relations.

   Finally from (4), $a = 0.640$.

A basic real business cycle (RBC) model

Simulations using a linear approximation to the equilibrium conditions

- Specify functional forms

$$U = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{(1-L_t)^{1+\eta}}{1+\eta}$$

$$F = Z_t N_t^{\alpha} K_{t-1}^{1-\alpha}$$

- First order conditions become (with some simplification)

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \alpha Z_t N_t^{\alpha-1} K_{t-1}^{1-\alpha} = \alpha \left(\frac{Y_t}{N_t} \right)$$

$$C_t^{-\sigma} = \beta \mathrm{E}_t \left[(1-\alpha) \left(\frac{Y_t}{K_{t-1}} \right) + 1 - \delta \right] C_{t+1}^{-\sigma}$$

$$Y_t = Z_t N_t^{\alpha} K_{t-1}^{1-\alpha}$$

$$Y_t = C_t + K_t - (1-\delta) K_{t-1}$$

A basic real business cycle (RBC) model

Steady state for Y, C, K, N, R:

$$\frac{\chi N^{\eta}}{C^{-\sigma}} = \alpha \left(\frac{Y}{N} \right)$$

$$1 = \beta R$$

$$R \equiv (1-\alpha) \left(\frac{K}{N} \right)^{-\alpha} + 1 - \delta$$

$$\left(\frac{Y}{N} \right) = \left(\frac{K}{N} \right)^{1-\alpha}$$

$$Y = C + \delta K$$

Linearizing around a $Z = 1$ steady state (lower case denotes % deviation)

$$\eta n_t + \sigma c_t = z_t + y_t - n_t$$

$$c_t = \mathrm{E}_t c_{t+1} - \sigma^{-1}(1-\alpha)\left(\frac{Y}{K}\right)\beta \mathrm{E}_t(y_{t+1} - k_t)$$

$$y_t = z_t + \alpha n_t + (1-\alpha)k_{t-1}$$

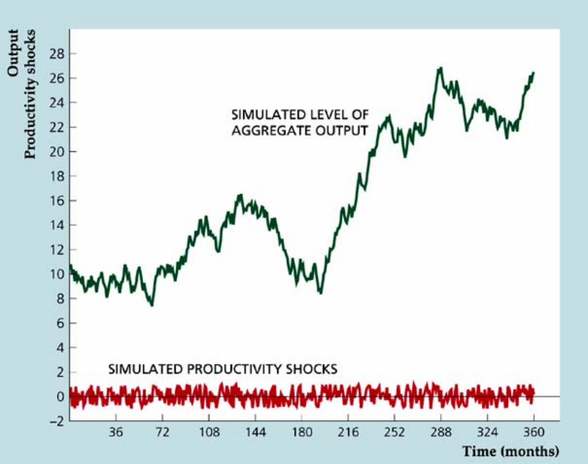
$$y_t = \left(\frac{C}{Y}\right)c_t + \left(\frac{K}{Y}\right)[k_t - (1-\delta)k_{t-1}]$$

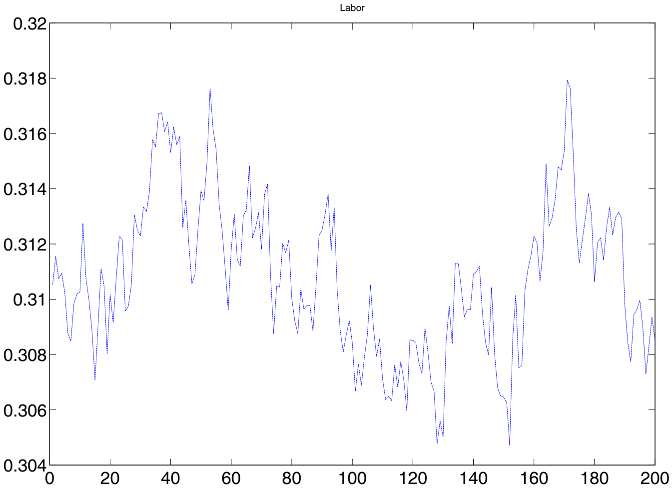
Assume

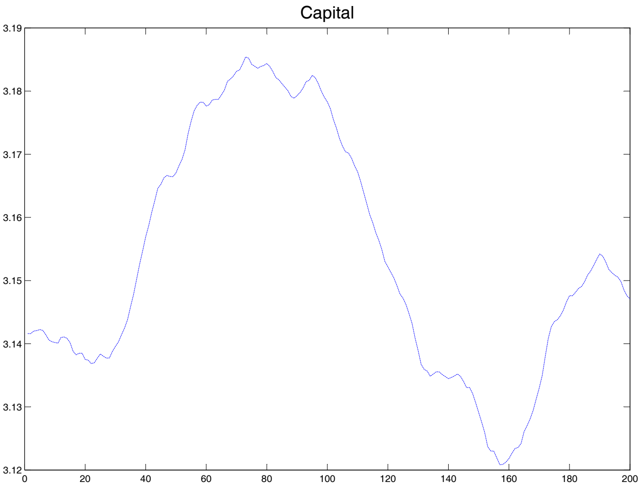
$$z_t = \rho z_{t-1} + e_t$$

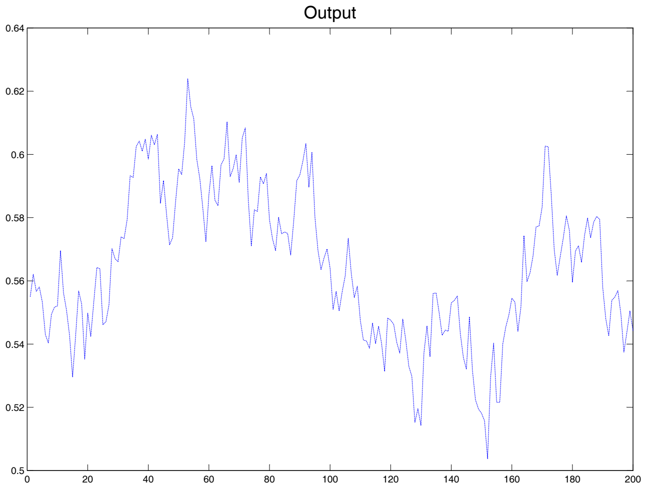
- Calibration
  - Parameters: η, σ, σ, α, δ, ρ, σ_e^2
  - Do not use business cycle evidence to calibrate parameters
- Simulations

Figure 10.03 Small shocks and large cycles




Labor
0.32
0.318
0.316
0.314
0.312
0.31
0.308
0.306
0.304
0
20
40
60
80
100
120
140
160
180
200


Capital
3.19
3.18
3.17
3.16
3.15
3.14
3.13
3.12
0
20
40
60
80
100
120
140
160
180
200


Output
0.64
0.62
0.6
0.58
0.56
0.54
0.52
0.5
0
20
40
60
80
100
120
140
160
180
200

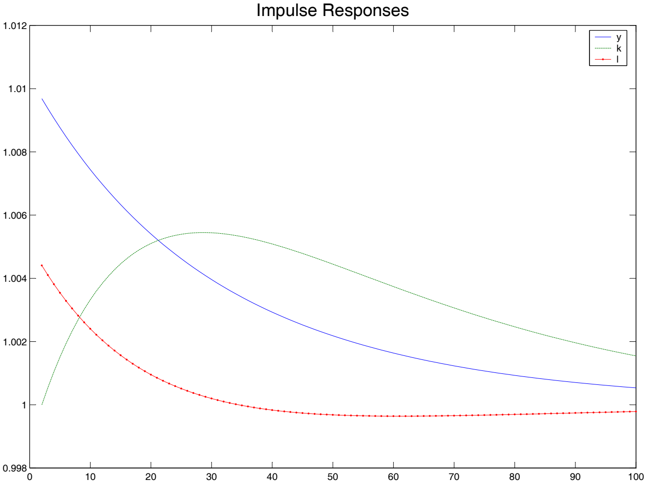

Impulse Responses
y
k
l
1.012
1.01
1.008
1.006
1.004
1.002
1
0.998
0
10
20
30
40
50
60
70
80
90
100

Figure 10.01 Actual versus simulated volatilities of key macroeconomic variables

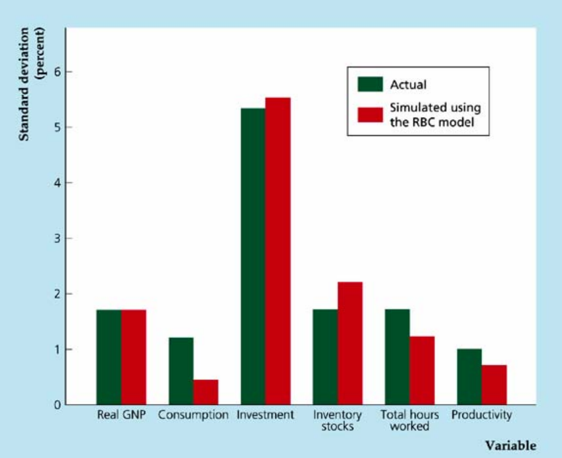
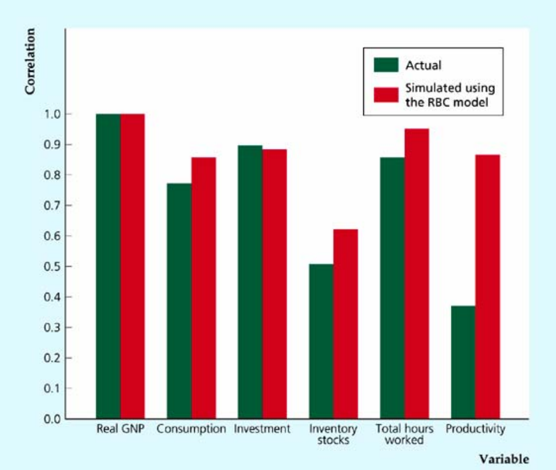


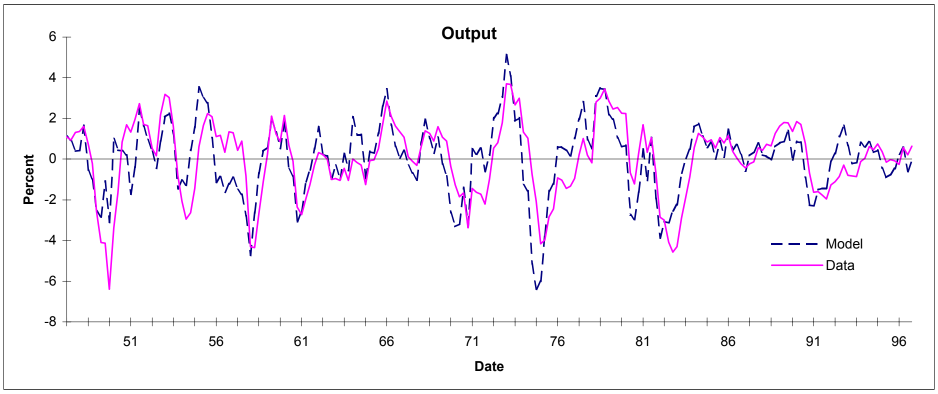
Figure 10.02 Actual versus simulated correlations of key macroeconomic variables with GNP

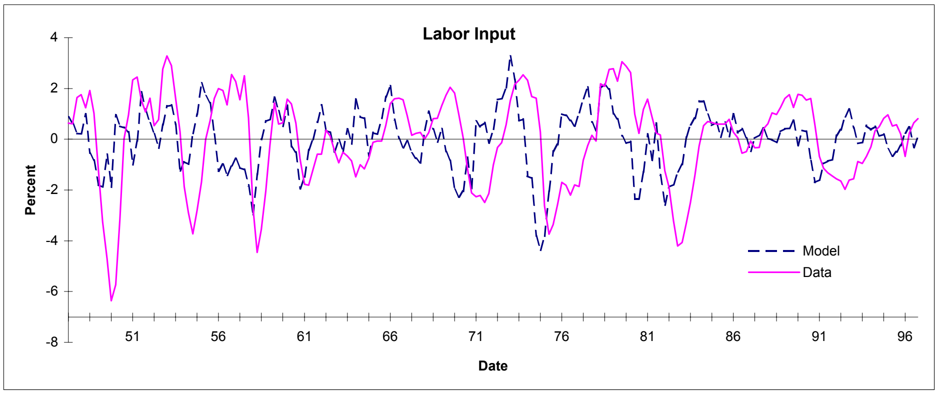


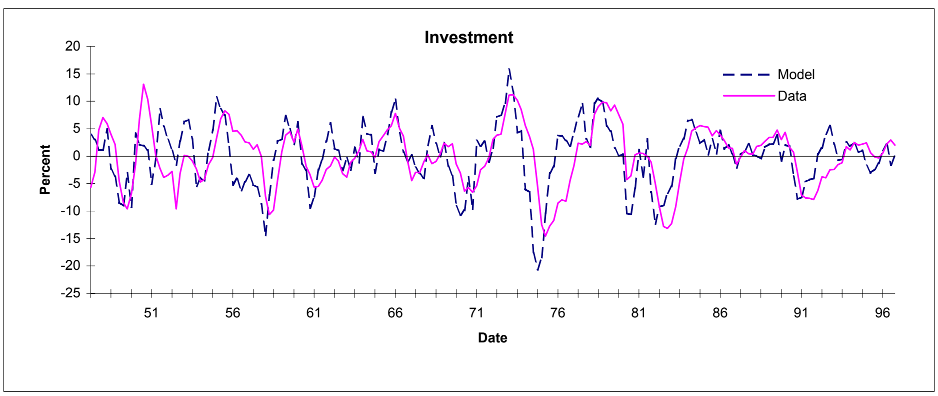
Assessment of the Basic Real Business Model

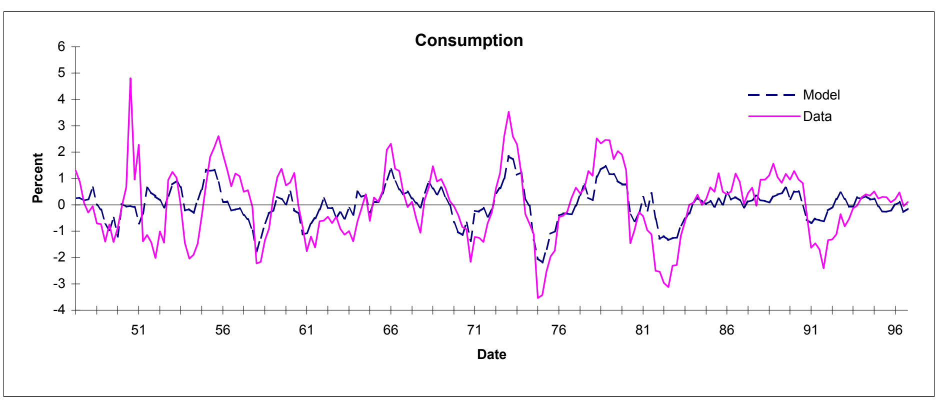
- It accounts for a substantial amount of the observed fluctuations. Accounts for the covariances among a number of variables. Has some problems accounting for hours worked.
- Are fluctuations in TFP really productivity fluctuations?
- Factor utilization rates vary over the business cycle. During recessions, firms reduce the number of shifts. Similarly, firms are reluctant to fire trained workers.
- Neither is well-measured. They show up in the Solow residual.
- There is no direct evidence of technology fluctuations.
- Is intertemporal labor supply really so elastic?
- All employment variation in the model is voluntary, driven by intertemporal substitution.
- Deliberate monetary policy changes appear to have real effects.

Figure 7



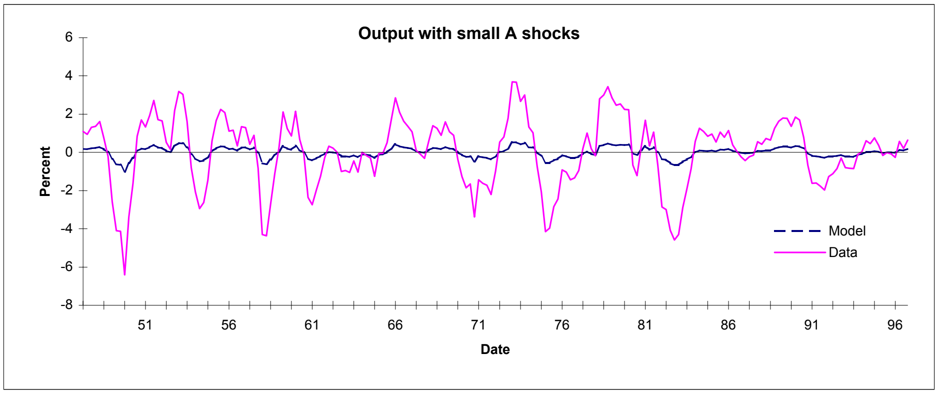


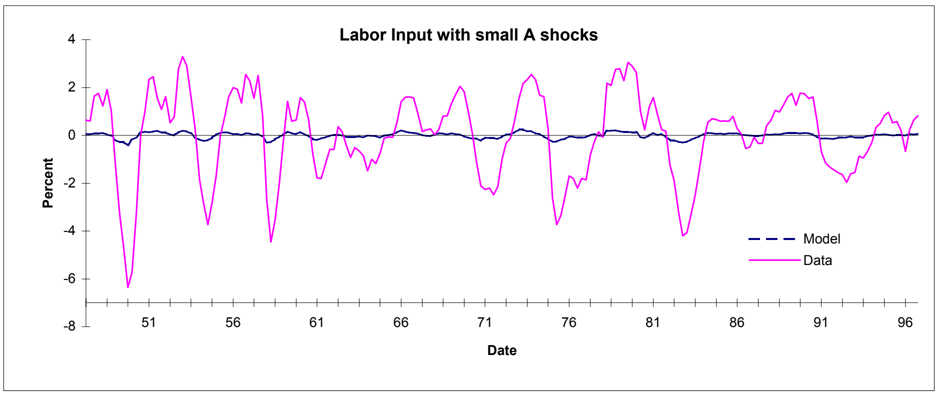


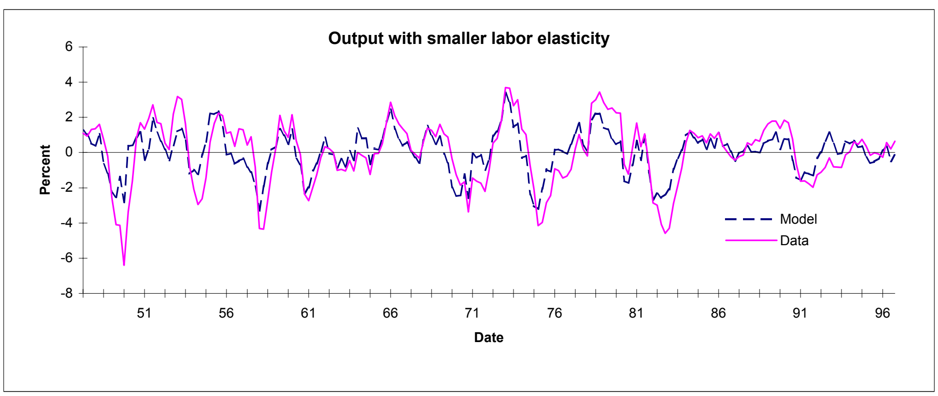


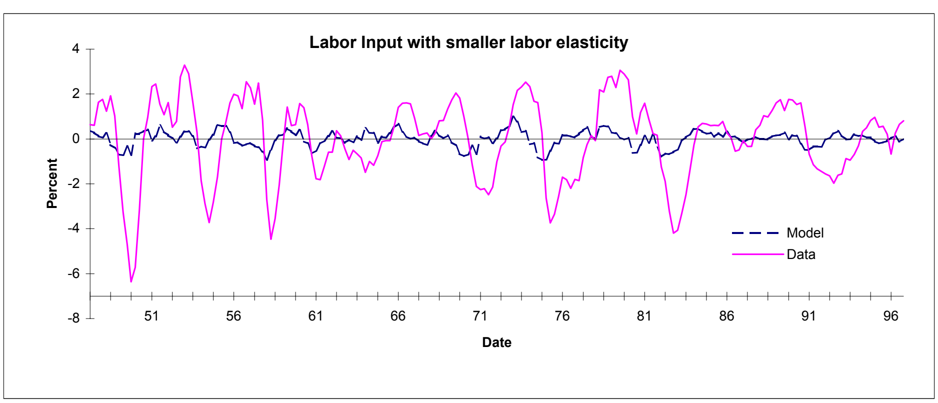
Note: Sample period is 1947:2 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.

Figure 8









Note: Sample period is 1947:2 - 1996:4. All variables are detrended using the Hodrick-Prescott filter.

Putting our Theory to Work I: the Great Depression

- Great Depression is a unique event in US history.
- Timing 1929-1933.
- Major changes in the US Economic policy: New Deal.
- Can we use the theory to think about it?

Data on the Great Depression

Year	u	Y	C	I	G	i	π
1929	3.2	203.6	139.6	40.4	22.0	5.9	–
1930	8.9	183.5	130.4	27.4	24.3	3.6	−2.6
1931	16.3	169.5	126.1	16.8	25.4	2.6	−10.1
1932	24.1	144.2	114.8	4.7	24.2	2.7	−9.3
1933	25.2	141.5	112.8	5.3	23.3	1.7	−2.2
1934	22.0	154.3	118.1	9.4	26.6	1.0	7.4
1935	20.3	169.5	125.5	18.0	27.0	0.8	0.9
1936	17.0	193.2	138.4	24.0	31.8	0.8	0.2
1937	14.3	203.2	143.1	29.9	30.8	0.9	4.2
1938	19.1	192.9	140.2	17.0	33.9	0.8	−1.3
1939	17.2	209.4	148.2	24.7	35.2	0.6	−1.6

Output, Inputs and TFP During the Great Depression

Theory

$$\frac{\dot{z}}{z} = \frac{\dot{Y}}{Y} - \alpha\frac{\dot{K}}{K} - (1-\alpha)\frac{\dot{N}}{N}$$

Data (1929=100)

Year	Y	N	K	z
1930	89.6	92.7	102.5	94.2
1931	80.7	83.7	103.2	91.2
1932	66.9	73.3	101.4	83.4
1933	65.3	73.5	98.4	81.9

Why did TFP fall so much?

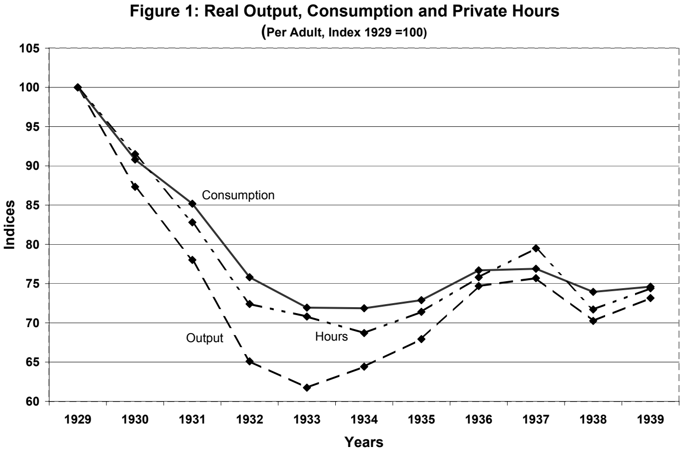
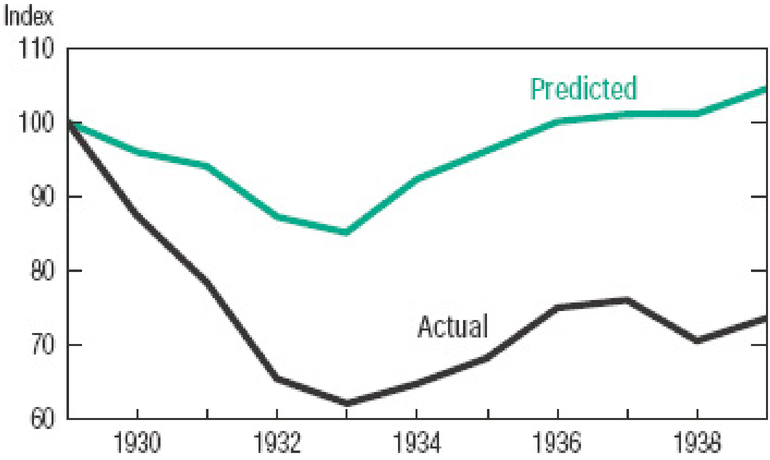


Figure 1: Real Output, Consumption and Private Hours
(Per Adult, Index 1929 =100)
105
100
95
90
85
80
75
70
65
60
Indices
Consumption
Output
Hours
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
Years

Predicted and Actual Output in 1929–39

Detrended Levels, With Initial Capital Stock in the Model Equal to the Actual Capital Stock in 1929



Potential Reasons

- Changes in Capacity Utilization.
- Changes in Quality of Factor Inputs.
- Changes in Composition of Production.
- Labor Hoarding.
- Increasing Returns to Scale.

Other Reasons for Great Depression

- Based on Cole and Ohanian (1999)
- Monetary Shocks: Monetary contraction, change in reserve requirements too late
- Banking Shocks: Banks that failed too small
- Fiscal Shocks: Government spending did rise (moderately)
- Sticky Nominal Wages: Probably more important for recovery

Output and Productivity after the Great Depression

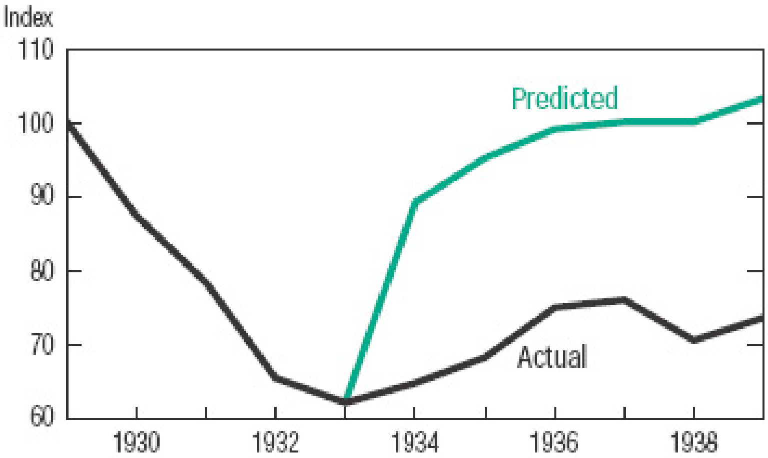
- Cole and Ohanian (2001). Data (1929=100); data are detrended

Year	Y	z
1934	64.4	92.6
1935	67.9	96.6
1936	74.4	99.9
1937	75.7	100.5
1938	70.2	100.3
1939	73.2	103.1

- Fast Recovery of z, slow recovery of output. Why?

Predicted and Actual Recovery of Output in 1934–39

Detrended Levels, With Initial Capital Stock in the Model Equal to the Actual Capital Stock in 1934



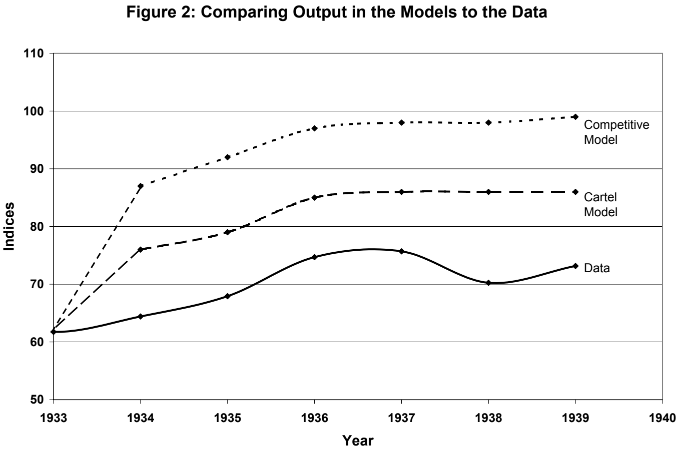


Figure 2: Comparing Output in the Models to the Data
110
100
90
80
70
60
50
Indices
1933
1934
1935
1936
1937
1938
1939
1940
Year
Competitive Model
Cartel Model
Data