Lecture 15: Equity Premium

Stochastic Growth Model

Economics 714, Fall 2014

1 Equity Premium

1.1 Characterization

Define $r^f = R - 1$, $\beta = \frac{1}{1+\delta}$, then (net) stock return r_{t+1} satisfies:

$$1 = E_t \left[\frac{1}{1+\delta} (1 + \Delta c_{t+1})^{-\gamma} (1 + r_{t+1}) \right]$$

Take 2nd order Taylor approximation of right side, unconditional expectations:

$$E(r) = \delta + \gamma E(\Delta c_t) + \gamma cov(r_t, \Delta c_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(\Delta c_t)$$

Which can be expressed as:

$$\frac{E(r_t) - r^f}{\sigma(r)} = \gamma \sigma(\Delta c_t) corr(\Delta c_t, r_t)$$

Left side known as Sharpe ratio

1.2 Attempted Resolutions

- Change preferences: recursive preferences, robustness, habit persistence
- Change constraints: Limited participation, transaction costs, incomplete markets
- Change shocks: disaster models, long-run risk, learning

2 Stochastic Growth/Real Business Cycle Model

Due to Brock-Mirman (1972), Kydland-Prescott (1982)

Add stochastic TFP to neoclassical/optimal growth model

2.1 Basic Model

Household problem:

$$\max_{\{c_t,k_t,n_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$

subject to:

$$c_t + k_{t+1} - (1-\delta)k_t = w_t n_t + r_t k_t$$

Firm problem:

$$\max_{K_t,N_t} [F(K_t, A_t N_t) - r_t K_t - w_t N_t]$$

Technology:

$$\log A_{t+1} = \rho \log A_t + \varepsilon_{t+1}$$

where $0 < \rho \leq 1$ and $\varepsilon_t \sim N(0, \sigma^2)$.

Feasibility:

$$C_t + K_{t+1} - (1-\delta)K_t = F(K_t, A_t N_t)$$

2.2 Recursive Competitive Equilibrium

Note firm problem is static, gives standard optimality conditions:

$$\begin{aligned} F_K(K, AN) &= r \\ AF_N(K, AN) &= w \end{aligned}$$

Can eliminate N and write these as $w = w(K, A)$, $r = r(K, A)$.

Household: individual capital k, aggregate K. Takes as given pricing functions, law of motion for aggregate K.

Bellman equation:

$$V(k, K, A) = \max_{c,n,k'} \{u(c, 1-n) + \beta E[V(k', K', A)|k, A, K]\}$$

subject to

$$\begin{aligned} c + k' - (1-\delta)k &= w(K, A)n + r(K, A)k \\ \log A' &= \rho \log A + \varepsilon' \\ K &= G(K, A) \end{aligned}$$

Solution determines household policies $k' = g(k, K, A)$, $c = c(k, K, A)$, $n = n(k, K, A)$.

Definition: A **recursive competitive equilibrium** is a value function $V(k, K, A)$ for the household, aggregate decision rules $C(K, A), N(K, A), K' = G(K, A)$ and price functions $r(K, A)$, $w(K, A)$ such that

(i) V solves the household problem, with decision rules g, c, n

(ii) The price functions r and w are consistent with the firm problem

(iii) individual and aggregate behavior are consistent: $g(K, K, A) = G(K, A)$, $c(K, K, A) = C(K, A)$, $n(K, K, A) = N(K, A)$

(iv) Aggregate feasibility is satisfied:

$$C(K, A) + G(K, A) - (1 - \delta)K = F(K, AN(K, A))$$

2.3 Characterization

$$V(k, K, A) = \max_{n,k'} \{u(w(K, A)n + r(K, A)k - k' + (1 - \delta)k, 1 - n) + \beta E[V(k', K', A)|k, A, K]\}$$

First order and envelope conditions:

$$\begin{aligned} u_c(c, 1 - n) &= \beta E[V_k(k', K', A)|k, A, K] \\ u_c(c, 1 - n)w(K, A) &= u_n(c, 1 - n) \\ V_k(k, K, A) &= u_c(c, 1 - n)[r(K, A) + 1 - \delta] \end{aligned}$$

Combining gives household Euler equation:

$$u_c(c, 1 - n) = \beta E[u_c(c', 1 - n')[r(K', A') + 1 - \delta]|k, A, K]$$

Impose equilibrium:

$$u_c(C, 1 - N) = \beta E\left[u_c(C', 1 - N')[F_K(K', A'N') + 1 - \delta]|k, A, K\right]$$

Also have intra-temporal optimality condition, in equilibrium:

$$\frac{u_n(C, 1 - N)}{u_c(C, 1 - N)} = AF_N(K, AN)$$