Lecture 16
More on Real Business Cycles

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Economics 712

A Quantitative Version

- Let's now suppose that we have an economy that is hit over time by productivity shocks with the same characteristics that the ones that hit the US economy.
- How does this economy behave? In particular, how do the variances and covariances of the main variables in our economy compare with those observed in the US economy?
- Basic real business cycle model due to Kydland and Prescott (1982). One of their two main contributions for which they won the Nobel prize in 2004.

Contribution of Kydland and Prescott

- In addition to its importance as a business cycle model, the Kydland-Prescott paper had a number of other methodological contributions.
  - Part of a then-new literature on rational expectations. Agents within the model understand fully the equilibrium laws of motion.
  - Used computer to solve and simulate the model to derive predictions to a broader extent then before.
  - Focused on calibration of the model rather than formal estimation.

Logic of the Model

- As described earlier, persistent shocks to TFP are the driving source of fluctuations in the model. No other randomness.
- Static effects of change in TFP: Implies higher labor productivity, increasing wages. Substitution effect leads to higher labor supply, thus increasing output.
- Dynamic effects of change in TFP: Part of increased output is consumed, but part is saved. The more persistent the effect, the more saved. Also greater returns to capital so more investment, yielding higher capital stock.
- So for extended period get greater output due to increases in labor and capital inputs as well as direct TFP effect.
- Effects of a single shock eventually die out, but they may be long-lived. However new shocks continually arrive.

Characterizing the Equilibrium

- This economy satisfies the conditions that ensure that both welfare theorems hold. Business cycles in the model are efficient.
- Fluctuations are the optimal response to a changing environment. They are not sufficient for inefficiencies or for government intervention. In this model the government can only worsen the allocation.
- The previous problem does not have a known “paper and pencil” analytic solution.
- Analysis of the model requires some approximations (such as linearization) or numerical analysis.

Solving the Model in a Special Case

- There is one known case where we can work out an explicit solution.
- Set $\delta = 1$ (full depreciation) use Cobb-Douglas production with $z = A^{1-\alpha}$, and log utility:

$$u(C, 1-N) = (1-a)\log C + a\log(1-N).$$

- Specialize the key equilibrium conditions:

$$\begin{aligned}
\frac{aC_t}{(1-a)1-N_t} &= (1-\alpha)\, z_t K_t^{\alpha} N_t^{-\alpha} \\
\frac{1}{C_t} &= \beta E_t \left[\frac{\alpha z_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}}{C_{t+1}} \right] \\
K_{t+1} &= z_t K_t^{\alpha} N_t^{1-\alpha} - C_t
\end{aligned}$$

- Make the following guesses:

$$C_t = (1-s)\,Y_t,\;\; N_t = \bar{N}$$

Constant saving rate s, constant labor supply $\bar{N}$.

- Substitute into conditions:

$$\begin{aligned}
\frac{a(1-s)z_t K_t^{\alpha}\bar{N}^{1-\alpha}}{(1-a)(1-\bar{N})} &= (1-\alpha)\,z_t K_t^{\alpha}\bar{N}^{-\alpha}.\\
\frac{1}{(1-s)z_t K_t^{\alpha}\bar{N}^{1-\alpha}} &= \beta E\left[\frac{\alpha z_{t+1}K_{t+1}^{\alpha-1}\bar{N}^{1-\alpha}}{(1-s)z_{t+1}K_{t+1}^{\alpha}\bar{N}^{1-\alpha}}\right]\\
&= \beta E_t\left[\frac{\alpha}{(1-s)K_{t+1}}\right]\\
&= \beta E_t\left[\frac{\alpha}{(1-s)sz_t K_t^{\alpha}\bar{N}^{1-\alpha}}\right]\\
\Rightarrow s &= \beta\alpha
\end{aligned}$$

Implications

- This special case is then similar to the Solow model: constant savings rate. Constant labor supply (no growth). Difference is random shocks.
- Now $K_{t+1} = sY_t$, so

$$\begin{aligned} Y_{t+1} &= z_{t+1} K_{t+1}^{\alpha} \bar{N}^{1-\alpha} \\ &= z_{t+1} \left(sY_t \right)^{\alpha} \bar{N}^{1-\alpha}. \end{aligned}$$

- Taking logs:

$$\begin{aligned} \log Y_{t+1} &= \mu + \log z_{t+1} + \alpha \log Y_t \\ &= \mu + \rho \log z_t + \alpha \log Y_t + \varepsilon_{t+1}. \end{aligned}$$

where $\mu = \alpha \log s + (1 - \alpha) \log \bar{N}$

Implications: Output Persistence

$$\log Y_{t+1} \quad = \quad \mu + \rho \log z_t + \alpha \log Y_t + \varepsilon_{t+1}.$$

- Output and technology together follow a (vector) $AR(1)$.
- Can simplify further, using:

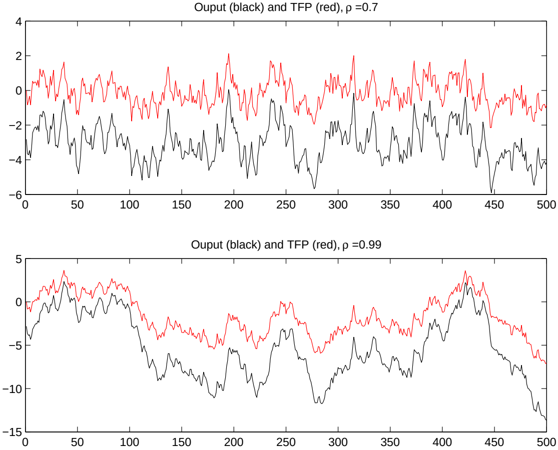
$$\log z_t = \log Y_t - \mu - \alpha \log Y_{t-1}$$

  So then:

$$\log Y_{t+1} = (1-\rho)\mu + (\rho + \alpha) \log Y_t - \alpha\rho \log Y_{t-1} + \varepsilon_{t+1}.$$

- Output follows an AR(2) process.
- Output is persistent because of the TFP shocks and because of capital accumulation.

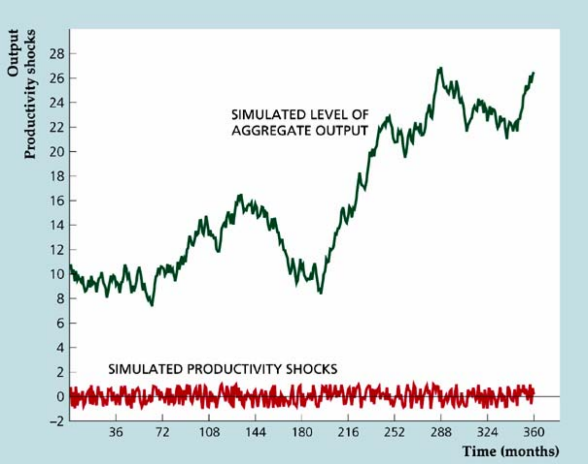
Output and TFP Comovements

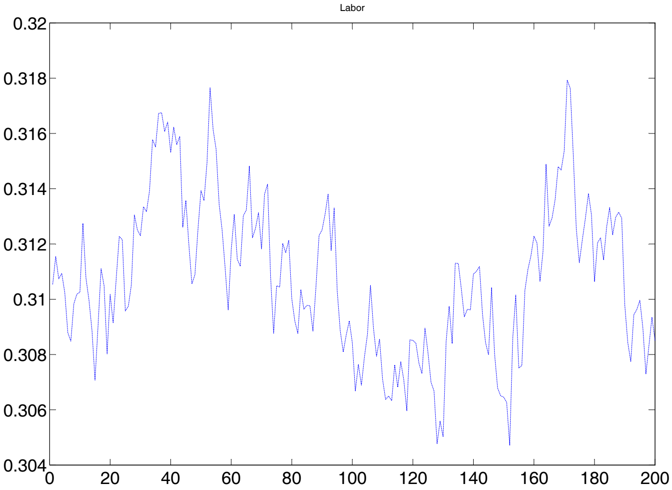


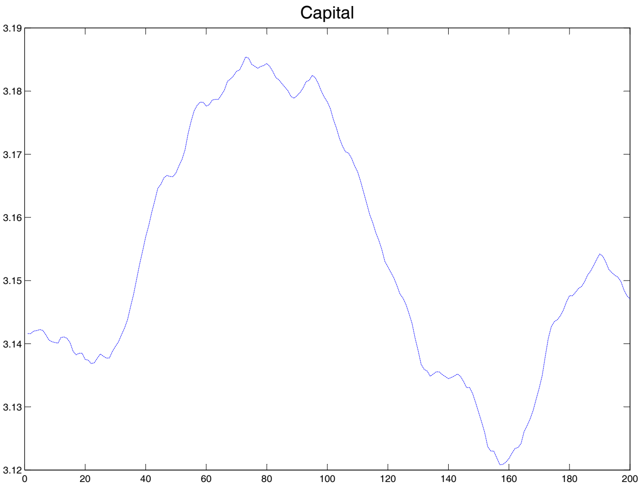
Simulations from a Quantitative Version

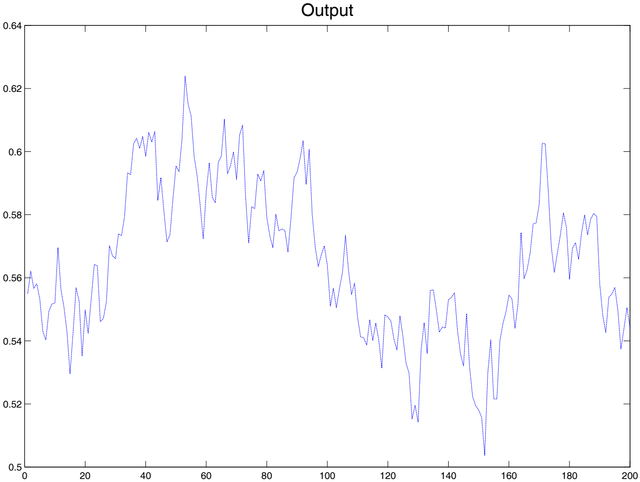
- We have seen the qualitative behavior of the model, showing that the real business cycle model is consistent with the data.
- Apart from the special case we studied, to fully solve the model we need to use numerical methods.
- Calibrate the model: choose parameters to match some key economic data.
  Example: set β so that steady state real interest rate matches US data.
- Program up on computer and simulate: use random number generator to draw technology shocks, feed them through the model.
- Compute correlations and volatilities and compare to US data.

Figure 10.03 Small shocks and large cycles




Labor
0.32
0.318
0.316
0.314
0.312
0.31
0.308
0.306
0.304
0
20
40
60
80
100
120
140
160
180
200


Capital
3.19
3.18
3.17
3.16
3.15
3.14
3.13
3.12
0
20
40
60
80
100
120
140
160
180
200


Output
0.64
0.62
0.6
0.58
0.56
0.54
0.52
0.5
0
20
40
60
80
100
120
140
160
180
200

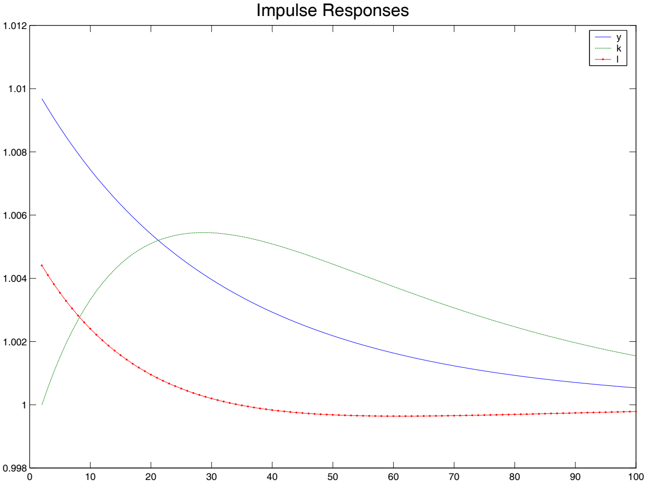

Impulse Responses
1.012
1.01
1.008
1.006
1.004
1.002
1
0.998
0
10
20
30
40
50
60
70
80
90
100
y
k
l

Figure 10.01 Actual versus simulated volatilities of key macroeconomic variables

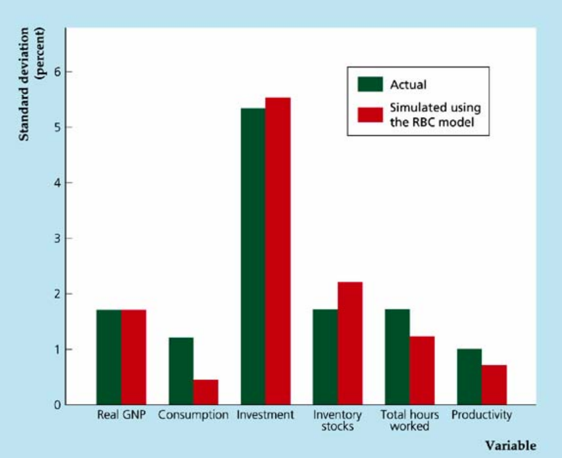
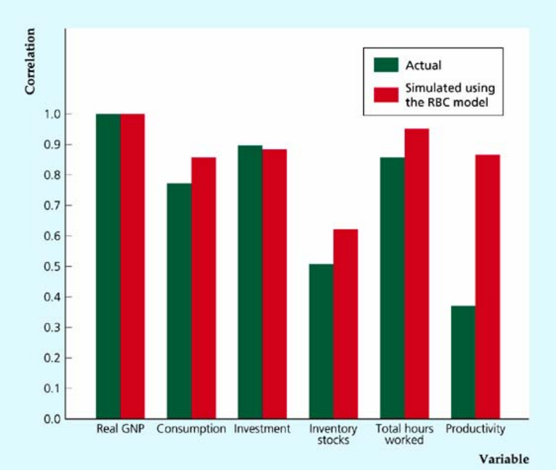


Figure 10.02 Actual versus simulated correlations of key macroeconomic variables with GNP



Assessment of the Basic Real Business Model

- It accounts for a substantial amount of the observed fluctuations. Accounts for the covariances among a number of variables. Has some problems accounting for hours worked.
- Are fluctuations in TFP really productivity fluctuations?
- Factor utilization rates vary over the business cycle. During recessions, firms reduce the number of shifts. Similarly, firms are reluctant to fire trained workers.
- Neither is well-measured. They show up in the Solow residual.
- There is no direct evidence of technology fluctuations.
- Is intertemporal labor supply really so elastic?
- All employment variation in the model is voluntary, driven by intertemporal substitution.
- Deliberate monetary policy changes appear to have real effects.