1 Equity Premium

1.1 Characterization

Define $r^f = R - 1$, $\beta = \frac{1}{1+\delta}$, then (net) stock return $r_{t+1}$ satisfies:

$$1 = E_t \left[ \frac{1}{1+\delta} (1 + \Delta c_{t+1})^{-\gamma} (1 + r_{t+1}) \right]$$

Take 2nd order Taylor approximation of right side, unconditional expectations:

$$E(r) = \delta + \gamma E(\Delta c_t) + \gamma \text{cov}(r_t, \Delta c_t) - \frac{1}{2} \gamma (\gamma + 1) \sigma^2(\Delta c_t)$$

Which can be expressed as:

$$\frac{E(r_t) - r^f}{\sigma(r)} = \gamma \sigma(\Delta c_t) \text{corr}(\Delta c_t, r_t)$$

Left side known as Sharpe ratio

1.2 Attempted Resolutions

- Change preferences: recursive preferences, robustness, habit persistence
- Change constraints: Limited participation, transaction costs, incomplete markets
- Change shocks: disaster models, long-run risk, learning
2 Stochastic Growth/Real Business Cycle Model

Due to Brock-Mirman (1972), Kydland-Prescott (1982)

Add stochastic TFP to neoclassical/optimal growth model

2.1 Basic Model

Household problem:

$$\max_{\{c_t, k_t, n_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$$

subject to:

$$c_t + k_{t+1} - (1 - \delta)k_t = w_t n_t + r_t k_t$$

Firm problem:

$$\max_{K_t, N_t} [F(K_t, A_t N_t) - r_t K_t - w_t N_t]$$

Technology:

$$\log A_{t+1} = \rho \log A_t + \varepsilon_{t+1}$$

where $0 < \rho \leq 1$ and $\varepsilon_t \sim N(0, \sigma^2)$.

Feasibility:

$$C_t + K_{t+1} - (1 - \delta)K_t = F(K_t, A_t N_t)$$
2.2 Recursive Competitive Equilibrium

Note firm problem is static, gives standard optimality conditions:

\[ F_K(K, AN) = r \]
\[ AF_N(K, AN) = w \]

Can eliminate \( N \) and write these as \( w = w(K, A), r = r(K, A) \).

Household: individual capital \( k \), aggregate \( K \). Takes as given pricing functions, law of motion for aggregate \( K \).

Bellman equation:

\[
V(k, K, A) = \max_{c, n, k'} \{ u(c, 1 - n) + \beta E[V(k', K', A)|k, A, K] \}
\]

subject to

\[
c + k' - (1 - \delta)k = w(K, A)n + r(K, A)k \\
\log A' = \rho \log A + \varepsilon' \\
K = G(K, A)
\]

Solution determines household policies \( k' = g(k, K, A), c = c(k, K, A), n = n(k, K, A) \).

**Definition:** A recursive competitive equilibrium is a value function \( V(k, K, A) \) for the household, aggregate decision rules \( C(K, A), N(K, A), K' = G(K, A) \) and price functions \( r(K, A), w(K, A) \) such that

(i) \( V \) solves the household problem, with decision rules \( g, c, n \)

(ii) The price functions \( r \) and \( w \) are consistent with the firm problem
(iii) Individual and aggregate behavior are consistent: 
\[ g(K, K, A) = G(K, A), c(K, K, A) = C(K, A), n(K, K, A) = N(K, A) \]

(iv) Aggregate feasibility is satisfied:
\[ C(K, A) + G(K, A) - (1 - \delta)K = F(K, AN(K, A)) \]

### 2.3 Characterization

\[ V(k, K, A) = \max_{n, k'} \{ u(w(K, A)n + r(K, A)k - k' + (1 - \delta)k, 1 - n) + \beta E[V(k', K', A)|k, A, K] \} \]

First order and envelope conditions:
\[ u_c(c, 1 - n) = \beta E[V_k(k', K', A)|k, A, K] \]
\[ u_c(c, 1 - n)w(K, A) = u_n(c, 1 - n) \]
\[ V_k(k, K, A) = u_c(c, 1 - n)[r(K, A) + 1 - \delta] \]

Combining gives household Euler equation:
\[ u_c(c, 1 - n) = \beta E[u_c(c', 1 - n')][r(K', A') + 1 - \delta]|k, A, K] \]

Impose equilibrium:
\[ u_c(C, 1 - N) = \beta E[u_c(C', 1 - N')[F_K(K', A'N') + 1 - \delta]|k, A, K] \]

Also have intra-temporal optimality condition, in equilibrium:
\[ \frac{u_n(C, 1 - N)}{u_c(C, 1 - N)} = AF_N(K, AN) \]